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Prime Normal Forms in Belief Merging

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Abstract

The aim of Belief Merging is to aggregate possibly conflicting pieces of information issued from different sources. The quality of the resulting set is usually considered in terms of a closeness criterion between the resulting belief set and the initial belief sets. The notion of distance between belief sets is thus a crucial issue when we face the merging problem. The aim of this paper is twofold: introducing a syntactical way to calculate distances and proposing the use of a distance based on prime implicants and prime implicates that considers the importance of each propositional symbol in the belief set.

Introduction

The goal of Belief Merging is to aggregate in a consistent way multiple beliefs usually represented as sets of logical statements so as to obtain a new set of statements (Liberatore and Schaerf 1998). Such process is applied in knowledge based systems when a cognitive agent receives a set of contradictory pieces of information from many sources, e.g. other agents or its own sensors or "mental states".

Belief Merging process is based on three main components (Konieczny, Lang, and Marquis 2002): a notion of distance between propositional models, a function to aggregate distances and a procedure to select the closest eligible resulting sets w.r.t. the aggregation stage. Usually, the Hamming distance is adopted in order to calculate the distance between models, where a propositional symbol is considered as minimal change unit (Dalal 1988). This distance has been widely considered in belief revision (Dalal 1988; Satoh 1988), belief update (Forbus 1989; Winslett 1988) and belief merging (Konieczny, Lang, and Marquis 2002). However, this consideration can not be minimal (Marchi, Bittencourt, and Perrussel 2010; Bittencourt, Perrussel, and Marchi 2004), because changing one propositional symbol truth value may lead to significant changes if this symbol frequently appears in the formulas of the initial belief bases. Thus, the common notion of minimal change is biased by the structure of the belief bases.

In that context, two main problems can be identified: first, the belief merging process is performed over set of models; and second, the minimal change unit may promote significant changes on the belief base. These issues have already been partially addressed in (Perrussel, Marchi, and Bittencourt 2008) where it was proposed a merging process that can satisfy the postulates characterizing belief merging proposed in (Konieczny, Lang, and Marquis 2002) in a partial way by choosing implicants (or implicates) among all the belief sets. As proposed, the merging process entails that some privileges are given to some specific formulas and these privileges are not compatible with the fairness principle which is a key one in belief merging: fairness states that all beliefs and all belief sets should be equally considered.

In this paper, we propose a new way to merge beliefs that avoid this problem by proposing a new way to define the implicants/implicates which will belong to the resulting base; and it is not required that these statements should belong to one of the initial belief sets. Hence, in that way, we take care of the fairness issue. We also avoid this limitation about postulates satisfaction.

This paper focuses on a syntactical way to perform the merging process where the belief bases are represented as sets of prime implicants, and by considering another minimal change distance based on prime implicants and prime implicates representation. These two representations enable us to correlate symbols in terms of models (prime implicants) and also in terms of structure (prime implicates). The key results are that the proposed framework (i) is fully compliant with the definition of belief merging and (ii) gives a new perspective on the closeness criterion for setting proximity between belief bases.

The paper is organized as follows: we first give the formal definitions which set the notion of prime forms (implicate and implicant) and we detail the quantum notation introduced by (Bittencourt 1998) which aims at relating implicate and implicant. Next, we formally present the concept of belief merging and present the set of associated postulates proposed by Konieczny and Péres (Konieczny and Pérez 2002). We also remind the definition of majority (Lin and Mendelzon 1999) and arbitration operators (Liberatore and Schaerf 1998). The next section presents the syntactical approach of the belief merging process. Next, we introduce the new minimal change unit and present its application on belief merging processes. We finally conclude the paper by presenting some considerations and future work.

Preliminaries

Let P be a finite set of propositional symbols, such that $P = \{p_1, \ldots, p_n\}$. Let $\{L_1, \ldots, L_{2n}\}$ be the set of their associated literals, where $L_i = p_j$ or $L_i = \neg p_j$. A *clause* is a *disjunction* of literals: $C = L_1 \vee \cdots \vee L_{k_C}$. A *term* is a *conjunction* of literals: $D = L_1 \wedge \cdots \wedge L_{k_D}$. The complement of a literal L is noted $\overline{L}$: if $L = p$ then $\overline{L} = \neg p$. The complement of a term D is noted $\overline{D}$, e.g. if $D = p_1 \wedge \neg p_2$ then $\overline{D} = \neg p_1 \wedge p_2$. The subtraction of two terms D and D' is noted $D - D'$ and results in literals of D that do not have correspondence with the literals of D', e.g. if $D = p_1 \wedge \neg p_2$ and $D' = p_1 \wedge p_2$, $D - D' = \{\neg p_2\}$.
Given a propositional logic language $\mathcal{L}(P)$ and a *formula* $\psi \in \mathcal{L}(P)$, ψ can be rewritten in a *conjunctive normal form (CNF)*, where $CNF_\psi = C_1 \wedge \cdots \wedge C_m$, or in a *disjunctive normal form (DNF)*, where $DNF_\psi = D_1 \vee \cdots \vee D_w$, such that $\psi \equiv CNF_\psi \equiv DNF_\psi$.
A clause C is an *implicate* (Jackson 1990; Kean and Tsiknis 1990) of a formula ψ iff $\psi \models C$, and it is a *prime implicate* iff for all implicates C' of ψ such that $C' \models C$, we have $C \models C'$. We define PI_ψ as a conjunction of prime implicates of ψ such that $\psi \equiv PI_\psi$. A term D is an *implicant* of a formula ψ iff $D \models \psi$, and it is a *prime implicant* iff for all implicants D' of ψ such that $D \models D'$, we have $D' \models D$. We define IP_ψ as a disjunction of prime implicants of ψ such that $\psi \equiv IP_\psi$. In propositional logic, prime implicates and prime implicants are dual notions, in particular, an algorithm that calculates one of them can also be used to calculate the other (Socher 1991; Bittencourt, Marchi, and Padilha 2003).
Alternatively, prime implicates and prime implicants can be defined as special cases of CNF (or DNF) formulas, that consist of the smallest sets of clauses (or terms) closed for inference, without any subsumed clauses (or terms), and not containing a literal and its negation. In the sequel, conjunctions and disjunctions of literals, clauses or terms are treated as sets.
We define a profile $\Psi = \{\psi_1, \ldots, \psi_n\}$ as a finite set of formulas, where each formula ψ_i represents a belief receive from the source i. We note $\bigwedge \Psi$, the conjunction of all formulas $\psi_1 \wedge ... \wedge \psi_n$; $\Psi_1 \sqcup \Psi_2$ denotes the union of each set belong to the sets Ψ_1 and Ψ_2, that is $\{\psi_{11} \cup \psi_{21}, \ldots, \psi_{1n} \cup \psi_{2n}\}$; and $\Delta_\mu(\Psi)$, the resulting belief merging set.
An interpretation is a function from P to $\mathbb{B} = \{true, false\}$. Let $\mathcal{W}$ be the set of all possible interpretations. An interpretation w is a model of a formula ψ ($w \models \psi$) iff ψ is true in w. For any formula ψ, $[\![\psi]\!]$ denotes the set of models of ψ.

Quantum Notation

Conjunctive and disjunctive normal forms, as well as prime implicants and prime implicates can be correlated through *quantum notation* (Bittencourt 1998). The quantum notation correlates each literal L that occurs in a formula ψ with the clauses C or terms D to which literal L belongs. In that sense, a conjunctive quantum of a literal $L \in \psi$, denoted by L^{F_c}, is a representation that correlates the literal L with its set of *conjunctive coordinates* $F_c \subseteq CNF_\psi$ that contains the subset of clauses in CNF_ψ to which literal L belongs. Dually, a disjunctive quantum of a literal $L \in \psi$, L^{F_d}, explicits the relation between the literal L and its set of *disjunctive coordinates* that contains the subset of terms in DNF_ψ to which literal L belongs.
In association with prime normal forms, *quantum notation* gives us a measure of the importance of a literal in a formula through the notion of *exclusive coordinates*. Given a term D and a literal $L_i \in D$, the exclusive conjunctive coordinates of L_i in D, defined by $\widehat{F}_c^i = F_c^i - \cup_{j=1, j\neq i}^k F_c^j$, are the clauses in set F_c^i, to which no other literal of D belongs; and given a clause C and a literal $L_i \in C$, the exclusive disjunctive coordinates of L_i in C, defined by $\widehat{F}_d^i = F_d^i - \cup_{j=1, j\neq i}^k F_d^j$, are the terms in set F_d^i, to which no other literal of C belongs. These sets represent clauses or terms that are supported only by the literal L.
An algorithm to calculate prime normal forms using quantum notation is presented in (Bittencourt, Marchi, and Padilha 2003). The following example illustrates the concepts of quantum notation and exclusive coordinates.

Example 1 *Consider formula ψ represented by PI_ψ and IP_ψ. Each literal in PI_ψ has a set of coordinates representing terms in IP_ψ to which the literal belongs, as well as each literal in PI_ψ has a set of coordinates representing clauses in PI_ψ to which the literal belongs. To facilitate the understanding, clauses and terms are identified by numbers.*

PI_ψ	IP_ψ
$1 : \neg p_3^{\{1,\boxed{3}\}} \vee \neg p_2^{\{1,\boxed{2}\}}$	$1 : \neg p_3^{\{1,\boxed{2}\}} \wedge \neg p_2^{\{1,\boxed{3,4}\}}$
$2 : \neg p_3^{\{\boxed{1},3\}} \vee p_4^{\{\boxed{2},3\}}$	$2 : \neg p_2^{\{\boxed{1,3},4\}} \wedge p_4^{\{\boxed{2},4\}}$
$3 : \neg p_2^{\{\boxed{1,2}\}} \vee \neg p_1^{\{\boxed{3}\}}$	$3 : \neg p_3^{\{\boxed{1},2\}} \wedge \neg p_1^{\{\boxed{3}\}} \wedge p_4^{\{2,\boxed{4}\}}$
$4 : \neg p_2^{\{\boxed{1},2\}} \vee p_4^{\{2,\boxed{3}\}}$	

The boxed coordinates indicate the exclusive coordinates of each literal. Consider the literal $\neg p_3$ in clause number 1 in PI_ψ. Its set of exclusive coordinates is given by:

$$\begin{aligned} \widehat{F}_d^{p_3} &= F_d^{p_3} - F_d^{\neg p_2} \\ \widehat{F}_d^{p_3} &= \{1,3\} - \{1,2\} = \{3\} \end{aligned}$$

The coordinate related to term 1 in IP_ψ is not exclusive to this literal, since it appears also in $\neg p_2$; but the coordinate related to term 3 in IP_ψ is exclusive, since any other literal in that clause has this coordinate.

Belief Merging

A belief merging process consists in aggregating multiple possible contradictory pieces of information in a single base. This process was identified by Revez (Revesz 1993), but a proper logical framework was introduced only in 2002 (Konieczny and Pérez 2002) when the specific nature of the belief merging process has been identified (Liberatore and Schaerf 1998; Lin and Mendelzon 1999). The proposed framework defines a belief merging process in a semantic way by considering the initial belief bases and an integrity constraint set in terms of models: the resulting belief base $\Delta_\mu(\Psi)$ is given by the models of the integrity constraint set μ that are closest to the initial profile $\Psi = \{\psi_1, ..., \psi_n\}$ according to some closeness criterion. This closeness criterion is represented with the help of a total pre-order $\leqslant_\Psi$ which is itself based on an underlying distance between interpretation and profile. That is: let $Dist$ be a function characterizing the distance between an interpretation w and a profile Ψ such that $Dist(w, \Psi) \geq 0$; then $w \leqslant_\Psi w'$ iff $Dist(w, \Psi) \leqslant Dist(w', \Psi)$ (i.e. $Dist(w, \Psi) = 0$ iff w is a model of each $\psi_i \in \Psi$). Based on the pre-order entailed by $Dist$, belief merging is characterized as follows:

$$[\![\Delta_\mu(\Psi)]\!] = \text{Min}_{\leqslant_\Psi}([\![\mu]\!])$$

Belief Merging can also be characterized by the following set of postulates:

(IC0) $\Delta_\mu(\Psi) \vdash \mu$

(IC1) If μ is consistent, then $\Delta_\mu(\Psi)$ is consistent

(IC2) If Ψ is consistent with μ, then $\Delta_\mu(\Psi) = \bigwedge \Psi \wedge \mu$

(IC3) If $\Psi_1 \leftrightarrow \Psi_2$ and $\mu_1 \leftrightarrow \mu_2$ then $\Delta_{\mu_1}(\Psi_1) \leftrightarrow \Delta_{\mu_2}(\Psi_2)$

(IC4) If $\psi \vdash \mu$ and $\psi' \vdash \mu$, then $\Delta_\mu(\psi \sqcup \psi') \wedge \psi \nvdash \perp \Rightarrow \Delta_\mu(\psi \sqcup \psi') \wedge \psi' \nvdash \perp$

(IC5) $\Delta_\mu(\Psi_1) \wedge \Delta_\mu(\Psi_2) \vdash \Delta_\mu(\Psi_1 \sqcup \Psi_2)$

(IC6) If $\Delta_\mu(\Psi_1) \wedge \Delta_\mu(\Psi_2)$ is consistent, then $\Delta_\mu(\Psi_1 \sqcup \Psi_2) \vdash \Delta_\mu(\Psi_1) \wedge \Delta_\mu(\Psi_2)$

(IC7) $\Delta_{\mu_1}(\Psi) \wedge \mu_2 \vdash \Delta_{\mu_1 \wedge \mu_2}(\Psi)$

(IC8) If $\Delta_{\mu_1}(\Psi) \wedge \mu_2$ is consistent, then $\Delta_{\mu_1 \wedge \mu_2}(\Psi) \vdash \Delta_{\mu_1}(\Psi) \wedge \mu_2$

The postulates describe the principles that belief merging operators should satisfy. Among these principles, syntax irrelevance (IC3), minimal change (IC2) and fairness (IC4) are the key postulates. Notice that fairness is an underlying principle of numerous other postulates (such as IC5 and IC6). Hence, Belief Merging can be expressed in an equivalent way in terms of selection of models and postulates.

There are two main belief merging operators: *Arbitration* (Revesz 1993; Liberatore and Schaerf 1998) and *Majority* (Lin and Mendelzon 1999). These operators follow the belief merging semantical definition. For both, distances are based on Dalal distance (Dalal 1988) which evaluates the closeness in terms of truth values of the propositional symbols.

$$\begin{aligned}\text{DIST}(w_i, w_j) = \quad & \{p \in P \mid p \in w_i \text{ and } p \notin w_j\} \cup \\ & \{p \in P \mid p \notin w_i \text{ and } p \in w_j\}\end{aligned}$$

Therefore, the distance between a model w and a belief base ψ is given by the minimal distance between w and a model $w_j \in \psi$.

$$\text{DIST}(w, \psi) = min(\mid \text{DIST}(w, w_j) \mid \ \mid w_j \in [\![\psi]\!])$$

Arbitration operator, noted Δ^{Max}, retains as much as possible the information of the initial belief bases. The distance between a model w and a profile Ψ is given by the maximum distance calculated between w and the bases ψ in Ψ:

$$Dist_{Max}(w, \Psi) = Max_{\psi_i \in \Psi} \mid \text{DIST}(w, \psi_i) \mid$$

The following total pre-order is assumed: the model w is preferred to the model w' iff the maximal distance from w to profile Ψ is smaller than the maximal distance from w' to Ψ.

$$w \leqslant_\Psi^{Max} w' \text{ iff } Dist_{Max}(w, \Psi) \leq Dist_{Max}(w', \Psi)$$

Majority operator, noted Δ^Σ, retains information that is believed by the majority. The distance between a model w and a profile Ψ is given by the sum of the calculated distances between w and ψ in Ψ:

$$Dist_\Sigma(w, \Psi) = \sum_{\psi_i \in \Psi} \mid \text{DIST}(w, \psi_i) \mid$$

The following total pre-order is assumed: the model w is preferred to the model w' iff the sum of the calculated distances from w to profile Ψ is smallest than the sum of the calculated distances from w' to Ψ.

$$w \leqslant_\Psi^\Sigma w' \text{ iff } Dist_\Sigma(w, \Psi) \leq Dist_\Sigma(w', \Psi)$$

Example 2 *Consider the merging of the belief bases* $\Psi = \{\psi_1, \psi_2, \psi_3\}$*, such that* $\psi_1 = \psi_2 = (p_1 \wedge p_2) \vee (p_2 \wedge p_3 \wedge p_4)$*,* $\psi_3 = (\neg p_3 \wedge p_4 \wedge \neg p_5)$*. Consider the integrity constraint* $\mu = (\neg p_1 \wedge \neg p_3 \wedge \neg p_4 \wedge p_5) \vee (\neg p_2 \wedge \neg p_3 \wedge p_5)$*. Table 1 presents the minimal distances calculated between models of* μ *and profile* Ψ*, as well as the values calculated using the Arbitration and Majority operators.*

The merging belief base is given by selecting the models of μ *according to the total pre-order of each operator.*

$$[\![\Delta_\mu^{Max}(\Psi)]\!] = [\![\Delta_\mu^\Sigma(\Psi)]\!] = \{\{p_1, \neg p_2, \neg p_3, p_4, p_5\}\}$$

$w \in [\![\mu]\!]$	ψ_1	ψ_2	ψ_3	Arb	Maj
$\{\neg p_1, p_2, \neg p_3, \neg p_4, p_5\}$	1	1	2	2	4
$\{\neg p_1, \neg p_2, \neg p_3, \neg p_4, p_5\}$	2	2	2	2	6
$\{\neg p_1, \neg p_2, \neg p_3, p_4, p_5\}$	2	2	1	2	5
$\{p_1, \neg p_2, \neg p_3, \neg p_4, p_5\}$	1	1	2	2	4
$\{p_1, \neg p_2, \neg p_3, p_4, p_5\}$	1	1	1	1	3

Table 1: Minimal distances between models μ and profile Ψ

Syntactical Belief Merging

In (Marchi, Bittencourt, and Perrussel 2010), the authors presented a syntactical approach to belief revision. We extend that approach to belief merging by considering the similarity between those processes. To perform the belief merging process in a syntactical way, we use the property of the prime implicants representation, i.e. each prime implicant represents a set of models. It means that once a prime implicant is changed, all related models are also changed.

The first step is to obtain the set of prime implicants of the profile $\Psi = (\psi_1 \ldots \psi_n)$, given by $IP_\Psi = (IP_{\psi_1}, \ldots, IP_{\psi_n})$, and the set of prime implicants of the integrity constraint μ, given by IP_μ.

In the second step, we extend each term D_μ^k in IP_μ with all contradictory literal of each term $D_{\psi_i}^j$ belonging to the base of IP_{ψ_i}.

$$\Gamma = \{D \mid \ D = D_\mu^k \cup (D_{\psi_i}^j - \overline{D_\mu^k})\}$$

The distance between terms is given by the contradictory literals between terms $D^j_{\psi_i}$ and D^k_μ:

$$\kappa(D^j_{\psi_i}, D^k_\mu) = D^j_{\psi_i} \cap \overline{D^k_\mu}$$

Example 3 *Consider merging of belief profile Ψ presented in example 2 represented by prime implicants $IP_\Psi = (IP_{\psi_1}, IP_{\psi_2}, IP_{\psi_3})$ and the integrity constraint μ, given by IP_μ, as follows:*

$$\begin{aligned} IP_{\psi_1} =& 1 : p_1 \wedge p_2, 2 : p_2 \wedge p_3 \wedge p_4 \\ IP_{\psi_2} =& 1 : p_1 \wedge p_2, 2 : p_2 \wedge p_3 \wedge p_4 \\ IP_{\psi_3} =& 1 : \neg p_3 \wedge p_4 \wedge \neg p_5 \\ IP_\mu =& 1 : \neg p_1 \wedge \neg p_3 \wedge \neg p_4 \wedge p_5, 2 : \neg p_2 \wedge \neg p_3 \wedge p_5 \end{aligned}$$

Where terms were numbered in order to indicate which terms D_{ψ_i} and D_μ compound each term in Γ. The following table presents terms in the set Γ as well as the set of contradictory literals κ for each term $D^j_{\psi_i}$ in the profile:

$D^j_{\psi_i} \times D^k_\mu$	$D = D^k_\mu \cup (D^j_{\psi_i} - \overline{D^k_\mu}) \in \Gamma$	$\kappa(D^j_{\psi_i}, D^k_\mu)$
$D^1_{\psi_1} \times D^1_\mu$	$\{\neg p_1, p_2, \neg p_3, \neg p_4, p_5\}$	$\{p_1\}$
$D^2_{\psi_1} \times D^1_\mu$	$\{\neg p_1, p_2, \neg p_3, \neg p_4, p_5\}$	$\{p_3, p_4\}$
$D^1_{\psi_1} \times D^2_\mu$	$\{p_1, \neg p_2, \neg p_3, p_5\}$	$\{p_2\}$
$D^2_{\psi_1} \times D^2_\mu$	$\{\neg p_2, \neg p_3, p_4, p_5\}$	$\{p_2, p_3\}$
$D^1_{\psi_2} \times D^1_\mu$	$\{\neg p_1, p_2, \neg p_3, \neg p_4, p_5\}$	$\{p_1\}$
$D^2_{\psi_2} \times D^1_\mu$	$\{\neg p_1, p_2, \neg p_3, \neg p_4, p_5\}$	$\{p_3, p_4\}$
$D^1_{\psi_2} \times D^2_\mu$	$\{p_1, \neg p_2, \neg p_3, p_5\}$	$\{p_2\}$
$D^2_{\psi_2} \times D^2_\mu$	$\{\neg p_2, \neg p_3, p_4, p_5\}$	$\{p_2, p_3\}$
$D^1_{\psi_3} \times D^1_\mu$	$\{\neg p_1, \neg p_3, \neg p_4, p_5\}$	$\{p_4, \neg p_5\}$
$D^1_{\psi_3} \times D^2_\mu$	$\{\neg p_2, \neg p_3, p_4, p_5\}$	$\{\neg p_5\}$

Table 2: Terms in Γ and the contradictory literals.

In a semantical way, a belief merging process consists of finding the models of the integrity constraint that are the closest to the models of the initial belief bases. In our context, syntactical belief merging using prime implicants means choosing the terms in Γ that have the smallest set of contradictory literals: the value of $\mid k(D_\psi, D_\mu) \mid$ is minimal. We perform this operation for each term D^k_μ and for each belief base IP_{ψ_i} of IP_Ψ.

$$d(D^k_\mu, IP_{\psi_i}) = \min(\{|k(D^j_{\psi_i}, D^k_\mu)| \mid D_{\psi_i} \in IP_{\psi_i}\})$$

To define syntactical Arbitration and Majority operators, we introduce a syntactical criteria of distance and a total pre-order. Our syntactical Arbitration operator $DNF_{\Delta^{Max}_\mu}$ is based on the following distance:

$$Dist_{Max}(D^k_\mu, IP_\Psi) = Max_{IP_{\psi_i} \in IP_\Psi} d(D^k_\mu, IP_{\psi_i})$$

and the following total pre-order:

$$D \leqslant^{Max}_{IP_\Psi} D' \text{ iff } Dist_{Max}(D, IP_\Psi) \leq Dist_{Max}(D', IP_\Psi)$$

Syntactical Majority operator, denoted by $DNF_{\Delta^\Sigma_\mu}$, implements the following distance:

$$Dist_\Sigma(D^k_\mu, IP_\Psi) = \sum_{IP_{\psi_i} \in IP_\Psi} d(D^k_\mu, IP_{\psi_i})$$

and the total pre-order is given by:

$$D \leqslant^\Sigma_{IP_\Psi} D' \text{ iff } Dist_\Sigma(D, IP_\Psi) \leq Dist_\Sigma(D', IP_\Psi)$$

To compose the merging belief base, we choose the terms in Γ associate with terms D^k_μ whose values are minimal according to the applied operator:

$$\Delta_\mu(\Psi) = Min_\leqslant(\Gamma)$$

Example 4 *Consider example 3. For terms D^k_μ and for belief bases IP_{ψ_i} we observe the distances given by the cardinality of the set κ, i.e. $\mid \kappa(D^k_\mu, D^j_{\psi_i}) \mid$. Table 3 presents the minimal distances $d(D^k_\mu, IP_{\psi_i})$. Applying the distances definitions and the total pre-order, Majority and Arbitration operators lead to the following results:*

Bold values indicate which term is selected by the operators w.r.t. the pre-orders. The resulting belief base is given by terms of Γ relate to D^2_μ with minimal $\mid \kappa(D^2_\mu, D^j_{\psi_i}) \mid$; that is:

D^i_μ	$d(D^i_\mu, IP_\Psi)$: ψ_1	ψ_2	ψ_3	$Dist_{Max}$	$Dist_\Sigma$
$\{\neg p_1, \neg p_3, \neg p_4, p_5\}$	1	1	2	2	4
$\{\neg p_2, \neg p_3, p_5\}$	1	1	1	**1**	**3**

Table 3: Minimal distances $d(D_\mu, IP_\Psi)$.

$$\begin{aligned} DNF_{\Delta^{Max}_\mu} = DNF_{\Delta^\Sigma_\mu} = \; & (p_1 \wedge \neg p_2 \wedge \neg p_3 \wedge p_5) \vee \\ & (\neg p_2 \wedge \neg p_3 \wedge p_4 \wedge p_5) \end{aligned}$$

The syntactical belief merging process gives different results that the ones obtained by the semantical process. This is due to the fact that each prime implicant represents a set of models and the syntactical operators will choose minimal subsets of models of profile IP_Ψ that are the closest to the integrity constraint. In the previous example 4, the result of the belief merging has three models, i.e:

$$\begin{aligned} \{&\{p_1, \neg p_2, \neg p_3, p_4, p_5\}, \\ &\{p_1, \neg p_2, \neg p_3, \neg p_4, p_5\}, \\ &\{\neg p_1, \neg p_2, \neg p_3, p_4, p_5\}\} \end{aligned}$$

against only one model returned by the semantical operators (example 2),

$$\{p_1, \neg p_2, \neg p_3, p_4, p_5\}$$

It means that literals $\{\neg p_1, \neg p_4\}$ were preserved.

This syntactical characterization satisfies the belief merging postulates previously presented:

Theorem 1 *For any profile Ψ and integrity constraint μ, the prime implicant-based merging process defined as $\Delta_\mu(\Psi) = Min_\leqslant(\Gamma)$ satisfies postulates (IC0)–(IC8).*

Sketch of the proof The proof is mainly based on the definition of a total pre-order over models entailed by the total pre-order defined over implicants. Once this pre-order is defined, it means that our merging operator can be expressed in terms of models as shown in the previous section.
The consequence is that our arbitration and majority operators satisfy postulates **(IC0)** through **(IC8)**.

A Quantum as Minimal Change Unit

One of the most important principles that guide the belief merging process is the minimal change. In general, the truth value of a propositional symbol is considered as a minimal change unit, as proposed by Dalal (Dalal 1988). This extralogical choice has a significant impact over the resulting belief base; this problem is getting clearer if the belief base is a conjunction of clauses. In (Marchi, Bittencourt, and Perrussel 2010; Bittencourt, Perrussel, and Marchi 2004), a new minimal change unit was introduced. Bittencourt *et al.* argued that the minimal change unit should be the literal and its context in a belief base, and they proposed to consider a clause in the set of prime implicates as the new minimal knowledge unit.
In this section, we investigate the behavior of the belief merging processes when using the minimal knowledge unit proposed by Bittencourt *et al.* In order to measure how many clauses are involved in a merging process, the set of exclusive conjunctive coordinates is taking in account: we only consider the *critical* clauses that are related with the literals in the contradictory set, given by $D^j_{\psi_i} \cap \overline{D^k_\mu}$. Therefore, we redefine the distance between terms κ as:

$$\widehat{\kappa}(D^j_{\psi_i}, D^k_\mu) = \cup_{n=1}^m \widehat{F}_c^n$$

where $\widehat{F}_c^n$ is the set of exclusive conjunctive coordinates associated with each literal $L_n \in D^j_{\psi_i} \cap \overline{D^k_\mu}$.
The minimal distance between term D^k_μ and belief base IP_{ψ_i} is given by smallest value of $\widehat{\kappa}$:

$$\widehat{d}(D^k_\mu, IP_{\psi_i}) = \min(\{|\widehat{\kappa}(D^j_{\psi_i}, D^k_\mu)| \mid D_{\psi_i} \in IP_{\psi_i}\})$$

Using this minimal change notion we can redefine syntactical Majority and Arbitration operators. The new Arbitration operator $DNF_{\widehat{\Delta}^{Max}_\mu}$ uses the following distance:

$$Dist_{Max}(D^k_\mu, IP_\Psi) = Max_{IP_{\psi_i} \in IP_\Psi} \widehat{d}(D^k_\mu, IP_{\psi_i})$$

The total pre-order is defined as before and is denoted by $\leqslant^{\widehat{Max}}_{IP_\Psi}$.
The distance criteria used by the syntactical majority operator based on the exclusive coordinates, noted $DNF_{\widehat{\Delta}^\Sigma_\mu}$, is:

$$Dist_\Sigma(D^k_\mu, IP_\Psi) = \sum_{IP_{\psi_i} \in IP_\Psi} \widehat{d}(D^k_\mu, IP_{\psi_i})$$

Again, we obtain a new total pre-order entailed by the new distance:

$$D \leqslant^{\widehat{\Sigma}}_{IP_\Psi} D' \text{ iff } Dist_\Sigma(D, IP_\Psi) \leq Dist_\Sigma(D', IP_\Psi)$$

We conclude the definition of the new merging operators by stating how the Belief Merging postulates are satisfied.

Theorem 2 *For any profile Ψ and integrity constraint μ, the implicant-based merging process defined as $\Delta_\mu(\Psi) = Min_\leqslant(\Gamma)$ such that $\leqslant = \leqslant^{\widehat{Max}}_{IP_\Psi}$ or $\leqslant = \leqslant^{\widehat{\Sigma}}_{IP_\Psi}$ satisfies postulates **(IC0)–(IC8)**.*

Sketch of the proof The proof is similar to the one given for theorem 1. That is, we build up a pre-order over interpretation which is entailed by the pre-orders over terms.

Example 5 *Consider the merging process performed over a profile $\Psi = \{\psi_1, \psi_2\}$ and integrity constraint μ, as presented bellow by prime implicants, prime implicates and quantum notation:*

$$IP_{\psi_1} = 1 : p_1^{\{1,2\}} \wedge p_2^{\{3\}}, 2 : p_2^{\{3\}} \wedge p_3^{\{1\}} \wedge p_4^{\{2\}}$$
$$PI_{\psi_1} = 1 : p_1^{\{1\}} \vee p_3^{\{2\}}, 2 : p_1^{\{1\}} \vee p_4^{\{2\}}, 3 : p_2^{\{1,2\}}$$

$$IP_{\psi_2} = 1 : p_1^{\{1\}} \wedge p_2^{\{2\}}, 2 : \neg p_3^{\{3\}} \wedge p_4^{\{4\}} \wedge \neg p_5^{\{5\}}$$
$$PI_{\psi_2} = 1 : p_1^{\{1\}}, 2 : p_2^{\{1\}}, 3 : \neg p_3^{\{2\}}, 4 : p_4^{\{2\}}, 5 : \neg p_5^{\{2\}}$$

$$IP_\mu = 1 : \neg p_1^{\{1\}} \wedge \neg p_3^{\{2\}} \wedge \neg p_4^{\{3\}} \wedge p_5^{\{4\}}, \quad 2 : \neg p_2^{\{1,2\}} \wedge \neg p_3^{\{3\}} \wedge p_5^{\{4\}}$$
$$PI_\mu = 1 : \neg p_1^{\{1\}}, 2 : \neg p_2^{\{2\}} \vee \neg p_3^{\{1\}}, 3 : \neg p_2^{\{2\}} \vee p_4^{\{1\}}, \quad 4 : p_5^{\{1,2\}}$$

The following table presents the set Γ calculated over IP_Ψ and IP_μ, as well as the contradictory sets of literals and the distances given by the exclusive coordinates associated with these sets:

$D = D^k_\mu \cup (D^j_{\psi_i} - \overline{D^k_\mu})$	$D^j_{\psi_i} \cap \overline{D^k_\mu}$	$\mid \cup_{n=1}^m \widehat{F}_c^n \mid$
$\{\neg p_1, p_2, \neg p_3, \neg p_4, p_5\}$	$\{p_1^{\{\boxed{1,2}\}}\}$	2
$\{\neg p_1, p_2, \neg p_3, \neg p_4, p_5\}$	$\{p_3^{\{\boxed{1}\}}, p_4^{\{\boxed{2}\}}\}$	2
$\{p_1, \neg p_2, \neg p_3, p_5\}$	$\{p_2^{\{\boxed{3}\}}\}$	1
$\{\neg p_2, \neg p_3, p_4, p_5\}$	$\{p_2^{\{\boxed{3}\}}, p_3^{\{\boxed{1}\}}\}$	2
$\{\neg p_1, p_2, \neg p_3, \neg p_4, p_5\}$	$\{p_1^{\{\boxed{1}\}}\}$	1
$\{\neg p_1, \neg p_3, \neg p_4, p_5\}$	$\{p_4^{\{\boxed{4}\}}, \neg p_5^{\{\boxed{5}\}}\}$	2
$\{p_1, \neg p_2, \neg p_3, p_5\}$	$\{p_2^{\{\boxed{2}\}}\}$	1
$\{\neg p_2, \neg p_3, p_4, p_5\}$	$\{\neg p_5^{\{\boxed{5}\}}\}$	1

Taking the minimal distances for each term D^k_μ, we have:

D^k_μ	$d(D^k_\mu, IP_\Psi)$ ψ_1	ψ_2	$Dist_{Max}$	$Dist_\Sigma$
$\{\neg p_1, \neg p_3, \neg p_4, p_5\}$	2	1	2	3
$\{\neg p_2, \neg p_3, p_5\}$	1	1	**1**	**2**

Where the bold values indicate the terms chosen by Majority and Arbitration operators. The merging belief base is given *by:*

$$DNF_{\widehat{\Delta}_{\mu}^{Max}} = DNF_{\widehat{\Delta}_{\mu}^{\Sigma}} = \begin{array}{l}(p_1 \wedge \neg p_2 \wedge \neg p_3 \wedge p_5) \vee \\ (\neg p_2 \wedge \neg p_3 \wedge p_4 \wedge p_5)\end{array}$$

Note that clause 3 of PI_{ψ_1} and clauses 2 and 5 of PI_{ψ_2} were changed by the merging process. Its means that clauses 1 and 2 of PI_{ψ_1} and clauses 1, 3 and 4 of PI_{ψ_2} were preserved. If the usual measure, that is, the number of contradictory literals was taken, both terms of IP_μ will be chosen and the resultant belief base will include, beyond terms present above, the following term: $(\neg p_1 \wedge p_2 \wedge \neg p_3 \wedge \neg p_4 \wedge p_5)$, that is minimal because literal p_1 of IP_{ψ_1} and, consequently, clauses 1 and 2 of IP_{ψ_1} and clause 1 of IP_{ψ_2} will also be changed.

Conclusion

In this paper we have extended the syntactical approach proposed by Bittencourt *et al.* (Marchi, Bittencourt, and Perrussel 2010) to belief merging processes. We redefine Majority and Arbitration operators in a syntactical way, by considering the belief merging over sets of models, that is, the prime implicants set of belief bases and integrity constraint. We show that this syntactical approach gives us different results than when we consider all models of the belief bases and the integrity constraint. This result proposes a new kind of belief merging which still takes care of the three key issues: consistency, minimal change and fairness; and at the opposite of (Perrussel, Marchi, and Bittencourt 2008), our approach of belief merging process satisfies all the belief merging postulates.

The advantage of our approach is that in general, we have less prime implicants than models and prime implicants is a natural way to handle minimal change (minimal set of relevant symbols) and fairness (combination of implicants).

We also presented the behavior of belief merging process when a new minimal change unit is considered. Using a special notation named *quantum notation*, we consider a clause in the set of prime implicates as minimal knowledge unit and we show that this choice allows to preserve more clauses compared to the the classical Dalal minimal distance. This new way to merge beliefs enable to stress up the minimal change issue. We have shown that this distance entails a well-behaved definition of belief merging operators.

References

Bittencourt, G.; Marchi, J.; and Padilha, R. S. 2003. A Syntactic Approach to Satisfaction. In Konev, B., and Schimidt, R., eds., 4^{th} *International Workshop on the Implementation of Logic (LPAR03)*, 18–32. University of Liverpool and University of Manchester.

Bittencourt, G.; Perrussel, L.; and Marchi, J. 2004. A syntactical approach to revision. In Mántaras, R. L., and Saitta, L., eds., *Proceedings of the 16th Europ. Conf. on Artificial Intelligence (ECAI'04)*, 788–792. Valencia, Spain: IOS Press.

Bittencourt, G. 1998. Concurrent inference through dual transformation. *Logic Journal of the IGPL* 6(6):795–834.

Dalal, M. 1988. Investigations Into a Theory of Knowledge Base Revision: Preliminary Report. In Rosenbloom, P., and Szolovits, P., eds., *Proceedings of the 7th National Conference on Artificial Intelligence (AAAI'98)*, volume 2, 475–479. Menlo Park, California: AAAI Press.

Forbus, K. D. 1989. Introducing Actions into Qualitative Simulation. In *Proceedings of the 11th International Joint Conference on Artificial Intelligence (IJCAI'89)*, 1273–1278.

Jackson, P. 1990. Computing Prime Implicants. In *Proceedings of the 10th International Conference on Automatic Deduction*, 543–557. Kaiserslautern, Germany: Springer Verlag LNAI 449.

Kean, A., and Tsiknis, G. 1990. An Incremental Method for Generating Prime Implicants/Implicates. *Journal of Symbolic Computation* 9:185–206.

Konieczny, S., and Pérez, R. P. 2002. Merging information under constraints: a logical framework. *Journal of Logic and Computation* 12(5):773–808.

Konieczny, S.; Lang, J.; and Marquis, P. 2002. Distance-based merging: a general framework and some complexity results. In *Proceedings of KR'02*, 97–108.

Liberatore, P., and Schaerf, M. 1998. Arbitration (or how to merge knowledge bases). *IEEE Transactions on Knowledge and Data Engineering* 10(1):76–90.

Lin, J., and Mendelzon, A. O. 1999. Knowledge base merging by majority. In *In Dynamic Worlds: From the Frame Problem to Knowledge Management*. Kluwer.

Marchi, J.; Bittencourt, G.; and Perrussel, L. 2010. Prime forms and minimal change in propositional belief bases. *Annals of Mathematics and Artificial Intelligence* 1–45. http://www.springerlink.com/content/cx18088m4r051280/fulltext.pdf.

Perrussel, L.; Marchi, J.; and Bittencourt, G. 2008. Quantum-based belief merging. In *Proceedings of the 11th Ibero-American Conference on AI (IBERAMIA'08)*, 21–30. Lisbon, Portugal: Springer-Verlag.

Revesz, P. Z. 1993. On the semantics of theory change: Arbitration between old and new information. In *In Proceedings of the Twelfth ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Databases*, 71–82.

Satoh, K. 1988. Nonmonotonic Reasoning by Minimal Belief Revision. In *Proceedings of the International Conference on Fifth Generation Computer Systems (FGCS'88)*, 455–462. Tokyo, Japan: Springer-Verlag.

Socher, R. 1991. Optimizing the Clausal Normal Form Transformation. *Journal of Automated Reasoning* 7(3):325–336.

Winslett, M. 1988. Reasoning About Action Using a Possible Models Approach. In *Proceedings of the 7th National Conference on Artificial Intelligence (AAAI'88)*, 89–93.