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Quasi-Topological Structure of Extensions in Logic of Determination of Objects (LDO) for Typical and Atypical Objects

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Abstract

This paper introduces and discusses a new algebraic structure, the *quasi-topologic* structure. The idea of this structure comes from language analysis on the one hand and from analysis of some real situations of clustering on the other. From the cognitive point of view, it is related to the Logic of Determination of Objects (LDO) and to the Logic of Typical and Atypical Objects (LTA) which is particular case of LDO. From the mathematical point of view, it is related to topology. By introducing the notion of *interna*l and *external border*, it extends the notion of border from classical topology.

Introduction

Cognitive problems of categorization, especially in the humanities, require a deeper analysis of their nature and a mathematical modelling approach different from existing classical mathematical models of categorization. The analysis of the categorization entails notions such as "object", "concept", "concept network", "typicality" and "atypicality". Existing mathematical models of categorization are not sufficient to account for some specific problems of typicality or exception. It is necessary to construct a more adequate mathematical to take in account the problems of conceptual distinctions between typical, atypical instances and exceptional occurrences of a concept inside a model that is useful for a logical analysis, a linguistic analysis and machine implementation of a software application for "automatic reasoning".

For this purpose, we propose in this article the notion of quasi-topological structure of the extension of a concept. The formalism of this model is described in the Logic of Determination of Objects (LDO) (Desclés, Pascu 2011) and in more restricted Logic of Typical and Atypical Objects (LTA) (Desclés, Pascu, Jouis, 2013; Desclés, Pascu 2014). As a logical-mathematical formalism, LDO is thought to precise the cognitive concept of typicality based formal relations between a concept (seen as more complex than a simple property) and objects which are instances of this concept. For this purpose, LDO have introduced distinctions as intension and essence of a concept in relation with extension and expansion of a concept.

The quasi-topology is an algebraic structure (which generalizes Kuratowski's algebra associated to a topological space) to precise the extensional structure of a concept, with typical instances and atypical instances. Roughly speaking, a quasi-topological structure defines a "thick boundary" (or "thick border") (having a topological meaning) with an "internal boundary" and an "external boundary".

In order to explain the cognitive features that link quasi-topology to common reasoning and common language, we present two examples: spatial relations between two places, and temporal change between the notion "young" and "told". To motive the distinction typical/atypical object analysed in the LDO framework, we recall the well known example (in AI) of the "ostrich" and an elementary mathematical example from basic arithmetic. For the notion of "exception", we present the different occurrences of "the inhabitants of a city" with the LTA formalism. Afterwards, we define the abstract quasi-topology structure and finally, we present a quasi-topological model associated with the LTA model.

Preliminary examples

a) First example: Spatial course (Desclés, Guenchéva 2009) with the aspectual notions "Still / Already" // "Not yet / No longer" (see Figure 1). Let us take two places: LOC_1 (for instance Paris) and LOC_2 (for instance Villejuif, located in the suburbs of Paris); an agent is going from the place (LOC_1) to the other place LOC_2). He starts by the interior of LOC_1 : it is inside LOC_1. Then, it passes into the area where it is *still* in LOC_1. The following area corresponds *to no-longer / not yet*: it is no longer in LOC_1 but not yet in LOC_2 Passing by the area corresponding to *already*, it arrives inside LOC_2.. This representation is double-oriented, that is the course is the same when one changes the orientation (see the figure 1).

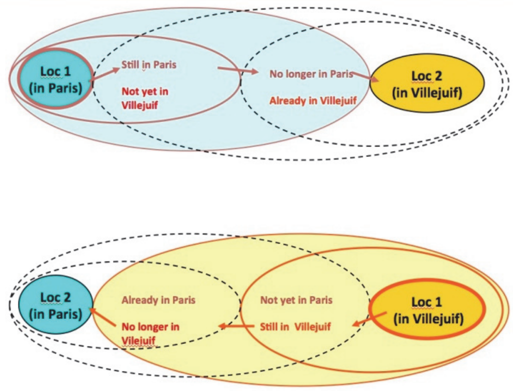

Figure 1.

b) Second example: a temporal course (Desclés, Pascu, 2016) from "Young to "Old" (see figure 2). In this case, the orientation has only one direction from "young" to **"**old**"**. The two areas corresponding to "young" and "old" are not disjointed. Inside each one, there is an area qualified by "still" and another qualified by "no longer". The area "young" can be considered as the "interior of to-be-young", the area corresponding to *still young* (not yet old), which is an "internal boundary" (or "border area" BA_1), and the area corresponding to *already old* (no longer young), which is an "external boundary" (or "border area" BA_2) (see the figure 2).

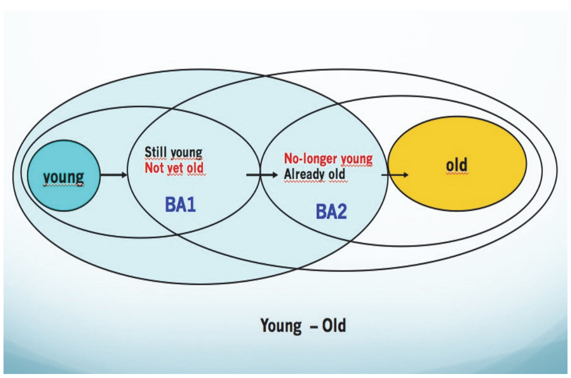

Figure 2.

c) Third example: a notional course (Desclés, Pascu, 2016) with the notions "Legal / Illegal" (see Figure 3). From "legal" to "illegal" one passes through notional areas such as "*still legal & not already illegal"*, "still illegal & not already legal" (see the figure 3).

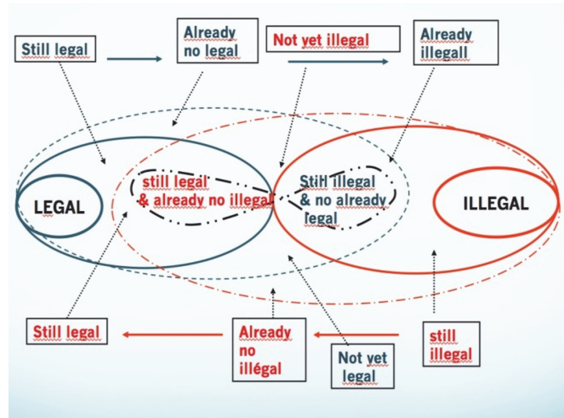

Figure 3.

These three examples are outcomes of express in the ordinary language on the one hand and from the different representations of movements or changes in the space, in the time (young old is the time related to a human being) and opposite notions – legal –illegal. The last one is very useful in the field of laws to establish the distinction between that which is permitted by law and that which is not permitted. All these examples show the necessity to formally define an internal border and an external border of a set (a place and an abs tract place) in order to be used in applications.

d) Fourth example: the ostrich (Desclés, Pascu, 2011) (see Figure 4). The problem of typicality atypicality is related to the "interior" and the "border of a set an abstract place" from the point of view of its modelling. It is the well-known network of concepts associated with the property "to-be-ostrich" is presented: the ostrich is a bird that does not fly, thus contradicting the "typical" property of a bird which has the capacity to fly..One aspect of the opposition typicality/atypicality problems comes from the treatment meaning of the logical negation among the properties of a concept. The distinction between *typical* and *atypical* occurrences of a property was introduced in a categorisation process by E.H. Rosch (Rosch, 1977; Rosch, Mervis, 1981). Knowledge Representation (KR), as part of Artificial Intelligence (AI) as well as Non-Monotonic Logic (Strassser, Antonelli, 2015) or Paraconsistent Logic (Da Costa, 1997; Béziau 2000, 2007) proposed different ways of solving the contradiction emerging in concept networks due to negation. The Logic of the Determination of Objects (LDO) (Desclés, Pascu, 2011) offers another solution based on the specific notion of / a deep study of logical relations between a the intension and essence of a concept and its extension presented by objects; this formal approach aims generates its own / a cognitive theory of typicality (Desclés, Pascu, 2011). In LDO, "*one ostrich*" designates "an atypical bird" because it inherits the negation of the property "to fly" but it belongs also to the set of birds.

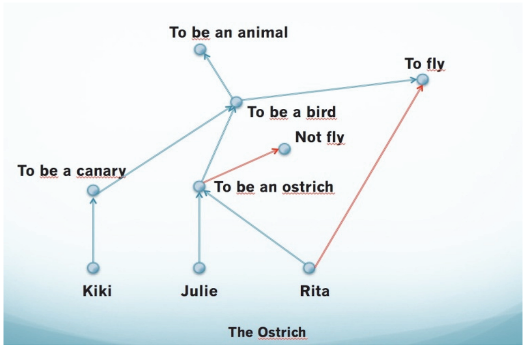

Figure 4.

e) Fifth example: the number 2 is atypical among prime numbers because it is even and all other prime numbers are odd.

f) Example six : The different uses of *The inhabitant of a city* (Desclés, Pascu, Jouis, 2013). The concept "inhabitant of X" includes the property "to be an inhabitant of X". To this property is associated a concept with its intension (a set of properties which determines/ characterize the concept) This concept has an intension - a set of properties which determines the concept); among these properties, there are "to have rights" and "to have duties". These properties imply others such as "to be protected" and "to respect the law". Not all inhabitants are characterized exactly by the same properties: some of them have citizenship while others do not. Situations also differ with respect to taxes and the right to vote. An inhabitant without identity papers is just an inhabitant of X but has none of the above properties: it is an exception among the inhabitants of X (see the figure 5).

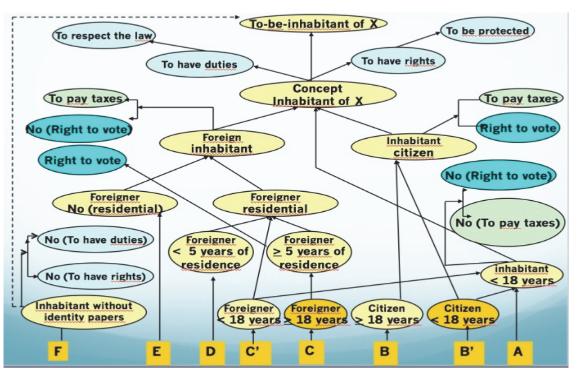

Figure 5.

These examples justify the need of an adequate formalisation of typicality- atypicality it is given in the framework of the LDO. The last example justifies the need to isolate exceptions in the extension of a property (following the LTA approach). As we are going show, the extensions, in LDO and LTA, are structured by a quasi-topology.

An important distinction: property / concept

In LDO (Desclés, Pascu, 2011), a property is defined as a "concept" (Frege,1893/1967): a function 'f' of a domain D of completely determined objects into the set {⊤, ⊥} of truth values. The relation 'f(x) = ⊤' means that the object x falls under property f and ' f(x) = ⊥' means that the object x does not fall under f. In the context of LDO, a property is exactly this function, but the notion of concept is defined from the notion of property. The concept defined from a property 'f' is designated by '^f'. Different networks of properties related to each other by the relation (noted '→') of inheritance (or comprehension), are characterizations of a given concept (Desclés, Pascu, 2011). The relation 'f → g' (reflexive and transitive) means that the property f inherits property g. Among all the properties inherited by the property f, we distinguish two classes: the class of properties that are necessarily inherited by f and a class of properties that are just inherited by f but which are not always necessary to it. The first class is the "essence of the concept ^f" (noted 'Ess[^f]'), the second class is the "intension of f" (noted 'Int[^f]) with the condition 'Int[^f] ⊇ Ess[^f]'. In LDO, an object is more than an argument of a property. The connection between a concept '^f' and extensional representative objects of the concept '^f' introduces the fully undetermined "typical object", designated by 'τf'. This object is as abstract object the best representative object of the concept 'f'. The other representative objects are more or less determinate objects and are generated from this abstract object 'τf' by means of determination operations. An determination operator, noted 'δ', associated to a qualitative property, for instance 'u', allows to construct, from a more or less determinate representative object, for instance 'x', another object, for instance 'y', more determinate than 'x', such that 'y = δ(u)(x). All representative objects constitute the "expansion" ("étendue" in French) of the concept '^f'; the "extension" is a part of the expansion, it is constituted by all completely determinate representative objects of the concept f. The extension is a part of the expansion.

The formalism of LDO is defined from a set F of properties and a set O of objects, with an inheritance relation '→' between properties and a determination relations between objects. A concept '^f' is defined by a property 'f' of F, with a specific essence Ess[^f] and a intension Int[^f]. A concept '^f' = < f, Ess[^f], Int[^f] >, where the abstract fully undetermined "typical object" $\boldsymbol{\tau}$(f) generates all objects of expansion and the extension by means of determination operations. The Expansion Exp [f] (or Etendue of f) and the extension Ext[f] are generated from by determinations associated to properties. The axiomatic relations and deduction rules between these constituents of LDO are given in (Desclés, Pascu, 2011). The typical objects of a concept '^f' inherit all the properties of the intension Int[^f]; the atypical objects of ^f do not inherit all the properties of the intension Int[^f] but they inherit all properties of the essence Ess [^f] ⊆ Int[f]. There are therefore typical objects and atypical objects and, correspondingly, the typical extension Ext_{τ}[^f] and the atypical extension Ext_{α}[^f].

The formalism of LTA is a particular case of LDO since LTA does not take in account more or less determinate objects generated from the abstract object '$\boldsymbol{\tau}$(f)' and, in this case, the expansion is reduced to the extension : all representative objects of the concept '^f' are completely determinate.

Notion of quasi-topology

A set E is structured by a quasi-topology when exist the pairs of sets $<F^1, O^1>$ and $<F^2, O^2>$, such that:

$$(*) \quad F^2 \supset F^1 \supseteq E \supseteq O^1 \supset O^2$$

where O^1 and O^2 are considered as "open" parts of E and F^1 and F^2 as "close" parts of E : O^1 is the largest part contained in E and O^2 is the largest part strictly contained in O^1; F^2 is the smallest part containing E and F^1 is the smallest part strictly containing F^2. The part O^2 is said the "strict interior" of E and O^1 the "(simple) interior" of E. The part F^2 is said the "large closure" of E and F^1 the "(simple) closure" of E. We define the internal and external boundaries of E by differences between parts around E:

IB (internal boundary) of E $=_{def}$ $F^1 - O^2$
EB (external boundary) of E $=_{def}$ $F^2 - O^1$

Let us consider a space X structured with two topologies T^1, T^2 defined on X. The set X is structured by a quasi-topology when for each subset E of X ($E \in P(X)$) there is an O^1 open regarding T^1 and O^2 open regarding T^2 such that we have the relations expressed by (*) above, where F^1 is the least closed in T^1 containing E, and F^2 is the least closed in T^2 containing F^2 and E. The "strict interior" of E is O^2 and the "interior" of E is O^1; the "closure" of E is F^1 and the "large closure" of E is F^2. We deduce the "internal boundary" and the "external boundary" of E.

For specific topologies, it is possible to define a quasi-topological structure on a topological space. We give an example with the topology of open intervals on the real line.

An example of quasi-topology. Let 'n' a enter number; a part E is an interval /- (n+1), + (n+1)/, i.e. the set of all numbers located between the two numbers '- (n+1) and '+ (n+1)'; this interval, noted /n+1/, is neither open, nor closed). For this part /n+1/, we define the open intervals:

$O^2 =_{def}$] - n, + n [
$O^1 =_{def}$] - (n+1), + (n+1) [

and the close intervals:
$F^2 =_{def}$ [- (n+2), + (n+2)]
$F^1 =_{def}$ [- (n+1), + (n+1)]
The internal and external boundaries are:
IB of /n+ 1/ = [-(n+1),-n] ∪ [+n, +(n+1)]
EB of /n+1/ = [-(n+2), -(n+1)] ∪ [+(n+1), +(n+2)]
(see the figure 6).

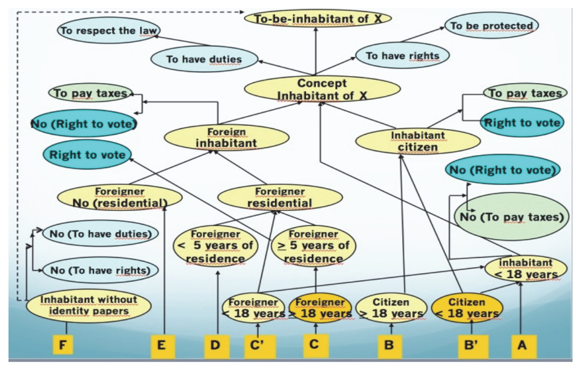

Figure 6.

Quasi-topology on completely determinate instances of a concept

In the framework of LDO and LTA (Desclés, Pascu, Jouis, 2013), from a given concept ^f = < f , Ess [^f] , Int [^f] >, we have used the following definitions and notations :

- Ext (Int [^f]) is the extension of all objects inheriting all the properties of intension Int[^f] (and, of course, all properties of Ess[^f]); all typical completely determinate instances of the concept '^f' belong to this extension, designated by Ext_{τ} [^f].

- Ext (^f) is the extension of all completely determinate objects which are typical or atypical instances of the concept ^f. All objects of this extension inherit necessarily all properties of the essence and also other properties not belonging to the essence but to the intension.

- Ext (Ess[^f]) is the extension of all completely determinate objects which inherit only all properties of the essence Ess [^f].

- Cl(Ext(Ess (^f))) is the closure of Ext(Ess^f)), that is the set of all completely determinate instances which do not inherit necessarily all properties of the essence.

- Ext(f) is the extension of all completely determinate objects falling only under the property f, that is the objects such that : f(x) = T.

A quasi-topology structure is defined on the different extensions of a concept '^f'. The "strict interior" of the extension Ext(Ess[^f]) of the essence Ess(^f) is Ext(Int[^f]), designated by Ext[^f] which is the extension of all typical instances of the concept '^f'; the "simple interior" is the extension Ext(^f) of the concept '^f'. The set Cl(Ext(Ess (^f))) is the closure of extension Ext(Ess[^f]) of the essence, and the "large closure" is the extension Ext (f) of the property 'f'. These different extensions are such that we have the following relations, as the relations given above by (*):

$$\text{Ext(f)} \supset \text{Cl(Ext(Ess (\^{}f)))} \supseteq \text{Ext(Ess(\^{}f))} \supseteq \text{Ext(\^{}f)} \supset \text{Ext(Int[\^{}f])}.$$

In the relations between the different extensions induced by a concept '^f', the extension Ext(f) of the property 'f' is the total space and is equal to its closure : Cl(Ext (f)) = Ext (f). The exceptions of the concept '^f' are located in the difference 'Ext(f) – Ext(Ess(^f))', that is the objects which fall under the property 'f' and which do not inherit some properties of the essence. The internal boundary is the difference 'Ext(Ess(^f)) – Ext(Int(^f))', that is the extension that is made all atypical insta nces of '^f'. .

A good example of quasi-topology is given by the analysis of the meaning of the notion "inhabitant of a city". A typical inhabitant is an inhabitant who has citizenship and who is more than 18 years old. This person has all the properties concerning rights and duties. An atypical inhabitant is, for example, an inhabitant who is less than 18 years old and who does not have the right to vote. The inhabitant without identity papers is just an inhabitant of a city but he has none of the above properties. He is an exception among the inhabitants of a city since he falls under no property of the network of properties, except the property "to be an inhabitant of the city". A "homeless inhabitant" falls also under this property 'f' but he inherit no property from the essence (he has no address) but he may have some properties from the intension. He is also an exception.

Conclusions

In this paper, we proposed the quasi-topology structure to organize the structure of the different extensions associated to a concept. From the conceptual point of view, quasi-topology refines the notion of topological boundary stated by the classical general topology, with two types of boundaries, the internal boundary and the external boundary. Quasi-topology structure can carry out finer categorizations in most contexts. We have applied this structure in the frameworks of LDO and LTA, giving a topological interpretation of the objects which are typical, atypical instances of a concept and exceptions and distinguishing explicitly a property and a concept constructed from a property given where the essence and intension of the concept are fixed.

We state that the quasi-topology structure is very important in the field of artificial intelligence, because with this structure it becomes possible to give a mathematical approach that can be applied in many fields of categorization where attributes of studied objects are not "precise" and are dependant of points of view, involving sometimes contradictions. The "imprecision" is formalized by the acceptance of internal boundaries and external boundaries of a category.

The examples presented in this paper show the fact that the notion of quasi-topology deserves to be studied and applied, especially in the theory of concepts and associated objects including typical and atypical representatives of the "common cognition" on one hand, and the representation of movements in the space, or changes of states taken by an object during an interval of time, or some specific type of cognition as "qualities" in the laws field – as legal / illegal –, on other hand. We claim that, in the semantic analysis of language, quasi-topology has an interpretative force.

As for its mathematical scope of quasi-topological structure, it remains a work to be done in order to establish its scope for establish a clear comparison with other models as rough sets or different types of topologies.

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