

Navigation, Cognitive Spatial Models, and the Mind

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Abstract

Because navigation produces readily observable actions, it provides an important window into how perception and reasoning support intelligent behavior. This paper summarizes recent results in navigation from the perspectives of cognitive neuroscience, cognitive psychology, and cognitive robotics. Together they argue for the significance of a learned spatial cognitive model. The feasibility of such a model for navigation is demonstrated, and important issues raised for a standard model of the mind.

Introduction

An embodied agent in the real world experiences a barrage of *percepts*, a continuous, multimodal stream of data. The agent's response to those percepts arises in the *mind*, the processes that determine intelligent behavior. While a model of the mind should not be merely a collection of task-specific behaviors, its application to a well-chosen task domain often highlights important issues. The premise of this paper is that *navigation*, the ability to move to an intended target, is such a task domain, for several reasons. Skilled navigation by an embodied agent employs both percepts and mental processes, including learning and goal-directed decision making. Navigation also supports more complex tasks, such as social interaction and self-defense. Moreover, navigation produces observable behavior that allows evaluation of the model itself.

A *standard model* of the mind is a computational entity “whose structures and processes are substantially similar to those found in human cognition” (Laird, Lebiere and Rosenbloom To appear). In science, such an overarching theory is best developed and validated through multiple lines of evidence. This paper therefore considers three perspectives on navigation: cognitive neuroscience, cognitive psychology, and cognitive robotics.

Because the neural system of an animate agent determines its behavior, one way to develop a standard model of the mind is to investigate it physiologically. *Cognitive neuroscience* studies the activity of *wetware*: individual neurons, ensembles of neurons, and brain areas. In particular,

recent work addresses observed physical responses in wetware before, during, and after navigation. The most accurate measurements on the neuronal level are invasive, that is, they insert instruments into the brain to record the responses of a live agent as it makes decisions. As a result, most experiments have been done on non-human animals. Nonetheless, this research has enlightened and inspired work with humans and artificial agents, and often foreshadowed its results. Thus, it is relevant here.

Because brain activity gives rise to the processes that determine thought, another way to develop a standard model of the mind is to study mental processes. As if a human mind were a black box, *cognitive psychology* studies memory and thought with cleverly designed experiments intended to understand how people experience their environment. Recent empirical research with human navigators verifies that they learn and exploit a *cognitive spatial model*, a summary that describes and explains cognitive processes that result from travel through the environment.

The third way to develop a standard model of the mind is to construct an embodied artificial agent that behaves the way a person would in similar circumstances. Indeed, autonomous robot navigators now confront many of the same challenges that human navigators do: the world is partially observable and changes in unanticipated ways, elapsed time and distance traveled may be conflicting criteria, and other mobile agents may be present. *Cognitive robotics* studies how a robot controller can learn and reason in such a complex world. In particular, recent work with a cognitive architecture demonstrates that navigation is feasible without a metric map, if the robot learns a cognitive spatial model.

This paper focuses on the role of a cognitive spatial model in navigation. The next section describes related work on navigation from the perspectives of cognitive neuroscience, cognitive psychology, and robotics. Subsequent sections describe how one cognitive architecture supports decision making for autonomous robot navigation, and the learned cognitive spatial model that improves its performance. The final section integrates these perspectives and

relates them to a standard model of the mind (Laird et al. To appear).

Perspectives on Navigation

Navigation and Cognitive Neuroscience

Significant discoveries on the neuronal level about how the mind navigates often result from invasive experiments with rodents. *Head direction cells* in the subiculum orient an animal globally, like a compass (Ranck 1985). *Place cells* in the hippocampus constantly revise their firing patterns to fit the animal's current location and its experience, (O'Keefe and Speakman 1987). An ensemble of place cells represents a specific location, and differentiates contexts hierarchically, with finer granularity (Smith and Mizumori 2006). *Grid cells* form a hexagonal network of neurons in the medial entorhinal cortex. They fire without visual input and, unlike place cells, provide an internal metric coordinate system (Langston et al. 2010). Grid cells connect to head direction cells, place cells, and border cells (which identify the edges of a closed environment).

Although this wetware was originally detected in rats and mice, similar cells have since been found in humans (Ekstrom et al. 2003). Moreover, there is evidence that head direction, place, and grid cells are sequentially activated offline, when the animal is quietly awake or asleep. The same neural firing sequences recorded during navigation repeat (*replay*), probably to consolidate memory (Wilson and McNaughton 1994). Such sequences, including some never experienced by the animal, also repeat before an animal begins to explore a new environment (*pre-play*). Neuroscientists suspect that such offline sequential activity is related to the animal's ability to represent space, to learn, and to plan (Buhry, Azizi and Cheng 2011).

Navigation and Cognitive Psychology

Cognitive psychology generalizes over human subjects to provide insights into the mind. For example, striking regularities appear in how pedestrians outdoors understand distance and direction, perceive proximity as dependent on context, and view direction as closely related to geometry (Worboys, Duckham and Kulik 2004). Moreover, research in cognitive psychology and cognitive neuroscience mutually inspire one another.

When Tolman observed rats in his mazes, he noted that they appeared to learn a *cognitive map*, a mental representation of the environment (Tolman 1948). This directly opposed the behaviorists' prior dictum that complex skills emerge simply from sequences of sensory-motor responses. The 2014 Nobel prize for the discovery of place and grid cells, however, confirmed Tolman's premise (Kiehn and Forssberg 2014). Meanwhile, psychologists have used

virtual reality to explore a wide range of structures and content that could appear in human cognitive maps (Foo et al. 2005). These included routes (single designated paths), graphs (to record connectivity and support the construction of novel detours), *labeled graphs* (with metric labels for distances and angles), and *surveys* (precise metric maps in an allocentric coordinate system).

Another example of how animal and human navigation are related comes from a desert ant as it searches for food. The ant effectively counts its steps from its nest and uses the sun as a compass, along with a mental clock to compensate for the sun's movement. This produces a continuous, remarkably accurate representation of the nest's location relative to the ant as it travels (Grah, Wehner and Ronacher 2005; Muller and Wehner 1988; Wittlinger, Wehner and Wolf 2006). There is, it now appears, a perceptual analog in people. The somatosensory-motor system reports *podokinetic* data, information about changes at the surface or inside a person's body when she walks, but not when she is transported (e.g., by wheelchair). Recent results in virtual hedge mazes (Chrastil and Warren 2013; Chrastil and Warren 2014) found that people who were deprived of podokinetic data by wheelchair transport could not actively learn accurate metric knowledge. The researchers concluded that the cognitive map learned by a human navigator is a labeled graph, not a survey.

Navigation and Robotics

In AI, an *autonomous agent* repeatedly executes a sense-decide-act loop. As embodied agents, robots are subject to both *sensor error* (percepts that provide a partial, noisy version of the ground truth) and *actuator error* (imprecise execution of a command). Although robots perceive continuous space and their hardware allows a broad range of possible actions, most robot controllers discretize both space and their action set to make computation tractable.

A significant challenge in autonomous navigation is *localization*, the robot's perpetual need to determine precisely where it is in space. Many robots have sensors that support only two spatial dimensions. They describe the robot's state as a *pose* $\langle x, y, \theta \rangle$ in an allocentric coordinate system, where (x, y) is the robot's *location* and θ is its *orientation* with respect to the origin. Localization is difficult because different locations may provide similar percepts (e.g., facing a corner) and the same location may provide different percepts given different orientations.

A robot can localize by *odometry*, which calculates from an initial pose and subsequent movements where it ought to be, given the actions it has taken. Over time, however, accumulated actuator error makes localization by odometry less and less accurate. The state-of-the-art in localization is *SLAM* (Simultaneous Localization and Mapping), which both localizes and builds a metric map at once from the ro-

bot’s percepts as it travels (Bailey and Durrant-Whyte 2006; Durrant-Whyte and Bailey 2006; Thrun and Montemerlo 2006). Given the uncertainties inherent in localization, SLAM is probabilistic, that is, it provides only likelihoods for the robot’s location.

Manufacturers now routinely ship their robots with *ROS*, the open-source, highly modular Robot Operating System (Quigley et al. 2008). *ROS* provides common functionalities, including SLAM. When a robot references a SLAM-generated map, however, it contends with the errors present during the map’s construction as well as the errors in the robot’s current localization. In realistic worlds, particularly those with other mobile agents, the need to learn and plan now drives a new line of research, cognitive robotics.

FORR

FORR (FOR the Right Reasons) is a cognitive architecture for learning and problem solving (Epstein 1994). *FORR*’s right reasons are *Advisors*, simple procedures that express opinions (*comments*) on the next action to take. To determine an agent’s next action, a decision cycle moves through the three-tier hierarchy shown in Figure 1, where short-term memory holds the current state and long-term memory holds plans and learned world knowledge.

In tier 1, Advisors are reactive, rule-like, and preordered. If a tier-1 Advisor comments, it either selects an action and thereby ends the decision cycle, or it eliminates one or more actions from consideration. Tier 1 Advisors mandate actions that satisfy the current goal immediately (e.g., go to a perceived target), avoid negative outcomes (e.g., do not hit walls), and eliminate actions inconsistent with any current plan. If only one action remains, it is executed.

Tier-2 Advisors are planners. In *FORR*, a plan need not be a complete, fixed action sequence determined in advance. It may be partial (e.g., “go forward 3m and turn

left”) and may expire or be abandoned. If there is no current plan, tier 2 constructs one, stores it, and ends the decision cycle. Otherwise, the actions that survived tier 1 are forwarded to tier 3.

Tier 3 combines the opinions of its heuristic Advisors on the remaining actions. To comment on action j , Advisor i assigns it a strength $s_{ij} \in \mathcal{R}$. To resolve disagreement and capitalize on the synergy among tier-3 Advisors, *voting* selects an action with maximum total comment strength $\text{argmax}_j \sum_i w_i s_{ij}$ where w_i is Advisor i ’s learned weight. Because tier 1 only forwards actions that support the current plan, tier 3’s decision always complies with it.

FORR shares many properties with the proposed standard model of the mind. It has been applied to multiple task domains, including game playing (Epstein 2001), constraint satisfaction (Epstein, Freuder and Wallace 2005), and human-machine dialogue (Epstein et al. 2012). Resource limits on its Advisors make it boundedly rational. *FORR* is task-independent, and its Advisors and learning mechanisms can operate in parallel. Its decision cycle is rapid; its long-term memory holds both declarative and procedural world knowledge, including synopses of its experience and metadata. In addition, *FORR*’s learning is online, incremental, and based on experience.

SemaFORR

SemaFORR is a platform-independent, *FORR*-based system for autonomous robot navigation (Epstein et al. 2015). Its long-term memory includes learned weights for tier-3 Advisors and a learned, cognitive spatial model. *SemaFORR* is implemented as a set of modules in *ROS*. It runs both in simulation and on real-world robots.

Short-term memory holds *SemaFORR*’s current state: the robot’s pose, the location of the current target, and the discretized action set. *SemaFORR*’s tier-1 Advisors direct the robot toward a perceived target, eliminate actions that would cause a collision, and ensure that the remaining actions support the current plan. Its tier-2 Advisors construct plans. Some of its tier-3 Advisors express deliberately disparate, narrowly-focused heuristics that represent navigational common sense and rely only on local perception. Individually they support proximity to the target, long steps, room to move about, circumnavigation of local obstacles, and movement to less familiar locations. The remainder of *SemaFORR*’s tier-3 Advisors construct their comments based on the learned cognitive spatial model described in the next section.

SemaFORR requires no map. It is fully compatible with SLAM and has made decisions for a variety of modern robots. Perceptual input varies with the platform (e.g., Fetch Robotics’ Freight has 660 laser scans at 15 Hz with a 25-meter range, distributed in a 220° arc). Independent copies

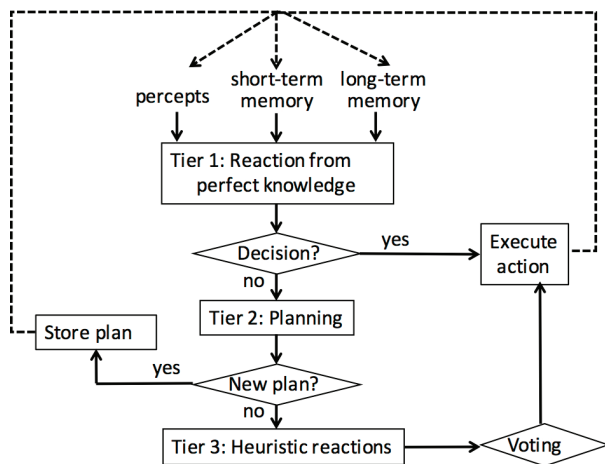


Figure 1: The FORR decision cycle

of SemaFORR have controlled eight robots as they pursued individual targets at once in the same space. In simulation, SemaFORR learns to navigate effectively in a complex office world as large as a city block. On a 1.2 GHz workstation, decision time there averaged 26.35 ms, including time to learn the spatial model. Even in a trade show world with many internal stationary obstructions, SemaFORR navigates successfully through a realistically simulated crowd of 1000 pedestrians (Aroor and Epstein 2017). In extensive testing, SemaFORR also compares well to an A* planner, but only when it learns and uses the cognitive spatial model described next (Epstein et al. 2015).

A Cognitive Spatial Model

Robots treat a map as a compendium of potential collisions, but people treat maps as a collection of opportunities for movement. The premise behind SemaFORR’s learned model is that the significant spatial features (*spatial affordances*) of a world facilitate, rather than obstruct, movement through it. SemaFORR’s cognitive spatial model is a set of spatial affordances that form a high-level description of the robot’s experience. These affordances are calculated from the robot’s percepts and revised each time it reaches a target. There are three basic spatial affordances: trails, conveyors, and regions.

A *trail* is a revision of the robot’s logged path to a target, as in Figure 2(a). The learning algorithm revises the path backward from the target, to smooth it and eliminate extraneous cycles. The result is a sequence of *trail markers*, visited locations with their sensor readings. A trail is typically suboptimal, but shorter and more direct than its original path, and requires less memory. In the worst case, the learning algorithm is quadratic in the number of decision points along the path. A tier-3 Advisor supports movement along a trail that leads to the vicinity of the target. In this way, trails are treated not as plans but as consolidated experience that supports reactive decisions.

In complex navigation environments, frequently visited zones often suggest useful connectivity (e.g., key intersections). SemaFORR superimposes a grid on the world, and

tallies the frequency with which a trail passes through each of its cells. A *conveyor* is a high-count cell in the grid. Conveyors serve as attractors to another tier-3 Advisor.

A *region* represents open space as a circle whose center is a location where the robot executed a decision cycle and whose radius is the smallest range value reported there by its sensors. Regions are distinct, and may grow and shrink with experience. Figure 2(b) shows the trails, regions, and conveyors in a learned cognitive model of a simple world.

Trails, conveyors, and regions not only summarize the robot’s experience, but also serve as building blocks for more powerful abstractions, including doors and a skeleton. Wherever a path crosses the perimeter of a region is an *exit*. With experience, regions stabilize in size and exits accumulate as clusters of discrete points. SemaFORR generalizes a cluster of exits into a continuous arc (a *door*). Doors provide a broader range of acceptable action to tier-3 Advisors that seek to enter or leave a region.

SemaFORR also learns a *skeleton* for a world, a graph whose nodes are regions, as in Figure 2(c). An edge in the skeleton indicates that a robot once moved between that pair of regions. A *leaf* is a region of degree one in the skeleton. Yet another tier-3 Advisor treats leaves as dead-ends to avoid unless the target lies there. (Other Advisors ensure that the robot reaches a target near the corner of a room, even if the region inscribed within the room excludes it.)

While full details on the nature of human cognitive maps are as yet incomplete, SemaFORR’s spatial model resembles it in several ways. The model consolidates experience (e.g., it engages in replay to learn a trail). The model is approximate, but has metric labels and clear relationships (e.g., sequences of trail markers, the skeleton) (Chrastil and Warren 2014). Moreover, the model’s continuing experience-based revisions (e.g., changes in regions) mirrors processes observed by neuroscientists in the human brain (Chadwick et al. 2015; Howard et al. 2014). Regions depict open space accessible through doors, a skeleton depicts global connectivity, and trails and conveyors consolidate experience into reusable chunks. The result, given the metrics on SemaFORR’s cognitive spatial model, is a useful and evolving graph labeled with some metric data.

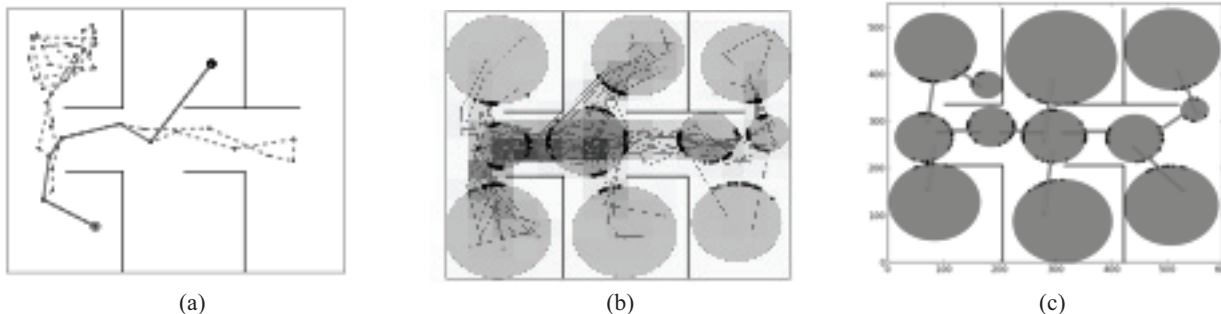


Figure 2: Without a map of the walls (a) dashed path from the lower left, and its (solid) derived trail (b) spatial model learned from navigation to 20 targets. Trail markers are dotted lines, conveyors are grid cells with darker shading, and regions are circles (c) skeleton with exits

Discussion

Cognitive neuroscience, cognitive psychology, and robotics address the physiology, processing, and engineering that underlie navigation. SemaFORR demonstrates the power autonomous robot navigation can derive from a cognitive spatial model. The opinions expressed in this section apply the material discussed thus far to further the development of a standard model of the mind.

Animals have much to contribute to a standard model of the mind that, we believe, is now overly anthropocentric. While the intent of the standard model is to simulate, not emulate, the mind, wetware results should not be ignored. The 2014 Nobel prize made clear that any brain that induces and supports a mind provides important clues to the nature of human-like thought. Wetware results from invasive experiments with animals inspire and inform subsequent, non-invasive experiments with people and, in turn, the development of artificial agents.

A standard model of the mind should include an explicit, learned, flexible, long-term model of the world in which that mind is embodied. Despite a continuous stream of percepts, physical and mental processes produce discrete, approximate world models. Such a model supports intelligent behavior and is subject to continuous revision, as learning compacts data for storage in long-term memory. SemaFORR's model demonstrates that, even for navigation, such a model can be purely symbolic. SemaFORR's successful navigation without a metric map argues that, in a dynamic, partially observable world, a learned, approximate cognitive spatial model is an appropriate, less demanding, more pragmatic, more flexible knowledge store.

A standard model of the mind should accommodate and integrate possibly conflicting sensory information along multiple modalities, and provide for multiple representations. Navigation in other animals often relies on simultaneous multimodal percepts: podokinetic data, sound, vision, olfaction, taste, and touch (Caprio et al. 2014). In people, virtual-reality experiments with wheelchair transport demonstrate that people rely on podokinetic data as well as vision. Moreover, recent work suggests that the human brain represents a person's intended goal direction separately from head direction (Chadwick et al. 2015). The human guidance system also changes the way it represents the distance to a goal based on how difficult it is to navigate there (Howard et al. 2014).

A standard model of the mind should support multiple criteria for its overall behavior. Reasoning over a world model applies a decision policy to select an action. While that policy could optimize a single criterion, the mind employs multiple decision criteria simultaneously (Ratterman and Epstein 1995). Navigation as a task domain highlights this. For example, whether or not to take the highway may depend not only on travel time, but also on scenic beauty,

tolerance for delay, access to renewable resources (e.g., fuel, food), and impact on hardware (e.g., distance, surface quality). Other criteria include cognitive load and the opportunity to refine the cognitive model itself.

A standard model of the mind should include curiosity. This reflects the well-known tradeoff in AI between exploration to acquire new knowledge and exploitation that applies it. A FORR-based agent typically has a tier-3 Advisor that deliberately seeks novel experiences (e.g., SemaFORR's EXPLORER). This is reactive exploration. Deliberate digression from a plan, or planners that intentionally digress, are alternative ways to incorporate curiosity.

Reactive planning may be the best way for a standard model of the mind to plan. Planning reuses long-term memory to address a new task. This is sensible in a fully observable, static environment with a single agent. For example, people regularly plan when they navigate (Torrens et al. 2012). Despite SemaFORR's successes with an A* planner in tier 2, however, a robot in a crowd of people who pursue their own targets travels more effectively and safely with a reactive planner that learns (Aroor and Epstein 2017).

A standard model of the mind should be able to provide human-friendly, natural-language explanations of its reasoning. WHY, a recent addition to FORR, is a first step (Korpan et al. under review). In less than 3 ms, WHY provides rich, nuanced statements about why a system chose an action, why an alternative action was less acceptable, and how confident it is in its decision. WHY's responses are based on the world model and its Advisors' rationales. WHY's output about navigation in a large, complex world has included: "I decided to take a hard left because I want to go far and I really want to stay away from that wall" and "The target isn't in this area, so I want to get out of here."

SemaFORR remains in development. It provides an example of how a mind might navigate in real time in large, complex worlds without a map. SemaFORR capitalizes on a synergy among multiple heuristic decision-making principles organized by reliability and high-level, readily replaced plans, but there may be other equally valid approaches. Work is needed on landmarks and on the automatic discovery of categories of higher-level structures (e.g., doors and skeletons).

In summary, this paper contends that navigation has significant import for a standard model of the mind. Because the conversion of percepts to mental experience is an approximation, and the consolidation of that experience is inductive generalization with its own errors, the mind necessarily reasons from noisy data. In a partially observable, dynamic world, human minds entertain multiple goal criteria simultaneously. Evolution has selected a changing, approximate world model as a pragmatic way to address such problems. Navigation makes these ideas explicit.

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