Preferential Semantics for Plausible Subsumption in Possibility Theory

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Abstract

Handling exceptions in a knowledge-based system has been considered as an important issue in many domains of applications, such as medical domain. In this paper, we propose several preferential semantics for plausible subsumption to deal with exceptions in description logic-based knowledge bases. Our preferential semantics are defined in the framework of possibility theory, which is an uncertainty theory devoted to the handling of incomplete information. We consider the properties of these semantics and their relationships. Entailment of these plausible subsumption relative to a knowledge base is also considered. We show the close relationship between two of our semantics and the mutually dual preferential semantics given by Britz, Heidema and Meyer. Finally, we show that our semantics for plausible subsumption can be reduced to standard semantics of an expressive description logic. Thus, the problem of plausible subsumption checking under our semantics can be reduced to the problem of subsumption checking under the classical semantics.

Introduction

Handling exceptions in a knowledge-based system has been considered as an important issue in many domains of applications, such as medical domain. Many nonmonotonic logics have been proposed to deal with this issue, among them are preferential logics (Kraus, Lehmann, and Magidor 1990). However, most of the preferential logics are based on propositional language, thus are not suitable to handle exceptions in applications where first-order logic is needed. As decidable fragments of first-order logic, description logics have received much attention as they are widely used in some applications, such as the Semantic Web and software engineering (see (Baader et al. 2007)). There have been some work on extending description logics with preferential semantics (Giordano et al. 2007; Britz, Heidema, and Meyer 2008). The work presented in (Britz, Heidema, and Meyer 2008) proposes two preferential subsumption relations in description logics and discusses their properties by adapting familiar properties of rational preferential entailment.

Possibility theory, which is developed from Zadeh's fuzzy set theory (Zadeh 1965), provides a powerful tool for performing nonmonotonic reasoning (Dubois, Lang, and Prade 1994; Benferhat, Dubois, and Prade 1997; 1998). It has been shown in (Benferhat, Dubois, and Prade 1998) that default rules (i.e., generic rules of the form "generally, if α then β", where α and β are propositional symbols) and hard rules (i.e., rules of the form "if α is observed, then β is always true") can be modelled in possibility theory. The authors show that the possibilistic inference based on default rules is equivalent to System P given in (Kraus, Lehmann, and Magidor 1990) subject to some minor difference. Considering the important role of possibility theory in nonmonotonic reasoning, one may wonder if we can apply it to deal with nonmonotonic reasoning in description logics. However, this is not a trivial problem because description logics, as fragments of first-order logic, have their own features. For example, in description logics, conjunction between two axioms in a knowledge base is not allowed. However, to interpret a propositional default rule or a propositional hard rule in possibility theory, we need to conjunct the propositional symbols.

In this paper, we propose several preferential semantics for plausible subsumption to handle exceptions in description logic-based knowledge bases. We extend an interpretation in description logics with a possibility distribution over its domain. With this extended interpretation, we define our preferential semantics. We first define a preferential subsumption which is inspired from the default rules interpreted in possibility theory in (Benferhat, Dubois, and Prade 1998). We call it a default preferential subsumption. We then define another preferential subsumption which is dual to the default preferential subsumption. We show that the default preferential subsumption and the dual default preferential subsumption are closely related to the preferential subsumption and the dual preferential subsumption given in (Britz, Heidema, and Meyer 2008) respectively. We further define a preferential subsumption which is the combination of both default preferential subsumption and dual default preferential subsumption. This preferential subsumption shows a strong causal relationship between two concepts. Finally, we define a preferential subsumption which is inspired from the hard rules interpreted in possibility theory in (Benferhat, Dubois, and Prade 1998). The relationships among different preferential subsumption relations are discussed. Entailment of these preferential subsumption relative to a knowledge base is then considered. Finally, we show that our semantics for plausible subsumption can be reduced to standard semantics of an expressive description logic. Thus, the problem of plausible subsumption checking under our semantics can be reduced to the problem of subsumption checking under the classical semantics.

The paper is organized as follows. We first provide some preliminaries on description logics and possibility theory, then present our preferential semantics for plausible subsumption and investigate their properties. After that, we consider the reduction of our semantics for plausible subsumption to standard semantics of description logics. We discuss related work before concluding this work.

Preliminaries

In this section, we introduce some background knowledge about description logics and possibility theory.

Description logics

We assume that the reader is familiar with Description Logics (DLs) and refer to Chapter 2 of the DL handbook (Baader et al. 2007) for a good introduction. Our method is independent of a specific DL language, and thus can be applied to any DL.

In DLs, elementary descriptions are *concept names* (unary predicates) and *role names* (binary predicates). Complex descriptions are built from them inductively using concept and role constructors provided by the particular DL under consideration. A DL-based knowledge base (or ontology) $O = (\mathcal{T}, \mathcal{R}, \mathcal{A})$ consists of a set $\mathcal{T}$ of concept axioms (TBox), a set $\mathcal{R}$ of role axioms (RBox), and a set $\mathcal{A}$ of assertional axioms (ABox). Concept axioms (or terminology axioms) have the form $C \sqsubseteq D$ where C and D are (possibly complex) concept descriptions, and role axioms are expressions of the form $R \sqsubseteq S$, where R and S are (possibly complex) role descriptions. The ABox contains *concept assertions* of the form $C(a)$ where C is a concept and a is an individual name, and *role assertions* of the form $R(a, b)$, where R is a role and a and b are individual names. The TBox and RBox are used to express the *intensional level of the ontology* while the ABox is used to express the *instance level of the ontology*.

An interpretation $\mathcal{I} = (\triangle^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of a non-empty domain set $\triangle^{\mathcal{I}}$ and an interpretation function $\cdot^{\mathcal{I}}$, which maps from individuals, concepts and roles to elements of the domain, subsets of the domain and binary relations on the domain, respectively. Given an interpretation $\mathcal{I}$, we say that $\mathcal{I}$ satisfies a concept axiom $C \sqsubseteq D$ (resp., a role axiom $R \sqsubseteq S$) if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ (resp., $R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$). Furthermore, $\mathcal{I}$ satisfies a concept assertion $C(a)$ (resp., a role assertion $R(a, b)$) if $a^{\mathcal{I}} \in C^{\mathcal{I}}$ (resp., $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$). An interpretation $\mathcal{I}$ is called a *model* of an ontology O, iff it satisfies each axiom in O.

In our work, we are interested in an extension of DL $\mathcal{ALC}$ (Schmidt-Schauß and Smolka 1991), which is a simple yet relatively expressive DL. The set of $\mathcal{ALC}$ concepts is the smallest set such that: (1) every concept name is a concept; (2) if C and D are concepts, R is a role name, then the following expressions are also concepts: $\neg C$ (full negation), $C \sqcap D$ (concept conjunction), $C \sqcup D$ (concept disjunction), $\forall R.C$ (value restriction on role names) and $\exists R.C$ (existential restriction on role names). The semantics of the constructors are defined as follows:

(1) $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$ and $\bot^{\mathcal{I}} = \varnothing$, $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$,
(2) $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$, $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$,
(3) $(\exists R.C)^{\mathcal{I}} = \{x | \exists\, y\ s.t.(x, y) \in R^{\mathcal{I}}$ and $y \in C^{\mathcal{I}}\}$,
(4) $(\forall R.C)^{\mathcal{I}} = \{x | \forall y (x, y) \in R^{\mathcal{I}}$ implies $y \in C^{\mathcal{I}}\}$.

Possibility theory

Possibility theory (Dubois and Prade 1986), which is developed from Zadeh's fuzzy set theory (Zadeh 1965), is based on the notion of a *possibility distribution*. A possibility distribution π is a mapping from the set (finite or infinite) Ω to a bounded, total ordered scale $(L, <)$, which is equipped with the maximum and the minimum operations and an order-reversing map that is taken as 1-$(\cdot)$. We follow the qualitative view of possibility theory, that is $(L, <)$ is merely taken as an ordinal scale. For example, we can take the unit interval [0,1] or its subsets as the range of possibility distributions. According to (Dubois and Prade 1998), the referential set Ω represents the universe of discourse and a possibility distribution represents a state of *knowledge* that distinguishes what is plausible from what is less plausible, what is the normal course of thing from what is not, what is surprising from what is expected. The inequality $\pi(\omega) \geq \pi(\omega')$ means that ω is a priori more plausible than ω'. A possibility distribution is said to be *normal* if $\exists \omega_0 \in \Omega$, such that $\pi(\omega_0) = 1$.

From a *possibility distribution* π, two measures can be determined: the possibility degree of subset A of ω, $\Pi(A) = sup\{\pi(\omega) : \omega \in A\}$ and the necessity degree of A, $N(A) = 1 - \Pi(\overline{A}) = inf\{1 - \pi(\omega) : \omega \notin A\}$, where $\overline{A}$ is the complement of A in Ω. The possibility measure Π satisfies the following properties: (1) $\forall\ A,\ B,\ \ \Pi(A \cup B) = max(\Pi(A), \Pi(B))$ (2) $\Pi(\varnothing) = 0$. Dually, the necessity measure N satisfies the following properties: (1) $\forall\ A,\ B,\ \ N(A \cap B) = min(N(A), N(B))$ (2) $N(\Omega) = 1$.

Preferential Semantics for Subsumption in Possibility Theory

In this section, we first define some preferential subsumption relations in possibility theory. We then consider entailment of these plausible subsumption relations relative to a knowledge base. Finally, we provide examples to illustrate some key notions.

Preferential semantics

In (Giordano et al. 2007) and (Britz, Heidema, and Meyer 2008), to define their preferential semantics of extended description logics, the authors extend a DL interpretation by attaching a partial order on its domain. This inspires us to define a *possibilistic interpretation*, which is a pair consisting of a DL interpretation and a possibility distribution over its domain. Following the convention of preferential logic (see the smoothness condition in (Kraus, Lehmann, and Magidor 1990)), we consider only those possibility distributions which are Noetherian, i.e., for any subset Ω' of Ω, there exists a $\omega \in \Omega'$ such that $\pi(\omega_i) \leq \pi(\omega)$ for all $\omega_i \in \Omega'$.

Definition 1. A possibilistic interpretation $(\mathcal{I}, \pi)$ consists of an interpretation $\mathcal{I} = (\triangle^{\mathcal{I}}, \cdot^{\mathcal{I}})$ and a possibility distribution π over its domain $\triangle^{\mathcal{I}}$.

Consider Definition 1, the requirement that a possibility distribution should be Neotherian ensures that the possibility measure Π defined on $\triangle^{\mathcal{I}}$ is well-defined, i.e., $\Pi(C^{\mathcal{I}})$ always exists for any concept C. Following the view of a possibility distribution in possibility theory, the possibility distribution of a possibilistic interpretation $(\mathcal{I}, \pi)$ gives a ranking on the domain $\triangle^{\mathcal{I}}$ of DL interpretation $\mathcal{I}$. We do not specify how to obtain such a possibility distribution on the domain of a DL interpretation because this is often application dependent. For example, in the medical domain, given a DL interpretation with finite domain, a possibility distribution on its domain can be obtained by counting the number of occurrences of each element of the domain in the available clinical databases (Britz, Heidema, and Meyer 2008).

The satisfaction of classical subsumption relation $\sqsubseteq$ relative to a possibilistic interpretation is defined as follows:

Definition 2. A possibilistic interpretation $(\mathcal{I}, \pi)$ satisfies $C \sqsubseteq D$, written as $(\mathcal{I}, \pi) \Vdash C \sqsubseteq D$, iff $\mathcal{I}$ satisfies $C \sqsubseteq D$.

We are now ready to define our first preferential subsumption, which is inspired from the default rules interpreted in possibility theory in (Benferhat, Dubois, and Prade 1998).

Definition 3. A possibilistic interpretation $(\mathcal{I}, \pi)$ satisfies the default preferential subsumption $C \sqsubseteq_D D$, written as $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$, iff $\Pi(C^{\mathcal{I}} \cap D^{\mathcal{I}}) > \Pi(C^{\mathcal{I}} \cap \neg D^{\mathcal{I}})$ is satisfied.

Definition 3 implies that $\Pi(C^{\mathcal{I}} \cap D^{\mathcal{I}}) > 0$ if $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$. In Definition 3, if we consider objects in the interpretation of a concept with highest possibility degree as its *kernel*[1], then the default subsumption $C \sqsubseteq_D D$ expresses that the *kernel* of C is in D (see Theorem 12). Definition 3 defined the satisfaction of the default preferential subsumption for a possibilistic interpretation. It will be used to define preferential subsumption entailment between a knowledge base and a preferential subsumption.

Dually, we define another preferential subsumption which expresses that the *kernel* of $\neg D$ is in $\neg C$.

Definition 4. A possibilistic interpretation $(\mathcal{I}, \pi)$ satisfies dual default preferential subsumption $C \sqsubseteq_{D^2} D$, written as $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$, iff $\Pi(\neg C^{\mathcal{I}} \cap \neg D^{\mathcal{I}}) > \Pi(C^{\mathcal{I}} \cap \neg D^{\mathcal{I}})$ is satisfied.

Definition 3 implies that $\Pi(\neg C^{\mathcal{I}} \cap \neg D^{\mathcal{I}}) > 0$ if $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$. According to Definition 3, we can see that our dual default preferential subsumption corresponds to the contraposition of the dual preferential subsumption.

Each of the mutually dual preferential subsumption relations $\sqsubseteq_D$ and $\sqsubseteq_{D^2}$ has its own feature (see next subsection). By combining these two subsumption relations, we can get a new preferential subsumption which shows stronger causal relationship between two concepts.

Definition 5. A possibilistic interpretation $(\mathcal{I}, \pi)$ satisfies the preferential subsumption $C \sqsubseteq_B D$ (combining Both $\sqsubseteq_D$ and $\sqsubseteq_{D^2}$), written as $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_B D$, iff $\Pi(C^{\mathcal{I}} \cap D^{\mathcal{I}}) > \Pi(C^{\mathcal{I}} \cap \neg D^{\mathcal{I}})$ and $\Pi(\neg C^{\mathcal{I}} \cap \neg D^{\mathcal{I}}) > \Pi(C^{\mathcal{I}} \cap \neg D^{\mathcal{I}})$ are satisfied.

One may think that our preferential subsumption relation $\sqsubseteq_B$ looks like a kind of default equivalence, thus it is expected to be symmetrical in the sense that, if $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_B D$ then $(\mathcal{I}, \pi) \Vdash D \sqsubseteq_B C$. However, this is not the case as can be seen from the following example.

Example 1 Let $(\mathcal{I}, \pi)$ be a possibilistic interpretation with $\mathcal{I} = (\triangle^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\triangle^{\mathcal{I}} = \{a, b, c, d\}$, $C^{\mathcal{I}} = \{a, b\}$ and $D^{\mathcal{I}} = \{b, c\}$, $\pi(a) = 0.3$, $\pi(b) = 0.5$, $\pi(c) = 0.9$ and $\pi(d) = 0.9$. Since $C^{\mathcal{I}} \cap D^{\mathcal{I}} = \{b\}$ and $C^{\mathcal{I}} \cap \neg D^{\mathcal{I}} = \{a\}$, we have $\Pi(C^{\mathcal{I}} \cap D^{\mathcal{I}}) > \Pi(C^{\mathcal{I}} \cap \neg D^{\mathcal{I}})$. So $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$. Similarly, we can check that $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$. Therefore, $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_B D$. Since $D^{\mathcal{I}} \cap \neg C^{\mathcal{I}} = \{c\}$, we do not have $\Pi(D^{\mathcal{I}} \cap C^{\mathcal{I}}) > \Pi(D^{\mathcal{I}} \cap \neg C^{\mathcal{I}})$. So $(\mathcal{I}, \pi) \Vdash D \sqsubseteq_D C$ does not hold. Thus $(\mathcal{I}, \pi) \Vdash D \sqsubseteq_B C$ does not hold as well.

Our final preferential subsumption, defined below, is inspired from the hard rules interpreted in possibility theory in (Benferhat, Dubois, and Prade 1998).

Definition 6. A possibilistic interpretation $(\mathcal{I}, \pi)$ satisfies the hard preferential subsumption $C \sqsubseteq_H D$, written as $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_H D$, iff $\Pi(C^{\mathcal{I}} \cap \neg D^{\mathcal{I}}) = 0$.

In Definition 6, if we take the objects with non-zero possibility degree as its *support*[2], then the hard preferential subsumption between concepts C and D expresses that the *support* of C is in D.

Theorems 7 and 8 show relationships among different preferential subsumption relations.

Theorem 7 *Given a possibilistic interpretation $(\mathcal{I}, \pi)$, for any concept C and D we have*

- $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$ *iff* $(\mathcal{I}, \pi) \Vdash \neg D \sqsubseteq_D \neg C$.
- $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_B D$ *iff* $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$ *and* $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$.
- *If* $\forall x \in \triangle^{\mathcal{I}}$, *we have* $\pi(x) = 1$, *then* $(\mathcal{I}, \pi) \Vdash C \sqsubseteq D$ *iff* $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$ *iff* $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$ *iff* $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_B D$ *iff* $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_H D$.

In Theorem 7, the first item shows the duality of $\sqsubseteq_D$ and $\sqsubseteq_{D^2}$ and the last item shows that if $\pi(x) = 1$ for all $x \in \triangle^{\mathcal{I}}$, then all the preferential subsumption relations are reduced to the classical subsumption relation.

Theorem 8 *Given a possibilistic interpretation $(\mathcal{I}, \pi)$, and any concepts C and D. If $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_H D$ then*

- $\Pi(C^{\mathcal{I}}) > 0$ *implies* $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$.
- $\Pi(\neg D^{\mathcal{I}}) > 0$ *implies* $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$.

[1]This term is borrowed from fuzzy set theory, where the *kernel of a fuzzy set is the set of the elements with membership degree 1.*

[2]This term is borrowed from fuzzy set theory, where the *support of a fuzzy set is the set of the elements with non-zero membership degrees.*

In the following, we investigate the properties of our preferential subsumption relations. We adapt properties for preferential subsumption relations given in (Britz, Heidema, and Meyer 2008) (we need to replace $(\mathcal{I}, \preceq)$ by $(\mathcal{I}, \pi)$). We summarize some of the results in Table 1.

Table 1: Properties of subsumption relations.

Properties \ Subsumption	$\sqsubseteq_D$	$\sqsubseteq_{D^2}$	$\sqsubseteq_B$	$\sqsubseteq_H$
And	$\surd$	$\surd$	$\surd$	$\surd$
Or	$\surd$	$\surd$	$\surd$	$\surd$
Left logical equivalence	$\surd$	$\surd$	$\surd$	$\surd$
Right logical equivalence	$\surd$	$\surd$	$\surd$	$\surd$
Left defeasible equivalence	$\surd$			$\surd$
Right defeasible equivalence		$\surd$		$\surd$
Right weakening	$\surd$			$\surd$
Rational right weakening	$\surd$	$\surd$	$\surd$	$\surd$
Cautious right weakening	$\surd$	$\surd$	$\surd$	$\surd$
Cautious monotonicity	$\surd$	$\surd$	$\surd$	$\surd$
Rational monotonicity	$\surd$	$\surd$	$\surd$	$\surd$
Cut	$\surd$			$\surd$

It is easy to check that all of our preferential subsumption relations are defeasible in the sense of the following term:

- Defeasibility: $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_X D$ does not necessarily imply $(\mathcal{I}, \pi) \Vdash C \sqsubseteq D$,

where $X \in \{D, D^2, B, H\}$.

Furthermore, it is easy to check that $\sqsubseteq_H$ is supraclassical, but other three preferential subsumption relations are restricted supraclassical in the following sense:

- if $(\mathcal{I}, \pi) \Vdash C \sqsubseteq D$ then $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_H D$.
- if $(\mathcal{I}, \pi) \Vdash C \sqsubseteq D$ and $\Pi(C^{\mathcal{I}}) > 0$, then $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$.
- if $(\mathcal{I}, \pi) \Vdash C \sqsubseteq D$ and $\Pi(\neg D^{\mathcal{I}}) > 0$, then $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$.
- if $(\mathcal{I}, \pi) \Vdash C \sqsubseteq D$ and $\Pi(C^{\mathcal{I}}) > 0$ and $\Pi(\neg D^{\mathcal{I}}) > 0$, then $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_B D$.

We can also check that the preferential subsumption relations $\sqsubseteq_{D^2}$ and $\sqsubseteq_H$ are monotonic:

- $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$ implies $(\mathcal{I}, \pi) \Vdash C \sqcap C' \sqsubseteq_{D^2} D$.
- $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_H D$ implies $(\mathcal{I}, \pi) \Vdash C \sqcap C' \sqsubseteq_H D$.

Whilst other two preferential subsumption relations are not monotonic in the sense of the following terms:

- $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$ does not necessarily imply $(\mathcal{I}, \pi) \Vdash C \sqcap C' \sqsubseteq_D D$.
- $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_B D$ does not necessarily imply $(\mathcal{I}, \pi) \Vdash C \sqcap C' \sqsubseteq_B D$.

All of our preferential subsumption relations except the hard preferential subsumption relation satisfy *restricted reflexivity (RR)*, which is adapted from a property for possibilistic consequence relations given in (Benferhat, Dubois, and Prade 1997).

- if $\Pi(C^{\mathcal{I}}) > 0$ then $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D C$.
- if $\Pi(\neg C^{\mathcal{I}}) > 0$ then $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} C$.
- if $\Pi(C^{\mathcal{I}}) > 0$ then $\Pi(\neg C^{\mathcal{I}}) > 0$, then$(\mathcal{I}, \pi) \Vdash C \sqsubseteq_B C$.

The hard preferential subsumption relation is *reflexive*, that is, for any concept C and possibility interpretation $(\mathcal{I}, \pi)$, we always have $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_H C$.

$\sqsubseteq_D$ and $\sqsubseteq_B$ satisfy *consistency preservation*, which is adapted from another property for possibilistic consequence relations in (Benferhat, Dubois, and Prade 1997):

- Consistency Preservation(CP): for any concept C, we have $(\mathcal{I}, \pi) \nVdash C \sqsubseteq_X \varnothing$, where $X \in \{D, B\}$.

The final property we will consider is called "Nihil ex absurdo", which is dual to consistency preservation:

- Nihil ex absurdo: for any concept C, we have $(\mathcal{I}, \pi) \nVdash \varnothing \sqsubseteq_X C$, where $X \in \{D, B\}$.

Both $\sqsubseteq_D$ and $\sqsubseteq_B$ satisfy this property.

Other Properties

In this subsection, we discuss the relationship between our preferential subsumption relations and the mutually dual preferential subsumption relations given in (Britz, Heidema, and Meyer 2008).

We redefine the mutually dual preferential subsumption relations given in (Britz, Heidema, and Meyer 2008) by an ordered interpretation $(\mathcal{I}, \leq)$, where $\leq$ is a Noetherian total pre-order over $\triangle^{\mathcal{I}}$. In (Britz, Heidema, and Meyer 2008), the authors take $\leq$ as a Noetherian modular partial order. Since there is a bijection between modular partial orders and total preorders on $\triangle^{\mathcal{I}}$, the order relation used in the following definitions can be a Noetherian modular partial order as well.

Definition 9. An ordered interpretation $(\mathcal{I}, \leq)$ satisfies the preferential subsumption $C \sqsubseteq_{ps} D$, written as $(\mathcal{I}, \leq) \Vdash C \sqsubseteq_{ps} D$ iff $C^{\mathcal{I}-} \subseteq D^{\mathcal{I}}$, where $C^{\mathcal{I}-} = \{x \in C^{\mathcal{I}} |$ for no $y \in C^{\mathcal{I}}$ is $x \leq y$ but $y \nleq x\}$.

Definition 10. An ordered interpretation $(\mathcal{I}, \leq)$ satisfies the dual preferential subsumption $C \sqsubseteq^*_{ps} D$, written as $(\mathcal{I}, \leq_\pi) \Vdash C \sqsubseteq^*_{ps} D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}+}$, where $D^{\mathcal{I}+} = D^{\mathcal{I}} \cup \{x \in \triangle^{\mathcal{I}} | \exists y \notin D^{\mathcal{I}}$ with $x \leq y$ but $y \nleq x\}$.

We provide a novel result showing that these two preferential subsumption relations can be defined by each other.

Theorem 11 *Given an ordered interpretation $(\mathcal{I}, \leq)$, for any two concepts C and D, we have*

$$(\mathcal{I}, \leq) \Vdash C \sqsubseteq_{ps*} D \text{ iff } (\mathcal{I}, \leq) \Vdash \neg D \sqsubseteq_{ps} \neg C.$$

Proof. We use $\Leftrightarrow$ to denote "if and only if". For any ordered interpretation $(\mathcal{I}, \leq)$, we have

$$\begin{aligned}
&(\mathcal{I}, \leq) \Vdash C \sqsubseteq_{ps*} D \\
\Leftrightarrow & C^{\mathcal{I}} \subseteq D^{\mathcal{I}} \cup \{x \in \triangle^{\mathcal{I}} | \exists y \notin D^{\mathcal{I}}, x \leq y\ , y \nleq x\} \\
\Leftrightarrow & C^{\mathcal{I}} \subseteq D^{\mathcal{I}} \cup \{x \in \triangle^{\mathcal{I}} - D^{\mathcal{I}} | \exists y \in \neg D^{\mathcal{I}}, x \leq y, y \nleq x\} \\
\Leftrightarrow & C^{\mathcal{I}} \subseteq D^{\mathcal{I}} \cup \{x \in \neg D^{\mathcal{I}} | \exists y \in \neg D^{\mathcal{I}}, x \leq y, y \nleq x\} \\
\Leftrightarrow & C^{\mathcal{I}} \subseteq D^{\mathcal{I}} \cup (\neg D^{\mathcal{I}} - (\neg D)^{\mathcal{I}-}) \\
\Leftrightarrow & C^{\mathcal{I}} \subseteq D^{\mathcal{I}} \cup (\neg D^{\mathcal{I}} \cap \neg(\neg D)^{\mathcal{I}-}) \\
\Leftrightarrow & C^{\mathcal{I}} \subseteq D^{\mathcal{I}} \text{ or } C^{\mathcal{I}} - D^{\mathcal{I}} \subseteq \neg(\neg D)^{\mathcal{I}-})
\end{aligned}$$

$\Leftrightarrow C^{\mathcal{I}} - D^{\mathcal{I}} \subseteq \neg(\neg D)^{\mathcal{I}-})$
$\Leftrightarrow(\neg D)^{\mathcal{I}-} \subseteq \neg(C^{\mathcal{I}} - D^{\mathcal{I}})$
$\Leftrightarrow(\neg D)^{\mathcal{I}-} \subseteq \neg C^{\mathcal{I}} \cup D^{\mathcal{I}}$
$\Leftrightarrow(\neg D)^{\mathcal{I}-} \subseteq \neg C^{\mathcal{I}}$(note $(\neg D)^{\mathcal{I}-} \subseteq D^{\mathcal{I}}$ always hold)
$\Leftrightarrow(\mathcal{I}, \leq) \Vdash \neg D \sqsubseteq_{ps} \neg C$ □

Let $\leq_\pi$ be an order relation on $\triangle^{\mathcal{I}}$, satisfying $\forall x, y \in \triangle^{\mathcal{I}}$, $x \leq_\pi y$ iff $\pi(x) \leq \pi(y)$. Then, $\leq_\pi$ is a Noetherian total order as we have assumed that π is Noetherian. We say that a set $C^{\mathcal{I}}$ is minimized relative to $\leq_\pi$ if all objects in $C^{\mathcal{I}}$ are minimal relative to $\leq_\pi$, i.e., for all $a \in C^{\mathcal{I}}$, there does not exist $b \in \triangle^{\mathcal{I}}$ such that $b \leq_\pi a$ but $a \not\leq_\pi b$, this is equivalent to say that $\Pi(C^{\mathcal{I}}) = 0$. The following theorem shows that our preferential subsumption relations $\sqsubseteq_D$ and $\sqsubseteq_{D^2}$ are equivalent to the preferential subsumption relations $\sqsubseteq_{ps}$ and $\sqsubseteq_{ps}^*$ respectively subject to some condition.

Theorem 12 *Given a possibilistic interpretation $(\mathcal{I}, \pi)$, for any two concepts C and D, we have*

- $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$ *iff* $(\mathcal{I}, \leq_\pi) \Vdash C \sqsubseteq_{ps} D$ *and* $C^{\mathcal{I}}$ *is not minimized.*
- $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$ *iff* $(\mathcal{I}, \leq_\pi) \Vdash C \sqsubseteq_{ps}^* D$ *and* $(\neg D)^{\mathcal{I}}$ *is not minimized.*

Proof. For the first item, we have
$(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$
$\Leftrightarrow \Pi(C^{\mathcal{I}} \cap D^{\mathcal{I}}) > \Pi(C^{\mathcal{I}} \cap \neg D^{\mathcal{I}})$
$\Leftrightarrow \{x \in C^{\mathcal{I}} | \pi(x) = \Pi(C^{\mathcal{I}})\} \subseteq D^{\mathcal{I}}$, and also $\Pi(C^{\mathcal{I}}) > 0$
$\Leftrightarrow \{x \in C^{\mathcal{I}} | \nexists y \in C^{\mathcal{I}} (\pi(x) \leq_\pi \pi(y) \wedge \pi(y) \not\leq_\pi \pi(x))\} \subseteq D^{\mathcal{I}}$, and $\exists x \in C^{\mathcal{I}} (\exists y \in \triangle^{\mathcal{I}}, x \geq_\pi y \wedge y \not\geq_\pi x)$
$\Leftrightarrow \{x \in C^{\mathcal{I}} | \nexists y \in C^{\mathcal{I}} (x \leq_\pi y \wedge y \not\leq_\pi x)\} \subseteq D^{\mathcal{I}}$, and $C^{\mathcal{I}}$ is not minimized
$\Leftrightarrow C^{\mathcal{I}-} \subseteq D^{\mathcal{I}}$, $C^{\mathcal{I}}$ is not minimized
$\Leftrightarrow (\mathcal{I}, \leq_\pi) \Vdash C \sqsubseteq_{ps} D$, $C^{\mathcal{I}}$ is not minimized.
For the second item, we have
$(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$
$\Leftrightarrow (\mathcal{I}, \pi) \Vdash \neg D \sqsubseteq_D \neg C$
$\Leftrightarrow (\mathcal{I}, \pi) \Vdash \neg D \sqsubseteq_{ps} \neg C$, and $\neg D^{\mathcal{I}}$ is not minimized (item 1)
$\Leftrightarrow (\mathcal{I}, \leq_\pi) \Vdash C \sqsubseteq_{ps}^* D$, and $\neg D^{\mathcal{I}}$ is not minimized. □

The relationship between our other preferential subsumption relations and the preferential subsumption relations $\sqsubseteq_{ps}$ and $\sqsubseteq_{ps}^*$ can be easily seen by Theorems 7, 8 and 12.

Based on Theorem 12, we are able to show the following relationship between our preferential subsumption relations $C \sqsubseteq_D$ and $C \sqsubseteq_{D^2}$ and the preferential subsumption relations defined in (Britz, Heidema, and Meyer 2008).

Corollary 1 Given a possibilistic interpretation $(\mathcal{I}, \pi)$, for any two concepts C and D, if $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$ (resp. $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$), then there exists a Neotherian, modular partial order $\leq$ such that $(\mathcal{I}, \leq) \Vdash C \sqsubseteq_{ps} D$ (resp. $(\mathcal{I}, \leq) \Vdash C \sqsubseteq_{ps}^* D$.

According to Theorem 12, we can also see that given a Neotherian, modular partial order $\leq$, $(\mathcal{I}, \leq) \Vdash C \sqsubseteq_{ps} D$ (resp. $(\mathcal{I}, \leq) \Vdash C \sqsubseteq_{ps}^* D$ may not imply that there exists a possibilistic interpretation $(\mathcal{I}, \pi)$ such that $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_D D$ (resp. $(\mathcal{I}, \pi) \Vdash C \sqsubseteq_{D^2} D$), unless we further assume that $C^{\mathcal{I}}$ (resp. $(\neg D)^{\mathcal{I}}$) is not minimized. This assumption is reasonable because if $C^{\mathcal{I}}$ (resp. $(\neg D)^{\mathcal{I}}$) is not minimized, then every object in $C^{\mathcal{I}-}$ (resp. $(\neg D)^{\mathcal{I}-}$) which is used to define the (dual) preferential subsumption can be considered as atypical. In this sense, our preferential subsumption relations are more reasonable than preferential subsumption relations given in (Britz, Heidema, and Meyer 2008).

Plausible subsumption entailment

We have defined the semantics of our preferential subsumption relations relative to a possibilistic interpretation. In this subsection, we consider plausible subsumption entailment from a knowledge base. We extend a DL knowledge base with our preferential subsumption statements, that is, a TBox consists of standard terminology axioms and subsumption statements of the form $C \sqsubseteq_D D$, $C \sqsubseteq_{D^2} D$, $C \sqsubseteq_B D$ and $C \sqsubseteq_H D$. We further extend the ABox to allow statements[3] of the form $a \preceq b$, which will be used to constrain π, formally defined as follows:

Definition 13. A possibilistic interpretation $(\mathcal{I}, \pi)$ satisfies an assertion $a \preceq b$ iff $\pi(a^{\mathcal{I}}) \leq \pi(b^{\mathcal{I}})$.

We need to point out that even if there does not exist any ABox assertion of the form $a \preceq b$, a possibility distribution can be still constrained by preferential subsumption statements in the DL knowledge base.

We call such an extended DL knowledge base as a possibility-based DL (PDL for short) knowledge base. We first define the notion of validity of the entailment of a preferential subsumption.

Definition 14. The preferential subsumption statement $C \sqsubseteq_X D$ is valid, written as $\models_P C \sqsubseteq_X D$, iff it is satisfied by all possibilistic interpretations $(\mathcal{I}, \pi)$, where $X \in \{D, D^2, B, H\}$.

The entailment of a preferential subsumption statement relative to a PDL knowledge base is defined as follows.

Definition 15. For a PDL knowledge base $\mathcal{K}$, $\mathcal{K}$ entails the preferential subsumption $C \sqsubseteq_X D$, written as $\mathcal{K} \models_P C \sqsubseteq_X D$, iff every possibilistic interpretation that satisfies $\mathcal{K}$ also satisfies $C \sqsubseteq_X D$ where $X \in \{D, D^2, B, H\}$.

According to Theorem 12 and the discussion about application of the entailment of preferential subsumption to inductive and abductive reasoning given in (Britz, Heidema, and Meyer 2008), we may propose $\mathcal{K} \models_P C \sqsubseteq_D D$ (resp. $\mathcal{K} \models_P C \sqsubseteq_{D^2} D$) as an apt constraint on "C is plausibly subsumed by D" in the context of an inductive (resp. abductive) stance on the instance checking problem. Furthermore, our preferential subsumption relations $\sqsubseteq_B$ and $\sqsubseteq_H$ can be used to capture stronger causal relationship between two concepts.

The properties of a preferential subsumption relation relative to a fixed possibilistic interpretation can be extended in the context of entailment relative to a PDL knowledge base. For example, property And of $\sqsubseteq_X$ relative to $\mathcal{K}$ reads as follows: if $\mathcal{K} \models_P C \sqsubseteq_X D$ and $\mathcal{K} \models_P C \sqsubseteq_X E$ then $\mathcal{K} \models_P C \sqsubseteq_X D \sqcap E$, where $X \in \{D, D^2, B, H\}$.

[3]This kind of statements was originally proposed in (Britz, Heidema, and Meyer 2008) to constrain an ordered interpretation.

Examples

In this subsection, we give two examples to illustrate our preferential semantics and plausible subsumption entailment. The first example illustrates satisfaction relations of our preferential subsumption. To simply notations, we write $\Pi(C)$ instead of $\Pi(C^{\mathcal{I}})$ when it is clear from the context.

Example 16 (adapted from the example about clinical record given in (Britz, Heidema, and Meyer 2008)) Suppose we have the following entry in a clinical record, where A, B and C are symptoms, and Y and Z are medical conditions: This entry says that patient 03154 had symptom A,

Number	Date	A	B	C	...	Y	Z
03154	17/06/04	1	-	0	...	-	1

did not have symptom C, and suffered from condition Z on day 17/06/04. The information about symptom B and condition Y is unknown. Following (Britz, Heidema, and Meyer 2008), we assume that symptoms A and C are relevant to condition Z and each complete profile (A, C, Z), i.e., each entry having values of 0 or 1 for A, C and Z, contributes to the generation of a possibility distribution on patient profiles. We do not discuss how the possibility distribution for the profile (A, C, Z) is generated, but refer the reader to (Britz, Heidema, and Meyer 2008) and (Britz, Heidema, and Meyer 2009) for some possible methods[4]. Suppose we have the following possibility distribution, denoted as π, for the profile (A, C, Z):

$$\begin{array}{ll} \pi(100) = 1.0 & \pi(010) = 0.8 \\ \pi(011) = 0.7 & \pi(111) = 0.7 \\ \pi(101) = 0.5 & \pi(000) = 0.5 \\ \pi(001) = 0.0 & \pi(110) = 0.0 \end{array}$$

We take all the eight profiles as the domain $\Delta^{\mathcal{I}}$ of an interpretation $\mathcal{I}$. A concept is interpreted under $\mathcal{I}$ as the set of all profiles that assign value 1 to the concept. For example, we have $A^{\mathcal{I}} = \{100, 111, 101, 110\}$ and $(A \sqcap \neg C)^{\mathcal{I}} = \{100, 101\}$. Profile 100 with highest possibility degree is considered as the most typical profiles, and profiles 001 and 110 with lowest possibility degree are considered as atypical ones. We show that $(\mathcal{I}, \pi) \Vdash Z \sqsubseteq_D C$. Since $C^{\mathcal{I}} = \{010, 011, 111, 110\}$ and $Z^{\mathcal{I}} = \{011, 111, 101, 001\}$, we have $(Z \sqcap C)^{\mathcal{I}} = \{011, 111\}$. We also have $(Z \sqcap \neg C)^{\mathcal{I}} = \{101, 001\}$. So we have $\Pi(Z \sqcap C) = 0.7 > \Pi(Z \sqcap \neg C) = 0.5$. It follows that $(\mathcal{I}, \pi) \Vdash Z \sqsubseteq_D C$. Similarly, we can show that $(\mathcal{I}, \pi) \Vdash Z \sqsubseteq_{D^2} A$. Let us consider the relationship between symptoms A and C and condition Z. Since $(A \sqcap C)^{\mathcal{I}} = \{111, 110\}$ and $(\neg Z)^{\mathcal{I}} = \{100, 010, 000, 110\}$, we have $(A \sqcap C \sqcap \neg Z)^{\mathcal{I}} = \{110\}$. So $\Pi(A \sqcap C \sqcap \neg Z) = 0$. If follows that $(\mathcal{I}, \Pi) \Vdash (A \sqcap C) \sqsubseteq_H Z$. This infers that $(\mathcal{I}, \Pi) \Vdash (A \sqcap C) \sqsubseteq_B Z$. Therefore, we can say that a patient having symptom A and testing positive for symptom C is highly possible to suffer from condition Z. Now suppose a patient having symptom A and testing negative for symptom C. Since $(A \sqcap \neg C)^{\mathcal{I}} = \{100, 101\}$, we have $(A \sqcap \neg C \sqcap Z)^{\mathcal{I}} = \{101\}$, thus $\Pi(A \sqcap \neg C \sqcap Z) \neq 0$. However, since $(A \sqcap \neg C \sqcap \neg Z)^{\mathcal{I}} = \{100\}$, we have $\Pi(A \sqcap \neg C \sqcap \neg Z) > \Pi(A \sqcap \neg C \sqcap Z)$, thus $(\mathcal{I}, \pi) \Vdash A \sqcap \neg C \sqsubseteq_D \neg Z$. We do not have $(\mathcal{I}, \pi) \Vdash A \sqcap \neg C \sqsubseteq_{D^2} \neg Z$. Thus the causal relationship between $A \sqcap \neg C$ and $\neg Z$ is weak.

[4]Remind that we consider qualitative possibility theory in our paper.

Next, we consider an example which is widely used to illustrate nonmonotonic logics.

Example 17 Suppose we have a knowledge base $\mathcal{K} = \{Bird \sqsubseteq_D Fly, Penguin \sqsubseteq_D \neg Fly, Penguin \sqsubseteq_H Bird\}$, which says that generally any bird can fly, generally penguin cannot fly, and all pengiums are birds. For simplicity, we use B, P and F to denote concepts $Bird$, $Penguin$ and Fly resepctively. For any interpretation $(\mathcal{I}, \pi)$, if $(\mathcal{I}, \pi)$ satisfies $\mathcal{K}$, then the possibility distribution π must satisfy the following constraints.

$$\Pi(B \sqcap F) > \Pi(B \sqcap \neg F), \;\; \Pi(P \sqcap \neg F) > \Pi(P \sqcap F),$$
$$\Pi(P \sqcap \neg B) = 0.$$

Therefore, $(\mathcal{I}, \pi)$ must satisfies $P \sqsubseteq_D \neg F$. But it does not satisfy $P \sqsubseteq_D F$. Thus the subsumption $P \sqsubseteq_D F$ is blocked from being inferred. We also have that $(\mathcal{I}, \pi)$ satisfies the default subsumption $P \sqcap B \sqsubseteq_D \neg F$, i.e, generally, any bird which is a penguin cannot fly. This can be shown as follows. Since $\Pi(P \sqcap \neg B) = 0$, we have $\Pi(P \sqcap \neg F) = max(\Pi(P \sqcap \neg F \sqcap B), \Pi(P \sqcap \neg F \sqcap \neg B)) = \Pi(P \sqcap \neg F \sqcap B)$ because $\Pi(P \sqcap \neg F \sqcap \neg B) \leq \Pi(P \sqcap \neg B) = 0$. We know that $\Pi(P \sqcap \neg F) > \Pi(P \sqcap F)$, thus $\Pi(P \sqcap \neg F \sqcap B) > \Pi(P \sqcap F) \geq \Pi(P \sqcap B \sqcap F)$. It follows that $(\mathcal{I}, \pi)$ satisfies $P \sqcap B \sqsubseteq_D \neg F$. Therefore, we have $\mathcal{K} \models_P P \sqcap B \sqsubseteq_D \neg F$. Note that we cannot infer this without the hard preferential subsumption $Penguin \sqsubseteq_H Bird$.

Translation to DL Reasoning

In this section, we present a translation from all statements expressible in possibility-based DL into statements of a classical DL. In this way, we provide a method for implementing the plausible entailment relations by using a classical DL reasoner. We consider possibility-based DL $\mathcal{ALC}$ in this work. We first add two role axioms to the language $\mathcal{ALC}$.

Definition 18. Given a role name R, we define a role axiom $Possibility(R)$ in a model theoretical way as follows: $\mathcal{I}$ is a model of $Possibility(R)$, written as $\mathcal{I} \Vdash Possibility(R)$, iff $R^{\mathcal{I}}$ is a total pre-order and is Noetherian. We define another role axiom $LBound(R)$ in a model theoretical way as follows: $\mathcal{I}$ is a model of $LBound(R)$, written as $\mathcal{I} \Vdash LBound(R)$, iff $R^{\mathcal{I}}$ has at least one minimal objects, that is, $R^{\mathcal{I}}$ is lower bounded.

In the following, we present a translation from all statements expressible in possibility-based DL $\mathcal{ALC}$ into an expressive DL. Given a specific instance L of the language for possibility-based DL $\mathcal{ALC}$, we define the translation of a statement expressible L as follows:

Definition 19. (Translation rules) For any statement ϕ in L, the *translation* ϕ^t of ϕ is defined as follows: for role name R not occuring in the alphabet of L, concepts C and D, and individuals a and b, If ϕ is of the form $a \preceq b$ then $\phi^t = \{R(a, b)\}$;

If ϕ is of the form $C \sqsubseteq_D D$ then $\phi^t = \{C \sqcap \forall R \sqcap \neg R^-.\neg C \sqsubseteq D,\ \exists(C \sqcap (\exists(R^- \sqcap \neg R).\top))\}$;

If ϕ is of the form $C \sqsubseteq_{D^2} D$ then $\phi^t = \{C \sqsubseteq D \sqcup \exists R \sqcap \neg R^-.\neg D,\ \exists(\neg D \sqcap (\exists(R^- \sqcap \neg R).\top))\}$;

If ϕ is of the form $C \sqsubseteq_B D$ then $\phi^t = \{C \sqsubseteq (\exists R \sqcap \neg R^-.C \sqcup D) \sqcap (D \sqcup \exists R \sqcap \neg R^-.\neg D),\ \exists(C \sqcap (\exists(R^- \sqcap \neg R).\top)),\ \exists(\neg D \sqcap (\exists(R^- \sqcap \neg R).\top))\}$;

If ϕ is of the form $C \sqsubseteq_H D$ then $\phi^t = \{C \sqcap \neg D \sqsubseteq \neg(\exists(R^- \sqcap \neg R).\top)\}$;

Otherwise $\phi^t = \{\phi\}$.

In Definition 19, the role R is used to capture a Noetherian total pre-order $\preceq$ and the role $R \sqcap \neg R^-$ is used to capture the asymmetry of $\preceq$, i.e., $\prec$. The transformation rules for preferential subsumption axioms can be explained as follows. For the translation of $C \sqsubseteq_D D$, $C \sqcap \forall R \sqcap \neg R^-.\neg C$ is interpreted as the maximally preferred objects in the extension of C and $\exists(R^- \sqcap \neg R).\top$ is interpreted as the objects which are not minimal (w.r.t. the preference order $R^{\mathcal{I}}$). For any concept C, $\exists C$ is called a concept existence axiom saying that there exists some object of C (see (Horrocks and Patel-Schneider 2003)). According to Theorem 10, $C \sqcap \forall R \sqcap \neg R^-.\neg C \sqsubseteq D$ and $\exists(C \sqcap (\exists(R^- \sqcap \neg R).\top))$ together capture the meaning of $C \sqsubseteq_D D$. $\sqsubseteq_{D^2}$ can be explained similarly. For the translation of $\sqsubseteq_B$, it is easy to understand it according to the fact that $\{C \sqsubseteq (\exists R \sqcap \neg R^-.C \sqcup D) \sqcap (D \sqcup \exists R \sqcap \neg R^-.\neg D)\}$ is semantically equivalent to $\{C \sqsubseteq \exists R \sqcap \neg R^-.C \sqcup D, C \sqsubseteq D \sqcup \exists R \sqcap \neg R^-.\neg D\}$ and $C \sqsubseteq (\exists R \sqcap \neg R^-.C \sqcup D)$ is equivalent to $C \sqcap \forall R \sqcap \neg R^-.\neg C \sqsubseteq D$. Finally, for the translation of $\sqsubseteq_H$, $C \sqcap \neg D \sqsubseteq \neg(\exists(R^- \sqcap \neg R).\top)$ means that any object which is in C but not in D are minimal.

The translation of a knowledge base in L is defined as follows.

Definition 20. Let $\mathcal{K}$ be a knowledge base expressed in L. The *translation* of $\mathcal{K}$, referred to as $\mathcal{K}^t$, is defined as follows: $\mathcal{K}^t = \cup_{\phi \in \mathcal{K}} \phi^t \cup \{Possibility(R), LBound(R)\}$.

In order to translate a knowledge base $\mathcal{K}$ in L, the translated knowledge base $\mathcal{K}^t$ must contain two new Rbox role axioms $Possibility(R)$ and $LBound(R)$, which are used to ensure that the newly introduced role name R in the translation of defeasible subsumption relations is a Noetherian total pre-order and is lower bounded. Reflexivity of roles, role transitivity and totality of roles are included in DL $\mathcal{SHOIQ}$, which underlies OWL 1.1. Note that we do not need all expressivity of that logic. In our translation, we also need inverse roles, role conjunctions, negation of atomic roles and concept existence axioms. Extending $\mathcal{ALC}$ with role conjunctions and negation of roles and other role constructors have been discussed in (Hustadt and Schmidt 1998). Concept existence axiom has been introduced in (Horrocks and Patel-Schneider 2003). However, as far as we know, there is no work discussing a DL which extends $\mathcal{ALC}$ with all these constructors. Noetherian roles are not included in any existing DL. We will investigate their impact on the complexity of entailment in the considered DLs which are at least as expressive as $\mathcal{ALC}$ plus reflexivity of roles, role transitivity, totality of roles, inverse roles, role conjunctions and negation of atomic roles, concept existence. Lower bounded roles are also out of scope of OWL DL. However, suppose that we are able to deal with Noetherian roles, then it should not be a problem to deal with lower bounded roles. This is because a lower bounded order is less constrained than a well-order which is the reverse of a Noetherian order.

Example 21 (Example 17 Continued) The translation of $\mathcal{K}$ is the following DL knowledge base $K^t = \{Bird \sqcap \forall R \sqcap \neg R^-.\neg Bird \sqsubseteq Fly, Penguin \sqcap \forall R \sqcap \neg R^-.\neg Penguin \sqsubseteq \neg Fly, Penguin \sqcap \neg Bird \sqsubseteq \neg(\exists(R^- \sqcap \neg R).\top), \exists(Bird \sqcap (\exists(R^- \sqcap \neg R).\top)), \exists(Penguin \sqcap (\exists(R^- \sqcap \neg R).\top)), Possibility(R), LBound(R)\}$.

We proceed to show that the above translation is correct.

Theorem 22 *For any knowledge base $\mathcal{K}$ expressed in L and any kind of statement ϕ in L, we have that $\mathcal{K} \models_P \phi$ iff $\mathcal{K}^t \models \psi$ for any $\psi \in \phi^t$.*

Proof. (Sketch) *Only if direction:* Suppose $K \models_P \phi$. For any interpretation $\mathcal{I}$, suppose $\mathcal{I} \models \gamma$, for any $\gamma \in \mathcal{K}^t$. Then there exists a role name R such that $R^{\mathcal{I}}$ is a Noetherian total pre-order and is lower bounded. We can construct a possibility distribution over $\triangle^{\mathcal{I}}$ from R as follows. Since R is a total pre-order and is lower bounded, we can use it to stratify $\triangle^{\mathcal{I}}$ in such a way as $\triangle^{\mathcal{I}} = S_1 \cup ... \cup S_n \cup ...$, where S_1 contains all the minimal objects in $\triangle^{\mathcal{I}}$, and any objects a and b in the same set S_i, we have $(a, b) \in R^{\mathcal{I}}$ and $(b, a) \in R^{\mathcal{I}}$, and for $a \in S_i$ and $b \in S_j$, where $i < j$, $(a, b) \in R^{\mathcal{I}}$ but $(b, a) \notin R^{\mathcal{I}}$. The possibility distribution π is then defined as $\pi(a) = k$ if $a \in S_k$. It is clear that $(a, b) \in R^{\mathcal{I}}$ iff $\pi(a) \leq \pi(b)$. Since $\mathcal{I} \models \gamma$, for any $\gamma \in \mathcal{K}^t$, we have $(\mathcal{I}, \pi) \Vdash \psi$ for any $\psi \in \mathcal{K}$. Therefore, $(\mathcal{I}, \pi) \Vdash \phi$. By Theorem 12, it is easy to see that $\mathcal{I} \models \psi$ for any $\psi \in \phi^t$. Similarly we can show the if direction. □

Theorem 22 tells us that checking whether a plausible subsumption (or a standard subsumption) follows from a knowledge base $\mathcal{K}$ expressed in L can be reduced to checking whether the translated DL statements follows from the translation $\mathcal{K}^t$ of $\mathcal{K}$.

Related Work

There have been some work on preferential reasoning in description logics, such as the work reported in (Britz, Heidema, and Meyer 2008; Giordano et al. 2007; 2008). As shown in our paper, the default subsumption $\sqsubseteq_D$ and dual default subsumption $\sqsubseteq_{D^2}$ are closely related to the preferential subsumption and dual preferential subsumption given in (Britz, Heidema, and Meyer 2008). A preferential description logic, called $\mathcal{ALC}$+**T**, is given in (Giordano et al. 2007) and is later extended to deal with typicality in (Giordano et al. 2008). Their work extends DL $\mathcal{ALC}$ with a "typicality" operator **T**, thus allows preferential subsumption which is similar to our default subsumption and the preferential subsumption given in (Britz, Heidema, and Meyer 2008). This work is also related to the work on nonmonotonic reasoning in description logics (Baader and Hollunder 1995a; 1995b; Bonatti, Lutz, and Wolter 2006; Donini, Nardi, and Rosati 2002; Governatori 2004). In (Baader and Hollunder 1995a), an extension of DL with Reiter's default logic is given and this work is further extended with priority in (Baader and Hollunder 1995b). The semantics of their logic given in (Baader and Hollunder 1995a) is a restricted semantics for open default theories where default rules are only applied to individuals that are explicitly mentioned in the ABox. The authors of the paper (Bonatti, Lutz, and Wolter 2006) propose nonmonotonic description logics by extending expressive DLs $\mathcal{ALCIO}$ and $\mathcal{ALCQO}$ with circumscription. In their work, some predicates (i.e. concept and role names) are identified as "abnormality predicates" that should be minimized, i.e., the extension of these predicates is required to be minimal w.r.t. set inclusion. Thus, the semantics of their logics are different from ours. The work presented in (Donini, Nardi, and Rosati 2002) extends DL $\mathcal{ALCK}$ with two modal operators. Their logics can capture default rules and other features in frame-based systems. Defeasible description logics are proposed in (Governatori 2004) by combining description logics and defeasible logic. They add to a DL knowledge base a set of defeasible logic rules and a preference relation over the rules.

This work is inspired from the work on applying possibilistic logic (Dubois, Lang, and Prade 1994) to deal with exceptional-tainted rules (Benferhat, Dubois, and Prade 1997; 1998). Our default subsumption and hard subsumption correspond to default rule and hard rule defined over propositional symbols in (Benferhat, Dubois, and Prade 1998). This work is different from the work on extending description logic with possibilistic logic (see, for example, (Qi, Pan, and Ji 2007)), where classical DL axioms are attached with weights. Unlike our logics, possibilistic description logics are mainly applied to deal with logical inconsistencies and uncertainty in description logics.

Conclusion and Future Work

This paper deals with the problem of preferential reasoning in description logics. Four preferential subsumption relations were defined and their relationships were discussed. The relationship between our preferential subsumption relations and preferential subsumption relations given in (Britz, Heidema, and Meyer 2008) was drawn. We also considered entailment of our preferential subsumption statements relative to a DL knowledge base extended with preferential subsumption statements and a special kind of role assertions. Two examples were given to illustrate these notions and show their potential application in practice. We provided a method for translating entailment of TBox statements (including classical subsumption statements and all preferential subsumption statements) and ABox statements (including classical ABox statements and statements of the $a \preceq b$) relative to an extended knowledge base to entailment of classical TBox statements and classical ABox statements relative to a DL knowledge respectively.

As a future work, we will extend this work by considering preferential subsumption statements attached with uncertainty degrees. This topic is interesting because the problem of dealing with uncertainty is closely related to the problem of nonmonotonic reasoning and possibility theory has been widely accepted as an important formalism to deal with both nonmonotonic reasoning and uncertainty (see (de Saint-Cyr and Prade 2006) for example). In (Benferhat, Dubois, and Prade 1997) and (Benferhat, Dubois, and Prade 1998), the authors argue that the possibilistic entailment defined by the default rules is too cautious and propose several more adventurous entailment relations. We will consider adapting these entailment relations to improve our default preferential subsumption. For example, when defining a plausible subsumption entailment, we may consider a single possibility distribution over the domain of a DL interpretation. We will also consider translating the new plausible subsumption entailment to a DL entailment. Another future work is to consider extending our preferential semantics by plausibility measures (Friedman and Halpern 2001). We have not considered the algorithmic aspects and complexity analysis of the entailment of our preferential semantics but leave them as future work.

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