Abstract
This paper studies aspecto-temporal representation and the temporal relations of discourse. We give a formal definition of different aspectual operators from the formal notions of states, events and process which are necessary for adequate analysis of aspects and tenses in natural languages. The aim of this work is not to present a deep comparison of the two models (SDRT and CAG) but to focus on the presentation of a cognitive and computational architecture of the CAG. Our study is specifically based on the computational and cognitive architecture with 7 levels of representations in the CAG in using Combinatory Logic. This architecture allows relating by formal calculus abstract semantic representations and linguistic observables. We focus on temporal relations of discourse and the insertion of a predicative relation in different temporal frameworks.

1. Introduction
Many works based on the analysis of temporal relations (Asher and Vieu 2005; Grosz and Sidner 1986; Lascarides and Asher 1993; Mann and Thompson 1987) assume that a text (or discourse) has hierarchical structures. But each method for representing a context is quite different. Our study is based on two representational methods of temporal relations: the Segmented Discourse Representation Theory (SDRT) and the model of Cognitive and Applicative Grammar (CAG). This paper presents a comparison of these two approaches about aspect and tense by an analysis of relations between events. We are not going to show all steps of SDRT’s representations, but we take a simple discourse (Asher and Lascarides 2003) and we analyze the same discourse with the framework of the CAG.

2. The Segmented Discourse Representation Theory (SDRT)
The Segmented Discourse Representation Theory (SDRT) (Asher 1993; Asher and Vieu 2005; Lascarides and Asher 1993) is anchored in the formal semantics to study the complex interplay between the semantic contribution of propositions with their components and the segmentation of discourse. SDRT is a dynamical and theoretical tool for the analysis of discourse, like Discourse Representation Theory (DRT) of Kamp (1981, 1984). The used basic elements to elaborate the discourses of structures are ‘segmented discourse representation structure’ (SDRS), which is basically compounded of elementary ‘discourse representation structure’ (DRS). SDRS is also recursively defined as a set of speech-act labels, $\pi_1, \ldots, \pi_n$, related by discourse relations $R$ such that each speech-act label is associated with a ‘discourse constituent’, which is either simple—the logical formula representing a simple clause—or complex—a SDRS representing a discourse segment. They are linked by an arrow which is labeled by discourse relations $R$. We represent SDRS in the form of boxes like DRS. To induce a temporal and hierarchical structure, SDRT distinguish discourse relations ‘coordinating’ from ‘subordinating’, therefore coordination and subordination affect the temporal order of text: the former indicate a continuation of some discourses pattern, like relations of ‘Narration’ or ‘Result’ in discourse segmentation, and the later indicate with types of information like relations of ‘Elaboration’ or ‘Explanation’. These relations are appeared where the clause $\alpha$ presents in the text before $\beta$. In the figure 1, when $R$ is a coordination the arrow is horizontal, while if $R$ is a subordination the arrow is vertical (oblique).
We list some discourse relations\(^1\) that are necessary to temporal structuring:

- **Narration**\((\alpha, \beta)\): The event described in \(\beta\) is a consequence of the event described in \(\alpha\);
- **Result**\((\alpha, \beta)\): the event described in \(\alpha\) caused the event or state described in \(\beta\);
- **Elaboration**\((\alpha, \beta)\): \(\beta\)’s event is part of \(\alpha\)’s;
- **Explanation**\((\alpha, \beta)\): the event describe in \(\beta\) explains why \(\alpha\)’s event happened.

As we mentioned, we take a discourse example of SDRT (Asher and Lascarides 2003):

(1) *Fred had a great evening last night* \((\pi_1)\). *He had a great meal* \((\pi_2)\). *He ate salmon* \((\pi_3)\). *He devoured lots of cheese* \((\pi_4)\). *He then won a dancing competition* \((\pi_5)\).

This narrative discourse \((1)\) describes the evening of Fred, which is elaborated on two sub-events, meal and dancing competition. These sub-events belong semantically to the main event, i.e. to the main sentence \((\pi_1)\) which represents the value of aspect ‘event’. The relations of discourse are described below:

- Elaboration links the first clause \((\pi_1)\) to the rest of the discourse \((\pi_2-\pi_5)\);
- Narration links the message of great meal to the dancing competition, i.e. \((\pi_2)\) and \((\pi_5)\);
- Elaboration links the message of great meal to two following clause, i.e. \((\pi_3)\) and \((\pi_4)\).

The following figure illustrates two ways of representation for SDRS which represents the discourse \((1)\)\(^2\). The notation \(K_\pi\) symbolizes the DRS which represents the \(\pi\)th sentence. The nodes represent the sentences (marked \(\pi\), and called “nodes of clauses”), the graph SDRT has the “nodes of ranges” (marked \(\pi’\), \(\pi”\)). In the box representation, a node of range labels a sub-SDRS.

![Figure 1: SDRS for discourse (1) and hierarchical structure](image)

Clauses \((\pi_1-\pi_4)\) elaborate the meal \((\pi_2)\), which in turn elaborates the evening \((\pi_1)\). \((\pi_5)\) also elaborates the evening, but unlike \((\pi_1-\pi_4)\) it doesn’t elaborate the meal. Rather, it forms a narrative with \((\pi_3)\). So \((\pi_5)\) shouldn’t be considered a part of the same “segment” as \((\pi_3-\pi_4)\). By the right frontier constraint\(^3\), we can’t attach \((\pi_5)\) to \((\pi_3)\)\(^4\). In the box representation of Figure 1, Narration \((\pi_5, \pi_3)\) and Elaboration \((\pi_5, \pi”)\) are on an equal footing in the sub-SDRS labeled \(\pi”\), where the distinction between coordination and subordination isn’t considered. On the other hand, for the graphic representation, the node of range \(\pi”\) dominates immediately the two arguments \(\pi_3\) and \(\pi_5\) of the relation coordinating, while this node dominates immediately only \(\pi_3\) of the relation subordinating Elaboration (this dominate \(\pi”\) but not immediately).

We show a DRS of one sentence (i.e. clause \((\pi_4)\), labeled \(K_{\pi_4}\)): there is three individual arguments \((x, y, u)\), in particular, ‘u’ expresses an anaphor of the Fred; an event represented by a formal and atomic logical expression; and a relation between event and a temporal constant ‘n (now)’ which gives an information that the event occurred before the moment of speaking, thus the past.

![Figure 2: DRS of \((\pi_4)\)](image)

Whether the relations are coordinating or subordinating, it depends on the circumstances of their use in the discourse.

### 3. Cognitive and Applicative Grammar (CAG)

The model of Cognitive and Applicative Grammar (CAG) (Desclés 1990a; 1990b; 2005) is an extension of Applicative Grammar of Shaumyan (1987); it adopts a cognitive and semantic view. The main hypotheses of the CAG model are: (i) the language is a cognitive activity which is not independent of other cognitive abilities: the lexical and grammatical categories of a natural language are anchored to cognitive categories built by activities of perception and action (Desclés 1990a); (ii) the different levels of analysis are expressed by typed-applicative expressions (Church’s functional types), \(\lambda\)-expressions and expressions of Combinatory Logic (CL) (Curry and Feys 1958); (iii) the relationships between levels are established by means of formal operators- called combinators - for composing more elementary operators.

---

\(^{1}\) There are also other discourse relations like: Background, Continuation, Parallel, Contrast, Topic, Precondition, Commentary, etc.

\(^{2}\) These diagrams are taken from Asher and Lascarides (2003).

\(^{3}\) For the “right-frontier-constraint”, see Polanyi (1988).

\(^{4}\) Commentaries took from Asher and Vieu (2005).
3.1. Computational Architecture of CAG

CAG is an analogue to a compiling program with 7 levels which are interrelated:
(i) morpho-syntactical configuration level (level 0) where the particular characteristics of a natural language are described (e.g. order of words, morphological cases, etc.);
(ii) logical-grammatical representation level expressed by typed applicative expression (level 1), where the formalism of Categorial Grammar (Biskri and Desclès 2005) gives a syntactical analysis of sentence and a decomposition of each sentence in operators and operands;
(iii) analysis of diatheses and topicalisation in using combinators (level 2);
(iv) analysis and representation of enunciation conditions for describing tenses, aspects and modalities (level 3);
(v) formal representation of the meaning of lexical predicate by Semantic-Cognitive Schemes (SCS, level 4);
(vi) integration of enunciative conditions with Semantic-Cognitive Schemes (level 5);
(vii) cognitive representation (for instance, by visual diagrams or iconic representations) in relation with a perception and an action (level 6) (Desclés and Ro 2011).

These levels are articulated by means of grammatical rules (relations between a definiendum - newly defined operator, and its definiens - complex operator) and elimination (respectively introduction) rules associated to combinators of CL in Gentzen style of natural deduction (Desclés 1990a; 2005). Let us some examples of elementary combinators by elimination rules for B, C, and C*:

\[
\begin{align*}
& B \frac{f(y,x)}{\forall y x} \\
& C \frac{f(x,y)}{\forall y x} \\
& C^* \frac{x f}{\forall x}
\end{align*}
\]

Derived combinators are defined from elementary combinators. We will present later (3.2.3) an example of a formal relation between a morpho-syntactical configuration (Fred devoured lots of cheese) and its grammatical aspecto-temporal semantic interpretation expressed at the level 6 of the architecture, in using the combinators B, C, C* and derived combinators.

3.2. Computational and Semantic Representation

Before starting our analysis of discourse (1), we present briefly some of technical and theoretical notions that we need for the representation of CAG.

3.2.1. Basic Notions of Aspect with topological boundaries. The aspectual theory in the CAG develops aspectual notions of different authors (Comrie 1976; Culio-li 1999; Desclès 1990b; 2005; Desclès and Guentchéva 2006; 2010). The temporality of language can’t be described without taking account of the aspectuality. All aspectual notions imply an underlying temporality; most of situations require topological boundaries of intervals compounded by instants.

To analyze semantically the expressions of linguistic temporality, we must take account of not only purely temporal relations, i.e. the concomitance (=), the temporal differentiation (≠) (anteriority or posteriority). A predicative relation, noted ‘Λ (lexis)’ (see Culio-li 1999), is aspectualized as a state, or an event, or a process (Comrie 1976; Desclès 1980; 1990b; Mourelatos 1981) in using aspectual operators \( \text{STATE}_0 \), \( \text{EVEN}_f \) and \( \text{PROC}_j \) which are actualized on topological intervals of instants:

(i) \( \text{STATE}_0 (\Lambda) \) is developed on the topological open interval ‘O’ and is true for each instant of ‘O’;
(ii) \( \text{EVEN}_f (\Lambda) \) is developed on the closed interval ‘F’ and is true at the right closed boundary ‘S(F)’.

\footnote{The combinators of Combinatory Logic (with functional types) of Curry (1958) are necessary to express exactly the definiens in terms of more elementary operators. For example, “I-am-saying” is a speech act operator; it is a result of a functional composition of the two operators: “I-SAY” and “PROC” where this later means that aspectual value of process developed during the interval ‘P’ of instants, with the right bound ‘T’}. The relation definiendum / definiens is formulated by means of the combinator B of the Combinatory Logic:

\[
\text{definiendum} \overset{=}{=} \text{B \ PROC}_0 (\text{I-SAY})
\]

Figure 3: Computational architecture of the CAG

158
(iii) PROC\textsubscript{j} (A) is developed on the interval ‘J’ with a left-closed boundary ‘\(\delta(J)\)’ and right-open boundary ‘\(\delta(J)\)’ and is true at each instant ‘t’ of ‘J’ before the right open boundary of ‘\(\delta(J)\)’ (\(t < \delta(J)\)).

3.2.2. Enunciative Scheme. The aspectualised predicative relations are located in an enunciative referential framework defined from the speech act of an speaker ‘I’ (enunciator of speaker) (Benveniste 1974; Culioli 1999); this speech act represented by the operator ‘I-SAY’ (see Harris 1982), it is developed as an unaccomplished process during the interval ‘\(J^0\)’ with the open right bound ‘\(\delta(J)\)’; ‘\(\delta(J)\)’ is not the “moment of speaking” but only “the first instant of not yet actualized situations”, it is an open right boundary.

Let us designate the general operator ‘ASP\textsubscript{j}’ of aspectuality whose operand is a lexis (or predicative relation); the value of ‘ASP\textsubscript{j}’ are STATE\textsubscript{0} or EVEN\textsubscript{F} or PROC\textsubscript{j}. To be enunciated, ‘ASP\textsubscript{j} (A)’ must be located in the temporal framework defined by the enunciator. Thus ‘ASP\textsubscript{j} (A)’ is concomitant or non concomitant with the speech act process; in the first case, the right bound ‘\(\delta(I)\)’ of ‘I’ is such that: \(\delta(I)=\delta(J)=\delta(\delta(J))\); in the second case, the right bound ‘\(\delta(I)\)’ is such that: \(\delta(I) > \delta(J)\). So we can define the enunciative scheme formulated by an applicative expression where the operator is always posited before its operand:

\begin{equation}
(2) \text{PROC}_{\text{j}}(\text{I-SAY}) \& (\text{ASP}_{\text{j}}(A) [\text{I REP} J^\text{j}])
\end{equation}

This scheme means that:

(2’) “the aspectual process ‘PROC\textsubscript{j}’ is applied on the result of the application of (I-SAY) on a conjunction of an aspectualized predicative relation ‘ASP\textsubscript{j} (A)’ and a temporal relation [I REP J\text{j}] where the temporal relation between on one hand, the interval ‘I’ related to the predicative relation and on the other hand, an interval ‘J^\text{j}’ related to enunciative process”.

The value of the relator ‘REP’ expresses the choice of a temporal value (‘\(=\)’, ‘\(<\)’ or ‘\(\geq\)’). The insertion of a predicative relation in the temporal framework of the enunciator contributes to specify its aspectual and temporal values.

3.2.3. Representation of Discourse with Asp ectual and Temporal Relations. Let us take the example (1). The general analysis of grammatical morphemes leads to represent the grammatical meaning of each proposition and of the adverbial expression (Last night) as follows:

(i) Morpho-Syntactical configuration:

\begin{itemize}
  \item \(\pi_1.1\). Last night (reform: All that follows occurred last night): Temporal Framework, STATE\textsubscript{0} (state)
  \item \(\pi_1.2\). Fred had a great evening : EVEN\textsubscript{F1} (event)
  \item \(\pi_2\). He had a great meal: EVEN\textsubscript{F2} (event)
  \item \(\pi_3\). He ate salmon: EVEN\textsubscript{F3} (event)
  \item \(\pi_4\). He devoured lots of cheese: EVEN\textsubscript{F4} (event)
  \item \(\pi_5\). He then won a dancing competition: EVEN\textsubscript{F5} (event)
\end{itemize}

(ii) Grammatical interpretation formulated by applicable expressions with \([x = Fred]\):

\begin{equation}
\pi_1.1. \text{PROC}_{\text{j}}(\text{I-SAY}) \& (\text{STATE}_{\text{j}} (\text{All that follows occurred last night}) [\delta(\text{I}) < \delta(J)])
\end{equation}

\begin{equation}
\pi_1.2. \text{PROC}_{\text{j}}(\text{I-SAY}) \& (\text{EVEN}_{\text{j}} (\text{have (a great evening)}) (Fred)) [\delta(\text{F}) < \delta(J)])
\end{equation}

\begin{equation}
\pi_2. \text{PROC}_{\text{j}}(\text{I-SAY}) \& (\text{EVEN}_{\text{j}} (\text{(have a great meal)}) (Fred)) [\delta(\text{F}) < \delta(J)])
\end{equation}

\begin{equation}
\pi_3. \text{PROC}_{\text{j}}(\text{I-SAY}) \& (\text{EVEN}_{\text{j}} (\text{(eat (salmon) (x)}) [\delta(\text{F}) < \delta(J)])
\end{equation}

\begin{equation}
\pi_4. \text{PROC}_{\text{j}}(\text{I-SAY}) \& (\text{EVEN}_{\text{j}} ((\text{devour (lots of cheese)}) (x))) [\delta(\text{F}) < \delta(J)])
\end{equation}

\begin{equation}
\pi_5. \text{PROC}_{\text{j}}(\text{I-SAY}) \& (\text{EVEN}_{\text{j}} ((\text{win (a dancing competition)}) (x))) [\delta(\text{F}) < \delta(J)])
\end{equation}

Each interpretative expression is built up from a morpho-syntactical configuration by means of on one hand, the equivalence between a \textit{definiendum} and its \textit{definizens} with different combinators which occur inside it and on the other hand, the elimination rules of combinators.

We detail one of these applicative expressions (\(\pi_4\)). We start with the configuration \(\pi_4\). The analysis of Extended Categorial Grammar (Biskri and Descles 2005) is applied to the passage from step 0 to 1. This Categorial Grammar builds up an applicative expression at the step 1 where this applicative expression is analyzed with the more synthetic terms: ‘\(P_5\)’-binary predicate \textit{devour} in the form of verbal infinitive; ‘\(A^2\)’ and ‘\(A^1\)’ - two actants \textit{lots of cheese} (\(A^2\)) and \textit{Fred} (\(A^1\)) (step 2).

0. Fred devoured lots of cheese
1. (devour-ed (lots-of-cheese)) (Fred)
2. past-suffix \(P_5\) \(A^2\) \(A^1\)
3. COMPLETE-EVENT-PAST \(P_5\) \(A^2\) \(A^1\)
4. \[\text{COMPLETE-EVENT-PAST} = X \& ((\delta(F)<\delta(J))) \]
5. \(\text{I-am-saying} \& (\text{EVEN}_{\text{j}} (P_5 \text{A}^2 \text{A}^1)) (\text{I-am-saying} \text{EVEN}_{\text{j}})\]
6. \(\text{I-am-saying} = B \text{ PROC}_{\text{j}} (\text{I-SAY})\)
7. \(\text{PROC}_{\text{j}}(\text{I-SAY}) \& (\text{EVEN}_{\text{j}} (P_5 \text{A}^2 \text{A}^1)) (\text{I-am-saying} (\text{EVEN}_{\text{j}})\))

The operator ‘\(\text{past-suffix}\)’ has a meaning which is analyzed by the operator ‘\(\text{COMPLETE-EVENT-PAST}\)’ whose operand is the predicative relation ‘\(P_5\)\(A^2\)\(A^1\)’ at the step 3. This aspectual operator ‘\(\text{COMPLETE-EVENT-PAST}\)’ is defined by a combinator X and the operator ‘\(\&\)’; ‘\(\text{I-am-saying}\)’, ‘\(\text{EVEN}\)’ and the temporal relations at the step 4. The combinator\(^6\) X expresses the functional composition of operators with a temporal relation. The successive eliminations of different combinators which are components of X lead to the expression 5. The operator ‘\(\text{I-am-saying}\)’ is a functional composition of the process actualized on ‘\(J^0\)’ and the enunciative operator ‘\(\text{I-SAY}\)’, hence the step 6. We obtain the final expression at the step 7. It expresses the meaning of the ‘\(\text{past-suffix}\)’ relative to the lexical predicate ‘\(P_5\)’. When we instantiate ‘\(P_5\)’, ‘\(A^2\)’, ‘\(A^1\)’ by their respective values, we obtain this final expression:

\(^6\) \([X = B_0 C_1 C_1 C_1 B_1]\) where \(B_0\) and \(C_1\) are combinators whose the actions are differed and \(B_1\) is a composition with \(B\) by itself (\(B_1 = BBB\)).
7’. PROCj0 ((I-SAY) (& (EVENF4 devour (lots-of-cheese) (Fred))) ([δ(F1)<δ(F0)]))

This expression 7’ is the aspectual meaning associated to the step 0.

For each proposition of the text, we obtain the aspectual representations of different grammatical suffixes (in this example we have only the events and one state). By analogue consideration of SDRT, we obtain the discursive structure of discourse (1) with different relations between the intervals of events and the interval of enunciative process (see the figure 4):

The linguistic adverbial expression ‘Last night’ is a deictic marker. This information leads to insert the narrative sequences in the enunciative referential framework (noted ‘REN’, in French: référentiel énonciatif) in locating the narration before the enunciative process. We represent the temporal relations in the figure 5.

3.2.4. Non Actualized Referential Framework. The notion of referential framework is necessary to understand to analyze different types of texts (see Desclés and Guentcheva 2010). Indeed, we modify the adverbial ‘Last night’ by ‘that day’. This adverbial is not a deictic marker, it indicates the following information: the sequence of events is not temporally related to the enunciative process but it is located in another referential framework: the non actualized referential framework (noted ‘RNA’). It is possible to change the tense and to use the simple present tense:

(2) Fred has a great evening that day (π1). He has a great meal (π2). He eats salmon (π3). He devours lots of cheese (π4). He then wins a dancing competition (π5).

In this case, the present tense is not the marker of a concomitance with enunciative process but it indicates a synchronization between the events of the narrative sequence of the referential ‘RNA’ with ‘T°’. Thus, the introduction of the new referential framework is linked to the enunciative framework ‘REN’ by a breaking relation (in French: rupture, noted ‘#’). The breaking between two different referential frameworks is compatible with synchronization (non concomitance) for expressing the narrative present. The synchronization permits to see the event of a narration as being in progress at the narrative index ‘t’ synchronized with the temporal locator ‘T°’ (see the figure 6).

With this modification, the clause (π1.2) represents the value of Resulting state which implies an occurrence of an event just before the state, i.e. it is contiguous to a previous event (π2-π5). This state is actualized on the open interval ‘O12’ adjacent to the close interval ‘F’.

4. Conclusion

There is also other referential frameworks like reported enunciative framework, for instance, possible framework, commentarial framework…, to represent the temporal rela-
tions of discourse (see Desclés and Guentchéva 2006; 2010).

By this short presentation based on illustrative example, we have not expressed all operations of analysis, in particular, the transfers from one level to another level in the computational architecture of the CAG: the lack of place is the reason. In this article, we showed two different methods for representing the temporal relations. We have explained only some steps of representation about temporal relations between predicative relations of a text in the model of CAG.

From aspectual and temporal relations, we have showed how a grammatical meaning can be construed from the identification of grammatical markers, after a syntactical analysis by Categorial Grammar. Our representation uses the topological concepts as open and close bounds, event, process, state, and resulting state... The applicative formalism and combinators of CL are useful to an approach by a computer science by means of functional programming languages (CAML, HASKELL, F#, etc.). By HASKELL, we have specific results with explicite calculi on types (see Desclés and Ro 2011; PhD in preparation of Ro).

We have not take into account the representation of the meaning of lexical predicate. Indeed, a lot of inferences in natural languages imply a representation of the meaning of lexical predicates. For instance, the understanding of «This table is in the kitchen. John moves the table into the garage» implies: «Now, the table is located in the garage». To resolve this problem, a model must represent the meaning of ‘to move’ and the temporal relations between a past situation and a present situation.

To analyze a text, it is necessary to relate the temporal meaning of lexical predicate with temporal aspectual meaning of grammatical units to build a Semantico-Cognitive representation (level 3, 5 and 6 in the figure 3) (see this topic Desclés 2005).

References


