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Knowledge Compilation in the Multi-Agent Epistemic Logic $\mathsf{K_n}$ *

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Abstract

In this paper, by employing logical separability, we propose an approach to knowledge compilation for the logic $\mathsf{K_n}$ by defining a normal form SDNF. We show that every epistemic formula can be equivalently compiled into a formula in SDNF, major reasoning tasks in SDNF are tractable, and formulas in SDNF enjoy the logical separability.

1 Introduction

It is crucial for an intelligent agent to be capable of representing and reasoning about high-order knowledge in the multi-agent setting. A general representative framework for these scenarios is multi-agent epistemic logics. However, many reasoning tasks, including satisfiability and forgetting, are intractable in such logics (Halpern and Moses 1992). These intractability results impede applications of multi-agent epistemic logics, *e.g.*, multi-agent epistemic planning (MAEP) (Kominis and Geffner 2015). Knowledge compilation provides an effective means to address the intractability problem (Darwiche and Marquis 2002).

The basic idea of knowledge compilation is to identify a normal form (*i.e.*, a fragment) of the given language such that each KB can be equivalently compiled into a KB in the normal form, reasoning tasks of interest in the latter are tractable, although the size of the compiled KB may expand to some extent.

However, existing normal forms have their shortcomings. The normal form S5-DNF (Bienvenu, Fargier, and Marquis 2010) is defined for only single-agent S5 but this normal form cannot be directly adapted to multi-agent epistemic logics. Two normal formas *cover disjunctive normal forms (CDNFs)* (ten Cate et al. 2006) and *prime implicate normal forms (PINFs)* (Bienvenu 2008) have been introduced for $\mathcal{ALC}$, a syntactic variant of $\mathsf{K_n}$. CDNF supports bounded conjunction while PINF does not. Moreover, in the worst case a compiled formula in PINF has double exponential expansion in size of the original formula. CDNF is relatively less succinct, *i.e.*, the CDNF representation is exponential large for some simple formulas.

Table 1: Succinctness of normal forms in $\mathsf{K_n}$. The symbol $\leq$ (or $\leq^*$) in the cell of row r and column c means that "the language $\mathcal{L}_r$ given at column r is at least as succinct as $\mathcal{L}_c$ given at column c (under the condition that $\mathcal{L}_0 \leq \mathcal{L}'_0$ in the case of $\leq^*$)". The symbol $\not\leq$ means that "$\mathcal{L}_r$ is not at least as succinct as $\mathcal{L}_c$".

$\mathcal{L}$	$\mathsf{SDNF}_{\mathcal{L}'_0}$	$\mathsf{SCNF}_{\mathcal{L}'_0}$	CDNF	PINF
$\mathsf{SDNF}_{\mathcal{L}_0}$	$\leq^*$	$\not\leq$	$\leq$	$\not\leq$
$\mathsf{SCNF}_{\mathcal{L}_0}$	$\not\leq$	$\leq^*$	$\not\leq$	$\leq$
CDNF	$\not\leq$	$\not\leq$	$\leq$	$\not\leq$
PINF	$\not\leq$	$\not\leq$	$\not\leq$	$\leq$

In this paper, in order to overcome shortcomings of existing normal forms, we provide a new approach for $\mathsf{K_n}$ by introducing two normal forms SDNF and SCNF. This is achieved by employing the concept of logical separability for epistemic terms. We then provide an almost complete knowledge compilation map for $\mathsf{K_n}$ by comparing the two new normal forms SDNF and SCNF with CDNF and PINF in terms of their succinctness and reasoning tasks preserved by the normal forms.

2 Separability-based Normal Forms for $\mathsf{K_n}$

In this section, we briefly introduce the syntax of multi-agent epistemic logic $\mathsf{K_n}$, and then introduce a general framework for defining normal forms DNF and CNF in $\mathsf{K_n}$ based on logically separability.

Let $\mathcal{A}$ be a set of n agents and P a countable set of variables. The set of epistemic formulas, $\mathcal{L}_\Box$ is obtained from the following grammer:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \Box_i\phi,$$

where $p \in P$ and $i \in \mathcal{A}$. The symbols $\vee$ and $\Diamond_i$, and the semantics of $\mathcal{L}_\Box$ formulas are defined as usual. We use $\mathcal{L}_0$ and $\mathcal{L}'_0$ for propositional fragments.

Definition 2.1. An epistemic term ϕ is a *separability-based term with* $\mathcal{L}_0$ ($\mathsf{STE}_{\mathcal{L}_0}$), if it is of the form $\alpha \wedge \bigwedge_{i\in\mathcal{B}}(\Box_i\beta_i \wedge \bigwedge_j \Diamond_i\gamma_{ij})$ s.t.

1. $\alpha \in \mathcal{L}_0$ and $\mathcal{B} \subseteq \mathcal{A}$;

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Table 2: Reasoning tasks for normal forms in $\mathsf{K_n}$. "✓" means that the language satisfies the polytime query (or transformation) property. "✕" means that the language does not satisfy the property, "○" means that the language does not satisfy the property unless $\mathbf{P} = \mathbf{NP}$, and "?" means that it is still open whether the property holds for the language. For the query $\mathbf{CE}_{\mathcal{L}_0'}$, we require that $\mathcal{L}_0'$ is dual to $\mathcal{L}_0$ in $\mathsf{SDNF}_{\mathcal{L}_0}$ and $\mathsf{SCNF}_{\mathcal{L}_0}$, and that $\mathcal{L}_0'$ is CL in CDNF and PINF. For the query $\mathbf{IM}_{\mathcal{L}_0'}$, the requirement of $\mathcal{L}_0'$ is the same as that in $\mathbf{CE}_{\mathcal{L}_0'}$ except that $\mathcal{L}_0'$ is TE in the case of PINF. $\mathsf{SDNF}_{\mathcal{L}_0}$ and $\mathsf{SCNF}_{\mathcal{L}_0}$ satisfy some query or transformation under certain conditions of $\mathcal{L}_0$. We list these conditions in the corresponding cell. For example, $\mathsf{SDNF}_{\mathcal{L}_0}$ satisfies $\mathbf{CO}$ if $\mathcal{L}_0$ does.

$\mathcal{L}$	CO	VA	SE	EQ	CE	$\mathbf{CE}_{\mathcal{L}_0'}$	IM	$\mathbf{IM}_{\mathcal{L}_0'}$	$\neg$C	$\wedge$C	$\wedge$BC	$\vee$C	$\vee$BC	CD	FO	SFO
$\mathsf{SDNF}_{\mathcal{L}_0}$	CO	○	○	○	○	CO, $\wedge$BC	○	○	✕	✕	$\wedge$BC	✓	✓	CD	FO	SFO
$\mathsf{SCNF}_{\mathcal{L}_0}$	○	VA	○	○	○	○	○	VA, $\vee$BC	✕	✓	✓	✕	$\vee$BC	CD	○	?
CDNF	✓	○	○	○	○	✓	○	○	✕	✕	✓	✓	✓	✓	✓	✓
PINF	✓	✓	✓	✓	○	✓	○	✓	✕	✕	✕	✕	✓	?	✓	✓

2. β_i's and γ_{ij}'s are disjunctions of $\mathsf{STE}_{\mathcal{L}_0}$'s;
3. $\gamma_{ij} \models \beta_i$ for any i and j.

Logical separable terms have the modularity property for satisfiability check and forgetting.

Proposition 2.1. *Let ϕ be an $\mathsf{STE}_{\mathcal{L}_0}$ of the form $\alpha \wedge \bigwedge_{i\in\mathcal{B}}(\Box_i\beta_i \wedge \bigwedge_j \Diamond_i\gamma_{ij})$ where $\mathcal{B} \subseteq \mathcal{A}$.*

- *ϕ is satisfiable iff all of α and γ_{ij}'s are satisfiable;*
- $\exists Q.\phi \equiv \bigwedge_{\alpha\in Prop(\phi)}(\exists Q.\alpha)\wedge$ $\bigwedge_{i\in\mathcal{B}}[\Box_i(\exists Q.\beta) \wedge \bigwedge_j(\Diamond_i(\exists Q.\gamma_{ij}))]$.

where $\exists Q.\phi$ denotes the result of forgetting Q in ϕ.

It is natural to obtain the definition of separability-based clauses that is dual to the notion of separability-based terms. Due to space limit, we do not give its definition.

We are ready to define separability-based DNF and CNF.

Definition 2.2. A formula ϕ is in *separability-based disjunctive (resp. conjunctive) normal form with $\mathcal{L}_0$ ($\mathsf{SDNF}_{\mathcal{L}_0}$ (resp. $\mathsf{SCNF}_{\mathcal{L}_0}$))*, if ϕ is a disjunction (resp. conjunction) of $\mathsf{STE}_{\mathcal{L}_0}$'s (resp. $\mathsf{SCL}_{\mathcal{L}_0}$'s).

3 A knowledge compilation map for $\mathsf{K_n}$

In this section, we present some major results: (1) $\mathsf{SDNF}_{\mathcal{L}_0}$ supports polytime test for clausal entailment ($\mathbf{CE}_{\mathcal{L}_0'}$), bounded conjunction ($\wedge\mathbf{BC}$) and polytime forgetting ($\mathbf{FO}$). (2) CDNF satisfies the same properties as $\mathsf{SDNF}_{\mathcal{L}_0}$ does, but is less succinct than $\mathsf{SDNF}_{\mathcal{L}_0}$. (3) PINF is tractable for sentential entailment check ($\mathbf{SE}$) and forgetting, but it fails to satisfy $\wedge\mathbf{BC}$. Thus, from the perspective of knowledge compilation, SDNF is more suitable for MAEP than the other three normal forms.

We first analyze the spatial complexity of SDNF and SCNF.

Proposition 3.1. *Any formula in $\mathcal{L}_\Box$ is equivalent to a formula in $\mathsf{SDNF}_{\mathcal{L}_0}$ (or $\mathsf{SCNF}_{\mathcal{L}_0}$) that is at most single-exponentially large in the size of the original formula.*

The succinctness of the four normal forms are summarised in Table 1. A language $\mathcal{L}$ is *at least as succinct as* $\mathcal{L}'$, if for any $\phi \in \mathcal{L}'$, there is an equivalent $\psi \in \mathcal{L}$ with size polynomial in $|\phi|$. Firstly, $\mathsf{SDNF}_{\mathcal{L}_0}$ (resp. $\mathsf{SCNF}_{\mathcal{L}_0}$) we propose are strictly more succinct than the existing normal form CDNF (resp. PINF). In addition, SDNF and SCNF are incomparable w.r.t. succinctness. This incomparability relation also holds for the other three pairs: (SDNF, PINF), (CDNF, SCNF), and (CDNF, PINF). Finally, CDNF and PINF are not at least as succinct as the other normal forms.

Theorem 3.1. *The results in Table 1 hold.*

In the following, we mainly discuss $\mathsf{SDNF}_{\mathcal{L}_0}$ against the class of reasoning tasks. Interestingly, given a suitable propositional sublanguage $\mathcal{L}_0$, $\mathsf{SDNF}_{\mathcal{L}_0}$ is tractable for all of reasoning tasks that DNF admits except the polytime clausal entailment check. However, it supports the polytime test for restricted clausal entailment.

Definition 3.1. A language $\mathcal{L}$ satisfies $\mathbf{CE}_{\mathcal{L}_0}$ (resp. $\mathbf{IM}_{\mathcal{L}_0}$), if there is a polytime algorithm for deciding whether $\phi \models \psi$ (resp. $\psi \models \phi$) for every $\phi \in \mathcal{L}$ and $\mathsf{SCL}_{\mathcal{L}_0}$ (resp. $\mathsf{STE}_{\mathcal{L}_0}$) ψ.

Now we elaborate on the results for reasoning tasks in Table 2.

Theorem 3.2. *The results in Table 2 hold.*

4 Conclusions

We have introduced a notion of logical separability for epistemic terms, which is a key property to guarantee that the satisfiability check and forgetting can be computed in modular way. Based on the logical separability, we have defined a normal form SDNF for the multi-agent epistemic logic $\mathsf{K_n}$, which can be seen as a generalization of the well-known propositional normal form DNF. As a dual to SDNF, we can define the SCNF for $\mathsf{K_n}$. More importantly, we have constructed a knowledge compilation map on four normal forms SDNF, SCNF, CDNF and PINF in terms of their succinctness and reasoning tasks.

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