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Formalism for Treatment of the Ambiguity in Front/Back Axis Expressions

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Abstract

In this paper we present a logical formalism for the treatment of pragmatic ambiguity in spatial expressions of the frontal axis (front/back). The ambiguity occurs because the same situation can be analyzed from different points of view. For this, we use frames of reference for the interpretation of front/back (intrinsic, extrinsic, deictic) together with formalisms of Qualitative Spatial Reasoning.

Introduction

In this paper, we will present a formalism for the treatment of the ambiguity in expressions of the lateral and frontal axes. These expressions have different ways of interpreting: for example when we tell someone that "an object X is on the left of another object Y", the listener could interpret this sentence in three ways:

a. on my left;

b. on your left (referring to the speaker);

c. the left side of Y.

The Figure 1 illustrates the example above. Thus, a spatial expression of the lateral or frontal axes (*projective expressions*) can be affected by pragmatic ambiguity. This arises because the *orientation relations* (that describe where objects are located relative to one another) are derived from the *mental models* and *frames of reference* that are used to determine the front side of an object. In the next Section, we address mental models and frames of references according to the works of Clementini, Di Felice, and Hernandez, Herskovits, Retz-Schmidt, Teixeira, and others authors.

Our purpose in this article is to use tools of Qualitative Spatial Reasoning together with a non-monotonic logical formalism for the development of our framework, that will be used in a multi-agent robotic system. The objective of this system is the verbal communication between humans and robots in stressful environments. The basis of our formalism is the Cardinal Directions Calculus presented in (Frank 1991). This calculus divides the area adjacent to the region occupied by an object in Euclidean space in nine tiles, as shown in Figure 2.

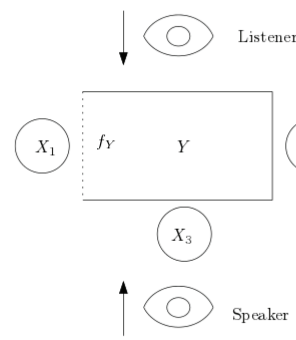

Figure 1: The possible locations of the object X relative to Y. The dotted line is the front (f_Y) of Y, and the arrows are the viewing directions.

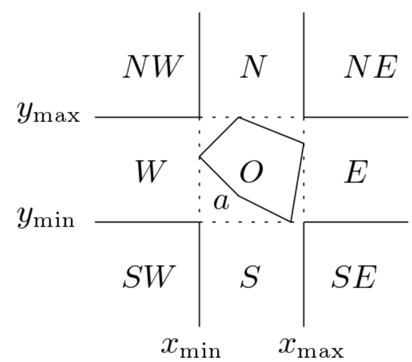

Figure 2: The tiles are defined based on the *minimal bounding box* (mmb) of a region. (The dotted line is the mmb of the region a). The tiles' name are the cardinal points: North (N), North-East (NE), North-West (NW), West (W), East (E), South (S), South-East (SE), and South-West (SW). The central tile is called O.

Frames of reference

According to Retz-Schmidt, the frame of reference for orientation determines the front side of an object. There are three basic types of reference frames: *intrinsic*, *extrinsic*, and *deictic*. Their definitions are given below, where the *Figure* is the object to be located, and the object in relation to which the Figure is located we called the *Ground*.

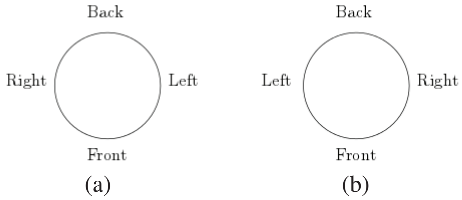

Figure 3: In (a), the Ground seen from inside. In (b), the Ground seen from outside.Adapted from (Retz-Schmidt 1988; Herskovits 1987).

- *Intrinsic*: in intrinsic perspective, the front, back, left, and right regions around the Ground object are determined by the type and/or properties of the Ground object itself. Retz-Schmidt suggests typical criteria for identification of the intrinsic front: the side situated in the direction of motion, the side containing the perceptual apparatus, or the side characteristically oriented to the observer. Once the intrinsic front of the Ground is determined, the other sides can be assigned in two ways (Figure 3): the Ground can be seen from the outside (buildings, for example) or inside (couch, for instance).
- *Extrinsic*: when an object does not have an intrinsic front, it can receive a *provisional front*. Vandeloise says that a not intrinsically oriented object can acquire a provisional front through other objects in its vicinity. Wunderlich argues that the provisional front depends on the accessibility: the front of an extrinsic object is the side that is accessible the soonest. Teixeira cites a mental model of the front attribution called mirroring. In this model, an object acquires a provisional front through of a mirroring process (Figure 4 (a)). Teixeira, based on cognitive experiments, says that a speaker tends to identify the left side as the back, and the right side as the front side. This is because of the discursive organization (from left to right).
- *Deictic*: when the orientation is given by the point of view from which the Ground object is seen by the person describing the situation. Retz-Schmidt claims that in deictic use, the reference frame can be oriented in two ways. One way is to have the front facing the point of view (mirroring). In a second, the Ground can see in the same direction as the speaker. Thus, the back is facing the point of view, and the front is on the far side of the Ground (this is referred as to *Tandem Principle*). (Retz-Schmidt 1988; Wunderlich 1985). The assignment of left and right regions to the Ground object is the same in both ways (See Figure 4).

  In the visibility mental model, the presence or absence of an accessibility/visibility trait can alter the perception of front/back and may cancel up to the intrinsic orientation of an object. (Teixeira 2001). In Figure 5 (a), the dresser has an intrinsic front, but in Figure 5 (b), under the cat's point of view, we can say "the mouse is behind the dresser".

Retz-Schmidt lists some papers dealing with the pragmatic ambiguity between reference frames: ambiguities between intrinsic and deictic use, where the point of view is omitted or it is intrinsic; among deictic uses (speaker or listener as the point of view); among intrinsic uses (different criteria in determining the intrinsic front); between intrinsic and extrinsic use (an object with intrinsic front can acquire a provisional front); among extrinsic uses (different extrinsic orientations).

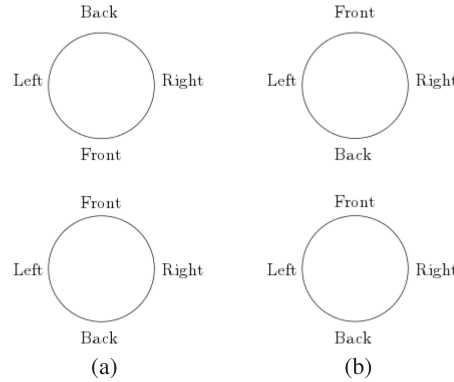

Figure 4: Mirroring process (a). Tandem Principle (b).(Retz-Schmidt 1988).

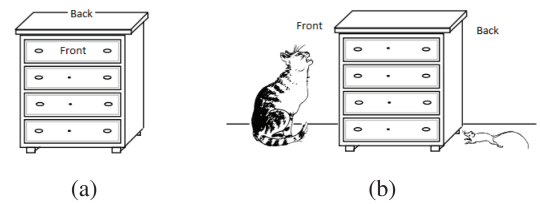

Figure 5: Intrinsic orientation (a). Visibility model (b). (Teixeira 2001).

The objective of our formalism is not to remove the ambiguity but instead consider all possible interpretations. Thus, when the formalism compares spatial descriptions with the arrangement of objects in the scenario, it will be able to group this objects according to the description and, through heuristics, to suggest the best case.

Frank (1991) proposed the use of a *Projection-based Model* for Cardinal and Ordinal directions. A two-dimensional Euclidean space containing an arbitrary single-piece region *a* is partitioned into nine tiles, as shown in Figure 2 above. For $\mathsf{T} \in \{\mathsf{N}, \mathsf{S}, \mathsf{W}, \mathsf{E}, \mathsf{NE}, \mathsf{NW}, \mathsf{SW}, \mathsf{SE}, \mathsf{O}\}$, $\mathsf{T}(a)$ refers to the tile T defined with respect to region *a*. Kor and Bennett proposed that the nine tiles were collapsed into six sets as shown in the Table 1, in a study called Horizontal and Vertical Constraints Model (HVCM).

And, in (Goyal and Egenhofer 2000) is presented a hybrid mereological, topological and cardinal direction relation model (HTCM), which provides the definitions of *whole* and *part* destination regions. Later, Kor and Bennett improved these definitions using RCC-5 JPED topological relations for regions (Cohn et al. 1997): $\mathsf{PP}(x,y)$ (x is a proper part of y), $\mathsf{PPi}(x,y)$ (y is a proper part of x), $\mathsf{EQ}(x,y)$ (x is identical with y), $\mathsf{PO}(x,y)$ (x partially overlaps y), and

WeakNorth: $\mathsf{WN}(a) \equiv \mathsf{NW}(a) \cup \mathsf{N}(a) \cup \mathsf{NE}(a)$
WeakWest: $\mathsf{WW}(a) \equiv \mathsf{SW}(a) \cup \mathsf{W}(a) \cup \mathsf{NW}(a)$
WeakSouth: $\mathsf{WS}(a) \equiv \mathsf{SW}(a) \cup \mathsf{S}(a) \cup \mathsf{SE}(a)$
WeakEast: $\mathsf{WE}(a) \equiv \mathsf{SE}(a) \cup \mathsf{E}(a) \cup \mathsf{NE}(a)$
Horizontal: $\mathsf{H}(a) \equiv \mathsf{W}(a) \cup \mathsf{O}(a) \cup \mathsf{E}(a)$
Vertical: $\mathsf{V}(a) \equiv \mathsf{S}(a) \cup \mathsf{O}(a) \cup \mathsf{N}(a)$

Table 1: Definition for HVCM.

$\mathsf{DR}(x,y)$ (x is discrete from y). Thus, the whole and part relations are defined as follow:

$$\omega_{\mathsf{T}}(b,a) \equiv \mathsf{PP}(b,\mathsf{T}(a)) \vee \mathsf{EQ}(b,\mathsf{T}(a)) \quad (1)$$

$$\rho_{\mathsf{T}}(b,a) \equiv \mathsf{PO}(b,\mathsf{T}(a)) \quad (2)$$

In Formula 1, the whole of b is in the T tile of a. In Formula 2, some part but not all of b is in the T tile of a. In the next section we shall present the formalism which is the main contribution of this article.

Formalism

Before presenting our formalism, we will show some necessary auxiliary definitions:

- $U = \{\mathsf{WN}(a), \mathsf{WS}(a), \mathsf{WW}(a), \mathsf{WE}(a)\}$ is the *set of weak tiles* of the region a defined in Table 1;
- $C = \{\mathsf{N}(a), \mathsf{S}(a), \mathsf{E}(a), \mathsf{W}(a)\}$ is the *set of cardinal tiles* of the region a;
- $E = \{\mathsf{NW}(a), \mathsf{NE}(a), \mathsf{SW}(a), \mathsf{SE}(a)\}$ is the *set of ordinal tiles* of the region a;
- $Z = U \cup C \cup E$;
- $\mathsf{mbb}(a)$ is the *minimal bounding box* of region a;
- $M = \{\mathsf{Northern}, \mathsf{Southern}, \mathsf{Western}, \mathsf{Eastern}\}$ is the *set of sides* of the $\mathsf{mbb}(a)$ defined, respectively, as the sides containing greater y-coordinate, lowest y-coordinate, lowest x-coordinate, and greater x-coordinate;
- $\theta : M \to U$ is the function that maps a side of $\mathsf{mbb}(a)$ to a weak tile. (Table 2);
- $\xi : M \to C$ is the function that maps a side of $\mathsf{mbb}(a)$ to cardinal tile. (Table 2);
- $\kappa : \{M \times M\} \to E$ is the function that maps a pair of sides of $\mathsf{mbb}(a)$ to ordinal tiles. (Table 3);
- $\alpha \subseteq M$ is the set of sides of the $\mathsf{mbb}(a)$ which share at least one point with front side of a. (Note that α contains 1 or 2 elements);
- $\tilde{\alpha} \subseteq M$ is the set of sides of the $\mathsf{mbb}(a)$ that are opposite in α. The symmetric relation of opposition ($\sim$) in M is given by $\mathsf{Northern} \sim \mathsf{Southern}$ and $\mathsf{Western} \sim \mathsf{Eastern}$. (Note that $\tilde{\alpha} \cap \alpha = \emptyset$);
- $\theta(\alpha) = \{\ldots, \theta(x_i)\}$ where $x_i \in \alpha$;
- $\phi(\alpha) = \{\ldots, \phi(x_i)\}$ where $x_i \in \alpha$ and $\phi = \xi$ if $|\phi(\alpha)| = 1$ or $\phi = \kappa$ otherwise;
- $d(b) \in M$ is the side of the $\mathsf{mbb}(a)$ that has the shortest distance from the $\mathsf{mbb}(b)$.

Side of $\mathsf{mbb}(a)$	θ	ξ
Northern	$\mathsf{WN}(a)$	$\mathsf{N}(a)$
Southern	$\mathsf{WS}(a)$	$\mathsf{S}(a)$
Western	$\mathsf{WW}(a)$	$\mathsf{W}(a)$
Eastern	$\mathsf{WE}(a)$	$\mathsf{E}(a)$

Table 2: Functions θ and ξ for the sides of $\mathsf{mbb}(a)$.

Pair of sides of $\mathsf{mbb}(a)$	κ
$\{\mathsf{Northern}, \mathsf{Eastern}\}$	$\mathsf{NE}(a)$
$\{\mathsf{Northern}, \mathsf{Western}\}$	$\mathsf{NW}(a)$
$\{\mathsf{Southern}, \mathsf{Western}\}$	$\mathsf{SW}(a)$
$\{\mathsf{Southern}, \mathsf{Eastern}\}$	$\mathsf{SE}(a)$

Table 3: Function κ.

A *projective structure* of a region a is a tuple $\langle a, Z, M, \mathcal{F}, \alpha, \mathcal{S}\rangle$, where:

- Z is the set of ordinal, cardinal and weak tiles of the a;
- M is the set of sides of the $\mathsf{mbb}(a)$ defined above;
- $\mathcal{F} = \{\theta, \xi, \kappa, \phi\}$ is the set of functions that map sides of $\mathsf{mbb}(a)$ to tiles in Z, defined in Tables 2,3;
- α, as defined above;
- $\mathcal{S}(b)$ is the function that provides the $d(b)$ side for each region b in the Euclidean space.

Thus, our formalism is specified in terms of an *orientation model* $\mathscr{M} = \langle D, \Sigma, W, R, \mathcal{P}, \Gamma\rangle$, where:

- D is a set of regions in Euclidean space. Each region represents an object in the scenario and is denoted by a lower case letter;
- Σ_a is the projective structure of the region a;
- W: ways of assigning front side (Tandem principle, for example) and left side (inside view, outside view). W is called set of *possible worlds*;
- $R = \{\mathsf{Front}(b,a), \mathsf{Back}(b,a), \mathsf{Left}(b,a), \mathsf{Right}(b,a)\}$ is the set of relationships (*predicates*) between two regions;
- $\mathcal{P} = \{\omega_{\mathsf{T}}(b,a), \rho_{\mathsf{T}}(b,a) \mid \mathsf{T} \in Z \cup \mathcal{F}\}$ as defined in Formulas 1,2.
- $\Gamma = \{\ldots, \Gamma_{w_i}, \ldots\}$ is the set of *predicate assignments*. The predicates in R are defined in relation to the formulas in $\mathcal{P}$. Thus, a predicate assignments $\gamma \in \Gamma_w$, in relation to a possible world $w \in W$, is a set of formulas with the following form

  $$R(b,a) \leftrightarrow \mathcal{P}_{\mathsf{T}}(b,a).$$

  Since each of these predicate assignment includes one definition of every predicate $R_i \in R$, Γ_w is the set of all predicate assignments γ defined for the possible world $w \in W$.

Interpretation function

Based on (Bennett 2011), we specify the interpretation function, $[\![R(b,a)]\!]^{w,\gamma}_{\mathscr{M}}$, that gives the denotation of a predicate R relative to a given model $\mathscr{M}$, a possible world $w \in W$ and a predicate assignment $\gamma \in \Gamma_w$. On the basis of the interpretation function, a semantic satisfaction relation can be defined by:

$$\mathscr{M}, \langle w, \gamma \rangle \Vdash R(b,a) \text{ iff } [\![R(b,a)]\!]^{w,\gamma}_{\mathscr{M}} = \mathbf{t}.$$

That is, the predicate R is true in model $\mathscr{M}$, at world w, with respect to predicate assignment γ. The *interpretation set* of a predicate relative to a model $\mathscr{M}$ is given by:

$$[\![R(b,a)]\!]_{\mathscr{M}} = \{\langle w, \gamma \rangle \mid (\mathscr{M}, \langle w, \gamma \rangle \Vdash R(b,a))\}.$$

This is the set of 2-tuple (*precisifications*) of the possible world/predicate assignment for which predicate R is evaluated as true. We say that R is true in the world w if R is true at least one predicate assignment $\gamma \in \Gamma_w$. So we can specify two operators with respect to a interpretation set $[\![R(b,a)]\!]_{\mathscr{M}}$:

- $\mathbb{C}_{\mathscr{M}} R(b,a)$ means that $[\![R(b,a)]\!]_{\mathscr{M}}$ contains all possible worlds W of the model $\mathscr{M}$. In other words, $R(b,a)$ is true in all possible worlds of $\mathscr{M}$. ($R(b,a)$ is *certainly* true).
- $\mathbb{P}_{\mathscr{M}} R(b,a)$ means that $[\![R(b,a)]\!]_{\mathscr{M}}$ is not empty, that is, it is true in at least one possible world of $\mathscr{M}$. ($R(b,a)$ is *possibly* true).

In the next section we will specify an orientation model for each of the frames of reference (intrinsic, extrinsic, and deictic).

Orientation models for frames of reference

In this section, we presented the orientation models for the reference frames. We consider that the scenario is like a photograph taken by the observer.

Intrinsic

In the intrinsic frame, the front of the Ground is already oriented. Therefore, the possible worlds in the intrinsic orientation model (denoted by $\mathscr{I}$) are those of assignment of left/right (inside view, outside view), according to Figure 3. Thus, the predicate assignments for the model $\mathscr{I}$ are given in Table 4, where w_1 is the inside view and w_2 is the outside view. Since the front of the Ground is determined beforehand, the Front/Back assignments are equal on both worlds and defined according to the front (α) and back ($\tilde{\alpha}$) sides of the $\mathsf{mbb}(a)$.

Relations	Inside		Outside	
	Strong	Weak	Strong	Weak
Front	$\omega_{\phi(\alpha)}$	$\rho_{\theta(\alpha)}$	$\omega_{\phi(\alpha)}$	$\rho_{\theta(\alpha)}$
Back	$\omega_{\phi(\tilde{\alpha})}$	$\rho_{\theta(\tilde{\alpha})}$	$\omega_{\phi(\tilde{\alpha})}$	$\rho_{\theta(\tilde{\alpha})}$
Left	ω_{E}	ρ_{WE}	ω_{W}	ρ_{WW}
Right	ω_{W}	ρ_{WW}	ω_{E}	ρ_{WE}

Table 4: Predicate assignments Γ_W for the model $\mathscr{I}$. The regions a, b are implied.

Extrinsic

In the extrinsic frame, the Ground receives a provisional front. Thus, the possible worlds in the extrinsic orientation model (denoted by $\mathscr{E}$) are the provisional fronts assignments: side that is accessible the soonest by observer (w_1); side obtained by mirroring process (w_2) with respect to another object; through of the discursive organization - left to right (w_3). The assignment of Left/Right is the same in all worlds. The Tables 5 and 6 show the predicate assignments Γ_W for the extrinsic model $\mathscr{E}$.

Relations	Soonest		Mirroring	
	Strong	Weak	Strong	Weak
Front	ω_{S}	ρ_{WS}	$\omega_{\xi(\mathcal{S}(b))}$	$\rho_{\theta(\mathcal{S}(b))}$
Back	ω_{N}	ρ_{WN}	-	-
Left	ω_{W}	ρ_{WW}	ω_{W}	ρ_{WW}
Right	ω_{E}	ρ_{WE}	ω_{E}	ρ_{WE}

Table 5: Predicate assignments Γ_W for the model $\mathscr{E}$ (the regions a, b are implied). Note that the function $\mathcal{S}(b)$ returns the side of the $\mathsf{mbb}(a)$ closest to $\mathsf{mbb}(b)$. In addition, the predicate Back is undetermined in w_2 because in the mirroring process the objects will be either next to each other (left or right) or facing each other.

Relations	Discursive	
	Strong	Weak
Front	ω_{E}	ρ_{WE}
Back	ω_{W}	ρ_{WW}
Left	ω_{N}	ρ_{WN}
Right	ω_{S}	ρ_{WS}

Table 6: Predicate assignments Γ_W for the model $\mathscr{E}$ (cont.).

Deictic

In deictic reference frame ($\mathscr{D}$), the objects are seen from the observer's point of view. Thus, in this model, there are two ways of assigning the front side to an object: Tandem principle (w_1) and mirroring process in relation to the observer (w_2). This frame is used even when the object has an intrinsic front.

Relations	Tandem		Mirroring	
	Strong	Weak	Strong	Weak
Front	ω_{N}	ρ_{WN}	ω_{S}	ρ_{WS}
Back	ω_{S}	ρ_{WS}	ω_{N}	ρ_{WN}
Left	ω_{W}	ρ_{WW}	ω_{W}	ρ_{WW}
Right	ω_{E}	ρ_{WE}	ω_{E}	ρ_{WE}

Table 7: Predicate assignments Γ_W for the model $\mathscr{D}$.

Visibility

We extend our formalism to relations between three regions. Thus, we have defined the predicate $\mathsf{Between}(c,b,a)$ and the visibility/accessibility function $\nu(c,b,a)$. This function indicates that the region c does not prevent the visibility or accessibility of b from the point of view of a, whereas the its negation ($\neg\nu$) states that b is invisible or inaccessible from the point of view of a. The definition of Between is given below:

$$\mathsf{Between}(c,b,a) \equiv \rho_{\mathsf{N}}(c,b) \wedge \rho_{\mathsf{S}}(c,a) \vee \rho_{\mathsf{S}}(c,b) \wedge \rho_{\mathsf{N}}(c,a) \vee \rho_{\mathsf{W}}(c,b) \wedge \rho_{\mathsf{E}}(c,a) \vee \rho_{\mathsf{E}}(c,b) \wedge \rho_{\mathsf{W}}(c,a). \tag{3}$$

The relation $\mathsf{Between}(c,b,a)$ can be interpreted as "c is between a and b." The function ν affects the way of analysing the relations between a,b and c, as shown in the Equations below:

$$\mathsf{Between}(c,b,a) \wedge \nu(c,b,a) \rightarrow [\![R(b,a)]\!]_{\mathscr{M}} \tag{4}$$

$$\mathsf{Between}(c,b,a) \wedge \neg\nu(c,b,a) \rightarrow [\![R(c,a)]\!]_{\mathscr{M}} \tag{5}$$

In the Equation 4, the presence of c not preventing visibility between a and b means that the predicates R can be applied for a and b in any orientation model $\mathscr{M}$, whereas in the Equation 5 we cannot relate a e b because c is obstructing the vision/access between them.

The Beside relation

Finally, we will include the relation $\mathsf{Beside}(b,a)$ in the set of predicates ($R' = R \cup \{\mathsf{Beside}\}$) . This relation refers the situation where all predicates of R are false in all 2-tuple worlds/predicate assignments. We choose to leave this relation at the end as it has the same definition for all worlds in all orientation models and can be defined as follows:

- $\mathsf{Beside}(b,a) \leftrightarrow \forall R_i \in R([\![R_i(b,a)]\!]_{\mathscr{M}} = \emptyset)$.

Examples

In this section, we will discuss the example given in Figure 6 (we adopted the side closest to the observer as the south side).

Let D be the set of regions of the Figure 6 and $a \in D$, the following sets are defined:

$$\mathbb{C}_{\mathscr{M}}R_a = \{b \in D \mid b \neq a, \mathbb{C}_{\mathscr{M}}R(b,a) = \mathbf{t}, \nexists c[\mathsf{Between}(c,b,a) \wedge \neg\nu(c,b,a)], c \in D\}. \tag{6}$$

$$\mathbb{P}_{\mathscr{M}}R_a = \{b \in D \mid b \neq a, \mathbb{P}_{\mathscr{M}}R(b,a) = \mathbf{t}, \nexists c[\mathsf{Between}(c,b,a) \wedge \neg\nu(c,b,a)], c \in D\}. \tag{7}$$

$$\mathbb{E}_{\mathscr{M}}R_a = \{b \in D \mid b \neq a, b \in \mathbb{P}_{\mathscr{M}}R_a, \omega_C(b,a) = \mathbf{t}\}. \tag{8}$$

$$S_a^{\mathscr{M}} = \{b \in D \mid b \neq a, \mathsf{Beside}(b,a) = \mathbf{t}\}. \tag{9}$$

In the *certainly set* $\mathbb{C}_{\mathscr{M}}R_a$ we have the regions that are certainly related to a by relation R; the *possibly set* $\mathbb{P}_{\mathscr{M}}R_a$ contains the regions that are possibly related to a by R; the *strictly set* $\mathbb{E}_{\mathscr{M}}R_a$ is a subset of the possibly set that contains the regions which are entirely enclosed in a cardinal tile; lastly, $S_a^{\mathscr{M}}$ is the set of regions whose relation $\mathsf{Beside}(b,a)$ is true in model $\mathscr{M}$.

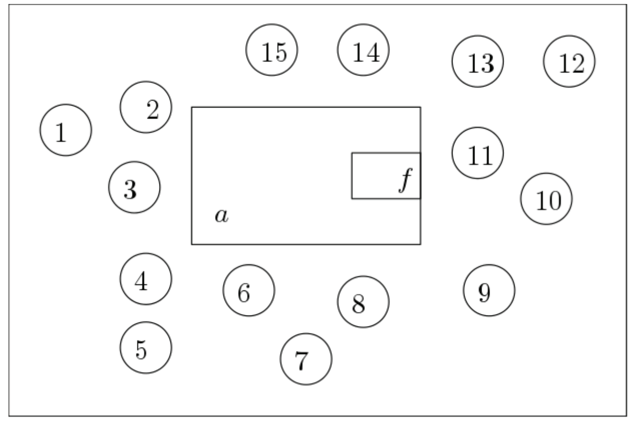

Figure 6: Scheme representing a building (a), whose front side is marked with f, surrounded by trees (numbered circles).

For example, for the region a, which is intrinsically oriented, the sets $\mathbb{C}_{\mathscr{I}}R_a$, $\mathbb{P}_{\mathscr{I}}R_a$, $\mathbb{E}_{\mathscr{I}}R_a$, and $S_a^{\mathscr{I}}$ shown below are given using the orientation model $\mathscr{I}$. Note that all trees (numbered circles) are accessible/visible from building a.

- $\mathbb{C}_{\mathscr{I}}\mathsf{Front}_a = \{9,10,11,12,13\}$;
- $\mathbb{P}_{\mathscr{I}}\mathsf{Front}_a = \{9,10,11,12,13\}$;
- $\mathbb{E}_{\mathscr{I}}\mathsf{Front}_a = \{10,11\}$;
- $\mathbb{C}_{\mathscr{I}}\mathsf{Back}_a = \{1,2,3,4,5\}$;
- $\mathbb{P}_{\mathscr{I}}\mathsf{Back}_a = \{1,2,3,4,5\}$;
- $\mathbb{E}_{\mathscr{I}}\mathsf{Back}_a = \{1,3\}$;
- $\mathbb{C}_{\mathscr{I}}\mathsf{Left}_a = \emptyset$;
- $\mathbb{P}_{\mathscr{I}}\mathsf{Left}_a = \{1,2,3,4,5,9,10,11,12,13\}$;
- $\mathbb{E}_{\mathscr{I}}\mathsf{Left}_a = \{1,3,10,11\}$;
- $\mathbb{C}_{\mathscr{I}}\mathsf{Right}_a = \emptyset$;
- $\mathbb{P}_{\mathscr{I}}\mathsf{Right}_a = \{1,2,3,4,5,9,10,11,12,13\}$;
- $\mathbb{E}_{\mathscr{I}}\mathsf{Right}_a = \{1,3,10,11\}$;
- $S_a^{\mathscr{I}} = \{6,7,8,14,15\}$.

The deictic model applied to the region a:

- $\mathbb{C}_{\mathscr{D}}\mathsf{Front}_a = \emptyset$;
- $\mathbb{P}_{\mathscr{D}}\mathsf{Front}_a = \{2,4,5,6,7,8,9,12,13,14,15\}$;
- $\mathbb{E}_{\mathscr{D}}\mathsf{Front}_a = \{6,7,8,14,15\}$;
- $\mathbb{C}_{\mathscr{D}}\mathsf{Back}_a = \emptyset$;
- $\mathbb{P}_{\mathscr{D}}\mathsf{Back}_a = \{2,4,5,6,7,8,9,12,13,14,15\}$;
- $\mathbb{E}_{\mathscr{D}}\mathsf{Back}_a = \{6,7,8,14,15\}$;
- $\mathbb{C}_{\mathscr{D}}\mathsf{Left}_a = \{1,3\}$;
- $\mathbb{P}_{\mathscr{D}}\mathsf{Left}_a = \{1,2,3,4,5\}$;
- $\mathbb{E}_{\mathscr{D}}\mathsf{Left}_a = \{1,3\}$;

- $\mathbb{C}_{\mathscr{D}}\mathsf{Right}_a = \{10, 11\}$;
- $\mathbb{P}_{\mathscr{D}}\mathsf{Right}_a = \{9, 10, 11, 12, 13\}$;
- $\mathbb{E}_{\mathscr{D}}\mathsf{Right}_a = \{10, 11\}$;
- $S_a^{\mathscr{D}} = \emptyset$.

Another example is given for the region (tree) 11. In this example, we adopted the extrinsic model since a tree is not intrinsically oriented. In addition, the building a or other trees can affect the visibility/accessibility. Thus, the sets are defined as follow:

- $\mathbb{C}_{\mathscr{E}}\mathsf{Front}_{11} = \emptyset$;
- $\mathbb{P}_{\mathscr{E}}\mathsf{Front}_{11} = \{a, 9, 10, 12, 13\}$;
- $\mathbb{E}_{\mathscr{E}}\mathsf{Front}_{11} = \{9, 13\}$;
- $\mathbb{C}_{\mathscr{E}}\mathsf{Back}_{11} = \emptyset$;
- $\mathbb{P}_{\mathscr{E}}\mathsf{Back}_{11} = \{a, 12, 13\}$;
- $\mathbb{E}_{\mathscr{E}}\mathsf{Back}_{11} = \{13\}$;
- $\mathbb{C}_{\mathscr{E}}\mathsf{Left}_{11} = \emptyset$;
- $\mathbb{P}_{\mathscr{E}}\mathsf{Left}_{11} = \{a, 9, 13\}$;
- $\mathbb{E}_{\mathscr{E}}\mathsf{Left}_{11} = \{9, 13\}$;
- $\mathbb{C}_{\mathscr{E}}\mathsf{Right}_{11} = \emptyset$;
- $\mathbb{P}_{\mathscr{E}}\mathsf{Right}_{11} = \{9, 10, 12, 13\}$;
- $\mathbb{E}_{\mathscr{E}}\mathsf{Right}_{11} = \{9, 13\}$;
- $S_a^{\mathscr{E}} = \emptyset$.

Note that in this example the regions related to 11 are limited by the visibility operator.

Heuristics

In this section we will establish a heuristic for analysis of the scenario by an order of precedence between the models, the sets certainly, possibly, and strictly and the predicates. Based on the findings of (Retz-Schmidt 1988) that the intrinsic frame prevails over the deictic and this over the extrinsic; and the (Teixeira 2001), that the frontal axis prevails over the lateral axis, the analysis of the scenario by the formalism will be given by the intrinsic model (when the Ground has intrinsic front) or deictic, otherwise. Once all the possibilities of a model are exhausted, the analysis is made in the next model. Thus, for a intrinsic Ground $\mathscr{I} \succ \mathscr{D}$; for a extrinsic Ground $\mathscr{D} \succ \mathscr{E}$, where $\succ$ denotes the order of precedence. The order of precedence between the sets and relations is given below:

1. $b \in \mathbb{E}_{\mathscr{M}}\mathsf{Front}_a$;
2. $b \in \mathbb{E}_{\mathscr{M}}\mathsf{Back}_a$;
3. $b \in \mathbb{E}_{\mathscr{M}}\mathsf{Left}_a \wedge \omega_{\mathsf{W}}(b, a)$;
4. $b \in \mathbb{E}_{\mathscr{M}}\mathsf{Right}_a \wedge \omega_{\mathsf{E}}(b, a)$;
5. $b \in \mathbb{C}_{\mathscr{M}}\mathsf{Left}_a \wedge \omega_{\mathsf{W}}(b, a)$;
6. $b \in \mathbb{C}_{\mathscr{M}}\mathsf{Right}_a \wedge \omega_{\mathsf{E}}(b, a)$;
7. $b \in \mathbb{P}_{\mathscr{M}}\mathsf{Left}_a \wedge \omega_{\mathsf{W}}(b, a)$;
8. $b \in \mathbb{P}_{\mathscr{M}}\mathsf{Right}_a \wedge \omega_{\mathsf{E}}(b, a)$;
9. $b \in \mathbb{C}_{\mathscr{M}}\mathsf{Front}_a$;
10. $b \in \mathbb{C}_{\mathscr{M}}\mathsf{Back}_a$;
11. $b \in \mathbb{P}_{\mathscr{M}}\mathsf{Front}_a$;
12. $b \in \mathbb{P}_{\mathscr{M}}\mathsf{Back}_a$;
13. $b \in \mathbb{E}_{\mathscr{M}}\mathsf{Left}_a \wedge \omega_{\mathsf{E}}(b, a)$;
14. $b \in \mathbb{E}_{\mathscr{M}}\mathsf{Right}_a \wedge \omega_{\mathsf{W}}(b, a)$;
15. $b \in \mathbb{C}_{\mathscr{M}}\mathsf{Left}_a \wedge \omega_{\mathsf{E}}(b, a)$;
16. $b \in \mathbb{C}_{\mathscr{M}}\mathsf{Right}_a \wedge \omega_{\mathsf{W}}(b, a)$;
17. $b \in \mathbb{P}_{\mathscr{M}}\mathsf{Left}_a \wedge \omega_{\mathsf{E}}(b, a)$;
18. $b \in \mathbb{P}_{\mathscr{M}}\mathsf{Right}_a \wedge \omega_{\mathsf{W}}(b, a)$.

Note that the set $S_a^{\mathscr{M}}$ will only be used in case all possibilities in both models fail. As was said previously, the purpose of this formalism is to address the ambiguity in projective expressions. In some languages, there are several terms that are used to describe the front/back relations. For example, in Brazilian Portuguese, "front of" can be expressed by "em frente a", "de frente a", "à frente de", "diante de", "defronte de", etc. Thus, a second stage of development of the project is to combine the presented formalism with the projective expressions in the chosen language by means of a cognitive research and a corpus analysis. In addition, the presented heuristic is generalist and must be adjusted according to the language.

In general, the formalism acts in two ways: first, when receiving a static image and a phrase containing a spatial description, the formalism must resort to heuristics and determine what object is referred to, considering that both agents, speaker and listener, are aware of the scenario. The point of view adopted will depend on the spatial expression in question (e.g., "my left" or "your left"). Secondly, the formalism will be able to give the precise description of one object in relation to another through heuristics. Now, we shall present a brief review of the papers that approach Human-Robotic interaction.

Human-Robotic interaction

(Milliez et al. 2014) present a situation assessment reasoner that generates information about the geometry of the environment with respect to relations between objects and humans. Therefore, they use a component called SPARK (*SPatial Reasoning and Knowledge*). This component is responsible for 3D environment management and spatial reasoning. (Kunze, Doreswamy, and Hawes 2014) implemented an approach for searching for objects on the basis of Qualitative Spatial Relations such as "left of" or "in front of" using an extend version of the ternary point calculus presented by (Moratz and Ragni 2008) (TPCC). This calculus is based on 24 JPED ternary relations of the distance and orientation. In addition, Moratz and Ragni presented an integration between TPCC and a robot.(Ros et al. 2010) proposed a strategy to solve possible ambiguities when referring to objects in a face-to-face interaction based on three mechanisms: visual perspective taking, spatial perspective taking and feature based descriptions.

Some papers make the Human-Robotic interaction using the OpenCV library. (van Delden and Umrysh 2011) presented a system for hand gesture recognition implemented in C# using OpenCV. (Pásztó and Hubinskỳ 2010) described the possibility of using visual systems for mobile robot navigation. The navigated mobile robot will not use any other sensors for scanning the environment, but only the visual information from the camera. (Koziol et al. 2009) presented a platform for commodity robots involving spatial-reasoning software and an OpenCV wrapper for vision-based capabilities.

Based on the work reported above our proposal to implement the formalism in a robotic system consists of a mobile robot with camera and no external sensor. We will divide our system in four layers:

1. Verbal communication interface to acquire the spatial expressions and natural language processing libraries for the extraction of the projective relation;
2. Computer vision interface for image capture;
3. Spatial-reasoning module based on the formalism presented;
4. Use of the heuristic to determine the object referred to in layer 1.

Therefore, a robot with embedded formalism will be able to address possible pragmatic ambiguities in projective expressions. Note that this is a generalist formalism and should be tailored for each language.

Conclusion

In this paper we address the question of the pragmatic ambiguity of the spatial expressions of the frontal and lateral axes, and propose a formalism based on Cardinal Direction Calculus and frames of reference to treat the ambiguity. Thus, formalism can be used both in the sense of interpreting a spatial expression and in describing one object in relation to another through the proposed heuristic. As future works we intend to include in the formalism the question of distance, introducing predicates such as “more ahead”, “more left”, etc.; conduct a cognitive survey with humans and compare the result with those presented by our formalism; to associate the formalism with spatial prepositions in Brazilian Portuguese.

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