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Research about 3-Color, 2 Direction Mobile Automata

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Abstract

This paper studies 3-state, 2-direction Mobile Automata. The results of this study show that although it is more difficult to find complexity in Mobile Automata than Cellular Automata, 3-color Mobile Automata can still be divided into four classes of complexity, thus producing complex behavior. There are 6^{27} number of 3-color Mobile Automata, which were studied and filtered to prove the complexity of Mobile automata. The results of this study infer that it is possible to observe complexity in systems that contain only one active cell, if the system has more then two states.

Stephen Wolfram (Wolfram 2002; 1984) has shown through experiments that among all kinds of Cellular Automata, it seems that the patterns which arise can almost always be assigned quite easily to one of just four basic classes illustrated below. These classes are conveniently numbered in order of increasing complexity, and each one has certain immediate distinctive features. These classes have been widely used in literature (Langton 1990; Dhar et al. 1995).
In class 1, the behavior is very simple, and almost all initial conditions lead to exactly the same uniform final state.
In class 2, there are many different possible final states, but all of them consist just of a certain set of simple structures that either remain the same forever or repeat every few steps.
In class 3, the behavior is more complicated, and seems in many respects random, although triangles and other small-scale structures are essentially always at some level seen.
And class 4 involves a mixture of order and randomness: localized structures are produced which on their own are fairly simple, but these structures move around and interact with each other in very complicated ways.

In A New Kind of Science (Wolfram 2002, pages 71-77) Wolfram defines Mobile Automata. These automata are similar to Cellular Automata except that instead of updating all cells in parallel, they have just a single active cell that gets updated at each step and they have rules that specify how this active cell should move from one step to the next. A more detailed definition of Mobile Automata is given in the next section. Wolfram's experiments with 2-color Mobile Automata showed that they can not be divided into the same 4 classes as Cellular Automata. Particularly, out of 65536 possible 2-color Mobile Automata explored, it was shown that not a single one produces any complex behavior.

In 2007, at the New Kind of Science Summer School Stephen Wolfram and Todd Rowland suggested that I explore the space of 3-color Mobile Automata. We hypothesized that with 3-colors more complex behavior may be produced. In this paper, we show that this hypothesis was correct. One of the biggest challenges was to find methods for exploring such a huge number of automata. As complex behavior can not be specified, there are no programs able to properly identify the desirable results, therefore human analysis was needed to explore 6^{27} Mobile Automata. Several programs were written in *Mathematica* (Software 1988) to identify and isolate potentially interesting automata, after which the output was manually checked in order to identify 3-color Mobile Automata with more complex behavior.

The results of the experiment showed that, although it is rare to see complex behavior in 3-color Mobile Automata, compared to Cellular Automata, 3-color Mobile Automata can still produce some complex behavior. Thus, according to the Principle of Computational Equivalence (Wolfram 2002), a 3-color Mobile Automata can produce as complex behavior as, for example, the Cellular Automata rule number 30.

Formal definition of Mobile Automata

A mobile automaton is a sequence of "colored" cells arranged in a line, where the color of each cell may change from one discrete time step to the next. In each step there is one cell, which is called the active cell, and only that cell is updated in the next step, according to a set of rules based on the colors its neighboring cells. The rules apply only to the active cell, and also specify how the active cell moves from one step to the next. All cells that are not active remain the

same from one step to the next. The rules are then applied iteratively for as many time steps as desired. Therefore, Mobile Automata are a class of automata similar to Cellular Automata, but in Cellular Automata all cells are active. Mobile Automata can also be considered a hybrid between Cellular Automata and Turing machines.
A graphical example of a 3-color Mobile Automaton is shown in the Figure 1.

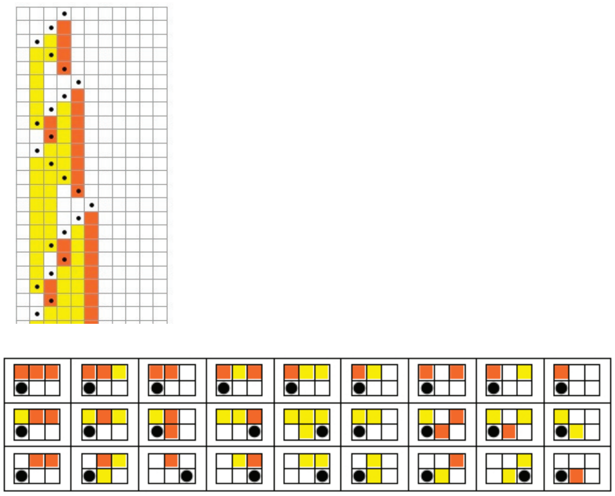


Figure 1: 3-color Mobile Automaton rule number 2000000008894. The black point shows the active cell in every step of evolution. The initial step, at the top, is a sequence of all white cells. Subsequent time steps are shown down the page, below the initial step. The rules for changing the color of the active cell and for moving the active cell are shown to the right of the evolution.

Enumerating 3-color Mobile Automata

In order to systematically study the 3-color Mobile Automata, a distinct non-negative integer is assigned to each automaton and is used to identify that automaton. The scheme for assigning a non-negative integer to the set of rules that define a 3-color Mobile Automaton is described as follows. Since each cell has 3 possible colors, there are $3^3 = 27$ possible combinations of colors for the active cell and its two neighbors. Let 0 represent white, let 1 represent light gray, and let 2 represent dark gray. If a is the number representing the color of the active cell's left neighbor, if b is the number representing the color of the active cell, and if c is the number representing the color of the active cell's right neighbor, then let $i = a3^2 + b3^1 + c3^0$. Note that a, b, and c are the digits of i in base 3 notation. Let d be the non-negative integer corresponding to the color that the active cell becomes on the next step when its neighborhood has the colors corresponding to a, b, and c. And let e be 0 or 1 if the active cell moves left or right, respectively. Define $x_i = 2d + e$. Note that d and e are the digits of x_i in a mixed-base notation, and that there are 6 possible values for x_i. A 3-color Mobile Automaton is identified by the number

$$n = x_{26}6^{26} + x_{25}6^{25} + \cdots + x_2 6^2 + x_1 6^1 + x_0 6^0$$

where x_{26}, x_{25}, ... , x_0 are the digits for n in base 6 notation. This enumeration of Mobile Automata was devised by Matthew Szudzik (Szudzik 2007). Note that n can range from 0 to $6^{27} - 1$.

Methods that were used used to study 3 Mobile Automata

As was shown in the previous section, there are 6^{27} 3-color Mobile Automata. This is a huge number. It is almost equal to the number of grains of sand on Earth, (10^{21}), as stated by *WolframAlpha* (WolframAlpha 2008). One can assume that in order to study this huge number of automata, computational software is necessary. In this study, all the computations were done with *Mathematica* (Software 1988).
As it was mentioned before, complex behavior can not be easily defined, so there are no programs which can properly identify this behavior. Therefore, human analysis was needed. On the other hand, it takes more then a lifetime for a person to look at every 3-color Mobile Automaton. So, a computer was first used to identify uninteresting Mobile Automata (in this case, "uninteresting" means Mobile Automata with repetitive and simple behavior), then, when a reasonable number of automata were left, they are manually checked, and the 3-color Mobile Automata which exhibited interesting complex behavior were identified.

In particular, the following steps were taken:
1. A program was run to exclude all 3-color Mobile Automata which demonstrated only simple, progressively changing behavior of the active cell. Figure 2 shows an example of this kind of behavior.

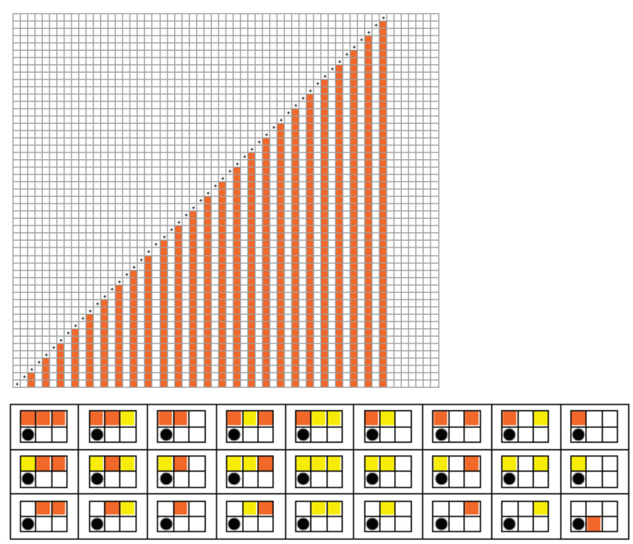


Figure 2: 3 Mobile Automata rule number 4. The initial state is a set of zeros.

2. All automata where the head zig-zagged back and forth, never moving more than one cell to the left or right, were ex-

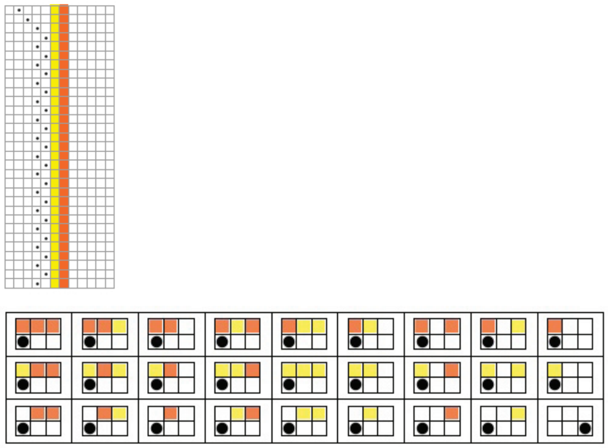


Figure 3: 3 Mobile Automata rule number 1. The initial state is a set of zeros and a 1.

cluded. Figure 3 shows an example of this kind of behavior.

3. Common patterns were identified and automata that repeated the same pattern over and over were excluded. Figure 4 shows an example of this kind of behavior. Some of these automated tests took days to run on a standard desktop computer. A reasonable number of automata remained after these tests, and they could be manually checked. Most of 3-color Mobile Automata that survived these tests, had very interesting complex behavior. Some examples are shown in the following sections.

3-color Mobile Automata classification into groups

The experiments of this study show that it is possible to classify 3-color Mobile Automata into four groups, and make analogies with the classification of Cellular Automata(Wolfram 1983), although Mobile Automata and Cellular Automata are two systems which have very different properties and overall behavior. In order to prove this, it is sufficient to show that for each group there exists one 3-color Mobile Automata which belongs to that group. This statement is true because, according to the classification definition, if a particular automata doesn't belong to any of the first 3 groups, then it belongs to the fourth group. In other words, if the behavior is not repetitive, pattern repetitive, or is not an interaction of repetitive patterns, then it is considered to be complex, thus it should be in the class number 4. Therefore, every 3-color Mobile Automata belongs to one and only one group in our classification. Now lets show that for every class there exists at least one 3-color Mobile Automata that belongs to that class.

3-color Mobile Automata in the first class This class contains all automata that produce very simple and repetitive behavior. It is very common behavior among 3-color Mobile Automata. An interesting observation was that a big number of 3-color Mobile Automata show interesting behavior during the first steps (from 200 to 600), but eventually become

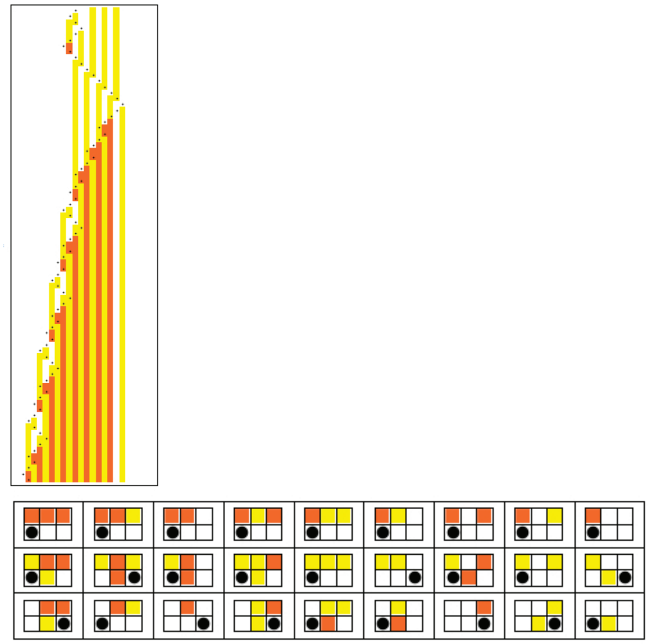


Figure 4: Rule number 50000000011832. The initial state is $\{0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0\}$.

repetitive and simple. Figures 5 and 6 show some examples of automata that belong to class 1.

3-color Mobile Automata in the second class This class contains automata that consist of a certain set of simple structures that either remain the same forever or repeat every few steps sometimes causing nested patterns. Figure 7 shows some examples of automata that belong to class 2. As one can notice, in all these examples the behavior of the active cell is eventually repetitive.

3-color Mobile Automata in the third class This class contains 3-color Mobile Automata whose behavior is more complicated and seems, in many respects, random, although

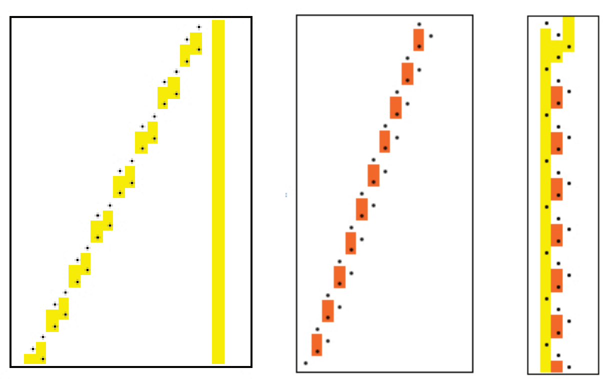


Figure 5: Examples of 3-color Mobile Automata that belong to class 1.

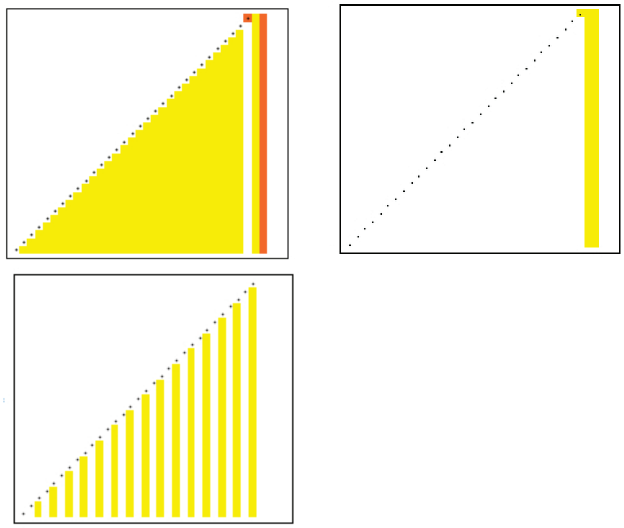


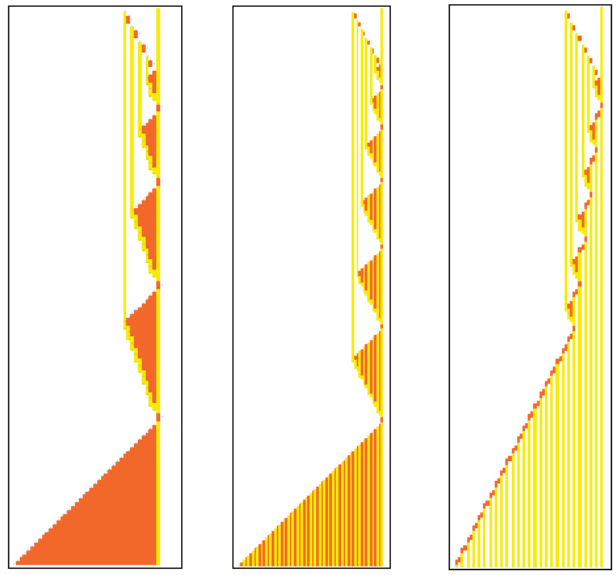
Figure 6: Most common behavior of 3-color Mobile Automata that belong to class 1.

triangles and other small-scale structures are seen at some levels. Figures 8 and 9 show some examples of 3-color Mobile Automata that belong to class 3.

The graph in Figure 9 shows the movement of the head during the evolution. The horizontal axis shows the number of steps, the vertical one the position of the active cell.
Figure 9 shows the 3-color Mobile Automata number 40000000011898 for 400 steps, the arrow shows the magnified version for 30 steps. In Figure 10 above, the initial position of the active cell is 1. Figure 12 shows the compressed forms of the same 3-color Mobile Automata. The graph on the right, shows only the steps where the active cell moved to the right, more then ever before. The results are shown for 1000 steps of evolution. The graph on the left shows only the steps where the active cell is changing its direction (from left to right or backwards). The picture shows 4000 steps of evolution. The magnified version for only 200 steps is shown at the center.

3-color Mobile Automata in the fourth class This class contains 3-color Mobile Automata whose behavior involves a mixture of order and randomness: localized structures are produced which on their own are fairly simple, but these structures move around and interact with each other in very complicated ways. The 3-color Mobile Automata shown in the Figures 13,14 and 15 is an example of this interesting behavior. The first impression about this 3-color Mobile Automata is that it produces a lot of similar patterns, but there is some interesting trend which indicates the unpredictability of the future evolution.The visually larger patterns increase during the evolution, causing all the remaining patterns to change their interactions, thus making the overall behavior irregular. Therefore this 3-color Mobile Automata is very interesting.

Figure 16 shows two compressed forms of the same



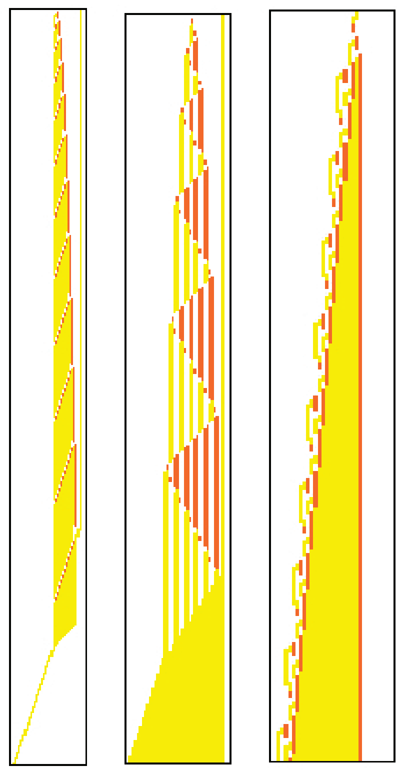


Figure 7: Examples of 3-color Mobile Automata that belong to class 2.

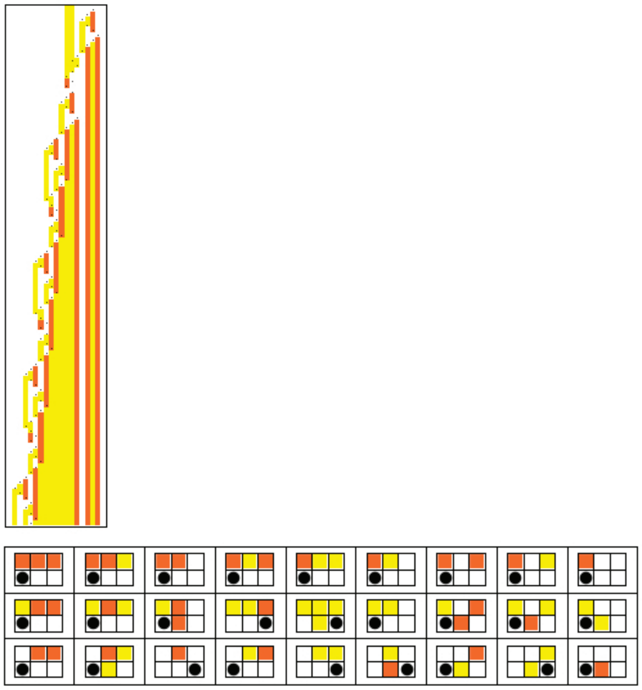


Figure 8: 3-color Mobile Automata class 3. Rule: 2000000001766. Initial state: $\{0, 0, 1, 1, 0, 0\}$

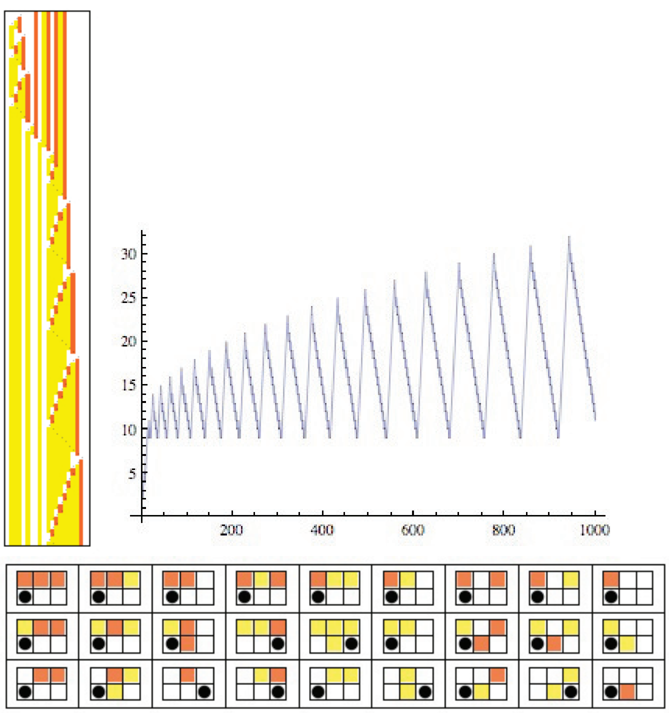


Figure 9: 3-color Mobile Automata class 3. Rule:2000000007814. Initial state: $\{0, 0, 0, 0, 2, 0, 1, 2, 0, 2, 1, 2\}$

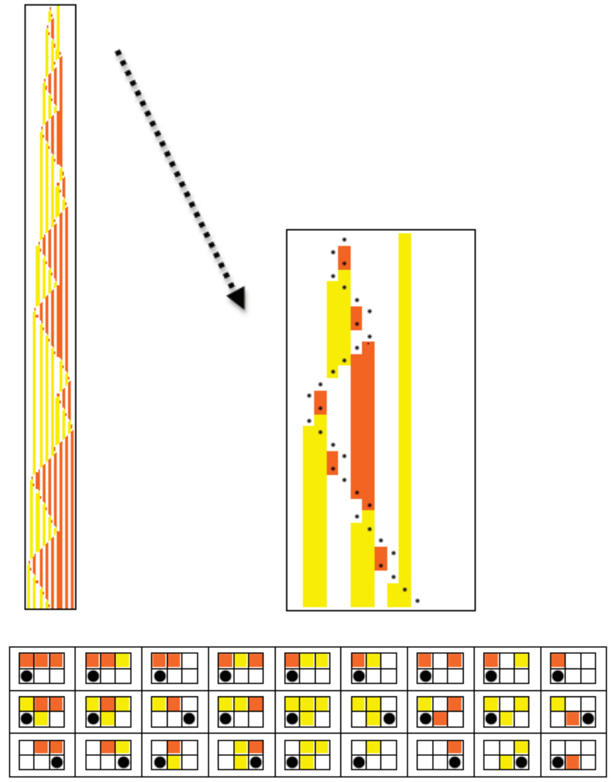


Figure 10: 3-color Mobile Automata class 3. Rule: 40000000011898. Initial state: $\{0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0\}$ Active cell position: 1.

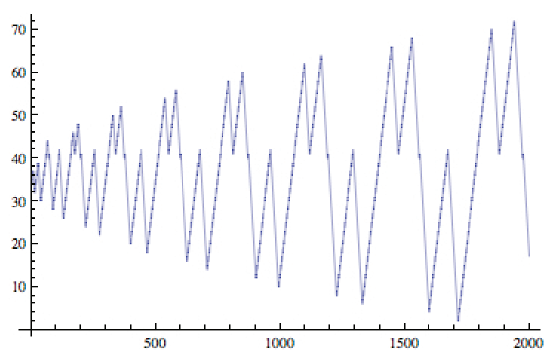


Figure 11: The activity of the head movement

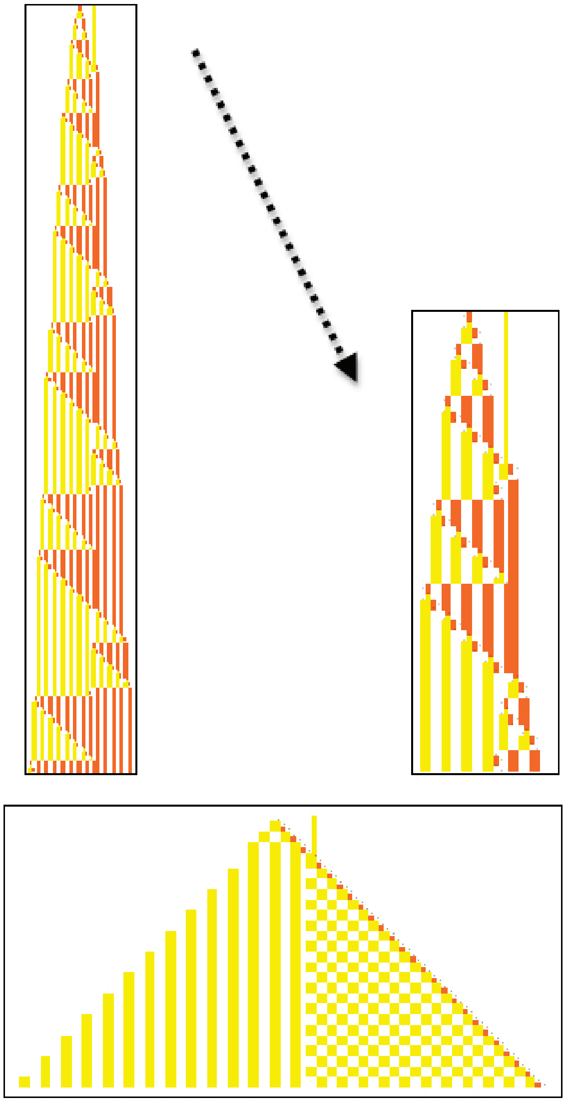


Figure 12: Two compressed forms of 3-color Mobile Automata class 3. Rule: 40000000011898.

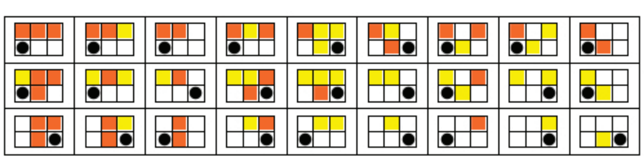


Figure 13: Rule icon of 3-color Mobile Automata class 4. Rule: 513555777855555777.

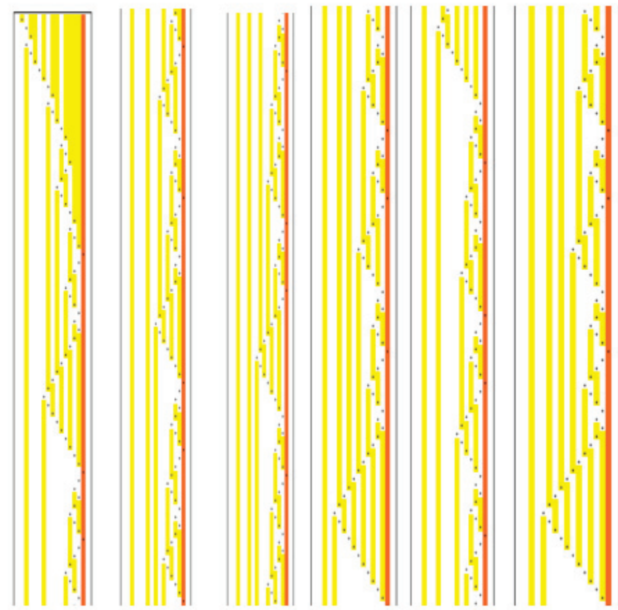


Figure 14: 3-color Mobile Automata class 4. Rule: 513555777855555777.Picture shows evolution for 400 steps.

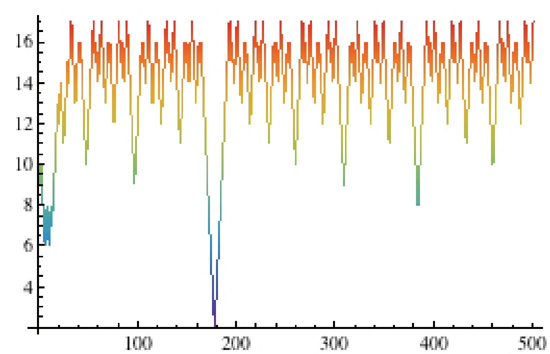


Figure 15: This graph shows the head movement of 3-color Mobile Automata rule 513555777855555777.

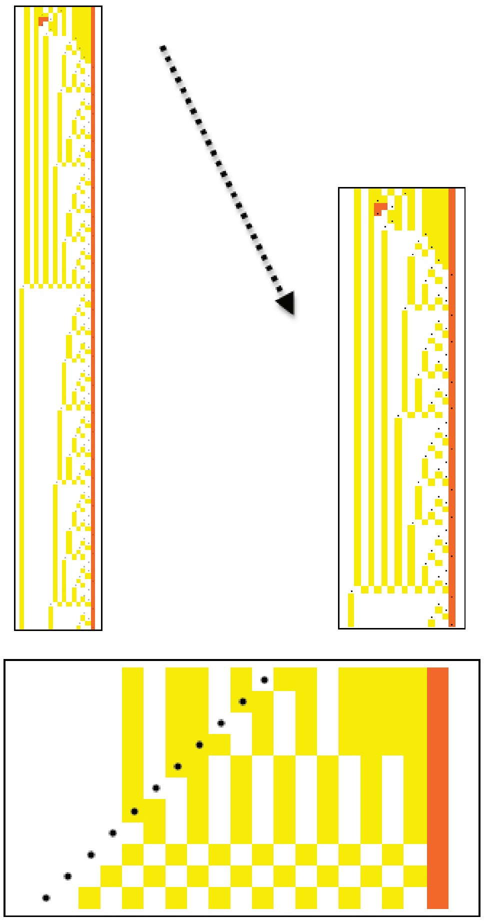


Figure 16: Compressed forms of 3-color Mobile Automata rule 513555777855555777.

3-color Mobile Automata number 513555777855555777. The graph on the right shows only the steps where the active cell moves to the left, more then ever before. The results are shown for 4000 steps of evoluation.The graph on the left shows only the steps where the active cell is changing its direction, either from left to right or vice versa. The picture shows 400 steps of evolution. The magnified version for only 100 steps is shown at the center.

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References

Dhar, A.; Lakdawala, P.; Mandal, G.; and Wadia, S. 1995. Role of initial conditions in the classification of the rule space of cellular automata dynamics. *Physical Review E* 51(4):3032.

Langton, C. 1990. Computation at the edge of chaos: Phase transitions and emergent computation. *Physica D: Nonlinear Phenomena* 42(1-3):12–37.

Software. 1988. Mathematica. Wolfram Research.

Szudzik, M. 2007. The enumeration of computational systems. NKS Summer School notes.

Wolfram, S. 1983. Cellular automata. *Los Almos Science*.

Wolfram, S. 1984. Cellular automata as models of complexity. *Nature* 311(5985):419–424.

Wolfram, S. 2002. *New Kind of Science*. Wolfram Media Inc.

WolframAlpha. 2008. http://www.wolframalpha.com. Computational engine.