

The Investigation of Problems of Healing of Cracks by Injection of Fluids with Inclusions in Various Thermoelastic Media

Alexander G. Bagdoyev, Gohar A. Manukyan

Institute of Mechanics of the National Academy of Sciences Armenia,
Yerevan, Armenia

bagdoyev@mechins.sci.am, avetisus@yahoo.com

Abstract

The purpose of this paper is to investigate the problem of healing of cracks in infinite thermo elastic surroundings by currents of fluids with inclusions. The process of healing is result of growing of layer of sediments on crack's surfaces due to transverse diffusion currents of crystalline inclusions which leads to closing of crack. The obtained analytical formulae for vertical displacements of boundary of crack and graphs of it allow determine coordinates and time moments of zero width of crack, i.e. process of healing of crack. The applications to biological and technological problems when as mixture is used fluid with camphor crystalline as well as to geothermal cracks with fluid currents containing silicon crystalline with application to geophysics, and results of calculations also can be used in meso mechanics. The investigation of obtained curves of dependence of times and coordinates of healing of cracks processes is done as by deterministic treatment, as well as on account of possible randomness, taking places for micro sizes of cracks by methods of nonlinear wave dynamics. The main methods of investigation of healing problem for cracks are taking into account of thermo diffusion and thermo elasticity stresses and displacements effects and tribological supplements in boundary condition on the cracks., and solution of posed problem of thermo elasticity by method of integral transformations of Laplace and Fourier, method of Winner-Hopf. The analytical solution for displacements of mentioned unsteady plane problem is brought to effective Smirnov-Sobolev form. These analytic and numerical methods based on dynamic thermo-elasticity approximation on account of diffusion currents of inclusions in fluid mixture in cracks allow determine time and coordinate of closing of cracks. Besides are examined obtained curves of healing processes by methods of nonlinear waves dynamics. These methods can be applied also to extremely processes of transition from meso level defects to synchronized processes of generation of macro cracks and fracture.

Introduction

Problems on building up of layer of inclusions, containing in fluid entering in crack, which is in infinite thermo elastic plane are considered. These problems were investigated in Lowell et.al (1993), as the problems of healing of fracture by injection of hot hydrothermal mixture with silica cristallines and their sedimentation on

fracture. It was assumed that current and temperature of fluid in point of entrance in fracture are constant on time. In Gliko (2008) is considered mentioned problem when as temperature in point of entrance in system as current of fluid can be function in time.

These problems were solved in Lowell et.al (1993), in Gliko (2008, Ghassemi et al (2005) and in numerous works of other foreign authors, in investigation of healing of cracks by thermo-diffusion effects of sedimentation of crystalline from hydrothermal mixture, by solution of thermo conductivity equations, in application to fracture in geophysical environment, without taking into account of elastic stresses.

Experimental curves of healing of strips, filled by fluid with inclusions in earth environment are done in Bhattacharia Subhamoy et.al. (2008). In Kukudzanov et.al (2010) it is investigated the problem of healing of plane cracks due to cyclic action of electrical current in metallic media. As result in Kukudzanov et.al. (2010) is shown increasing of strength of material under action of electrical current. These results also can be applied in examination of interrelation of nano, micro, meso and macro levels action on defects dynamics. In Panin (2000), in Bagdoyev et.al. (2010) is carried out application of general method of solution of mixed unsteady anti-plane, plane and space problems for cracks by solution of system of Winner-Hopf equations, derivation of formulae for stress-intensity coefficients and numerical calculations of them, which allow apply results by any way in fracture problem.

In present paper mathematical solution by mentioned methods is obtained for important for practice problems of healing of biological cracks with camphor- oil and for geothermal cracks by treatment of the coupled thermo elastic and diffusion mixed boundary problem for crack with fluid-crystalline mixture. Solution of unsteady mixed plane boundary value problem for thermo elasticity is obtained by integral transforms Laplace on time and Fourier on coordinate, is brought to Winner-Hopf equation with continuous coefficients. After reverse integral transformations solution for displacements is brought to

Smirnov-Sobolev integral form. The graphs of vertical displacements on crack surface are calculated numerically and the moments of healing of crack in some points are determined for various values of typical constants for biological, technological and seismological cases. Obtained curves of healing processes of cracks are examined by methods of nonlinear wave dynamics to obtain the probabilities distribution for them.

1. Statement of problem of crack in thermo elastic plane with fluid current.

The problem of semi-infinite crack in thermo elastic plane, when there is current of fluid with inclusions, entering in crack at initial moment $t = 0$, is considered. These problems are especially important for study of process of narrowing of crack due to action of crystallite inclusions in camphor -oil and in analogically by mathematical treatment problem of building up of cracks by inclusions in camphor- oil in corresponding problems of technology in Laboratory of Institute of Mashinovedenia, Pushkino, Moscow region., 2007, in Bhattacharia Subhamoy et al.(2008). In plane x, y equation of crack boundary is $y = \pm b_1(x, t)$, $b_1(x, t) = b_0 + U_y(x, y, t)$, $y \approx 0$, where thickness $2b_1$ is small and later is taken only upper sign, and is solved problem for $y \geq 0$, due to symmetry. U_x, U_y are components of displacements in elastic media. Let the temperature T of elastic plane and fluid are approximately the same, denoting by T_0 constant initial value of T , by $q = \rho_f v b_0$ the current of entering fluid in crack, v —fluid velocity along x axis of crack, ρ_f —density of fluid, i_0 —constant diffusion current of inclusions along y axis, which is supposed known.

In present paper is used treatment based on investigation of processes of healing of fracture, on account also of abovementioned one dimensional investigations in Lowell et.al. (1993), Gliko (2008), Ghassemi et.al. (2005), by taking into account both unsteady two dimensional thermo elastic stresses effects and thermo-diffusivity effects.

In Lowell et.al.(1993) is used thermodiffusion coefficient $\gamma = \frac{\partial c}{\partial T}$, entering in simple equation of mass balance, which supposed known constant, approximately taken for moment $t = 0$, $\gamma = \frac{\partial c_0}{\partial T_0}$. By K let us denote constant coefficient of building up of crack surface. It is written complex statement of problem of narrowing of crack surface on account of thermodiffusivity, tribological term with stresses and transverse diffusion

term of cristallines, one obtains equation on fracture surface $y = b_1(x, t)$, which since b_1 is small can be written on $y = 0$ axis of fracture

$$\rho_s \frac{\partial b_1}{\partial t} = q \gamma \frac{T - T_0}{l} \Big|_{y=0} - K \sigma_{yy} - i_0 H(x) H(t) \quad (1.1)$$

here $H(t)$ is Heaviside unit function, and initial conditions for unsteady problem are zero. The crack equation $y = b_1$, $y = b_0 + U_y(x, 0, t)$, $0 < x < \infty$.

One must put and solve corresponding problem of thermo elasticity by solution of the equations of motion of thermo elastic media (Landau et.al.1954) on account of the stresses components are (Landau et.al.1954) and $v_{x,y}$ and temperature $T - T_0$ relation

$$\begin{aligned} \frac{\sigma_{yy}}{\rho} &= (a^2 - 2b^2) \frac{\partial U_x}{\partial x} + a^2 \frac{\partial U_y}{\partial y} - \frac{K_0}{\rho} \alpha (T - T_0), \\ \frac{K_0}{\rho} &= a^2 - \frac{4}{3} b^2, \quad \frac{\sigma_{xy}}{\rho} = b^2 \left(\frac{\partial U_x}{\partial y} + \frac{\partial U_y}{\partial x} \right) \end{aligned} \quad (1.2)$$

where ρ is density of elastic media, a, b velocities of longitudinal and transverse waves, K_0 is volume modulus of media, α is coefficient of thermal expansion (Landau et.al.1954). Besides there is thermo conductivity equation (Landau et.al.1954)

$$\begin{aligned} \frac{\partial T}{\partial t} + \frac{c_\rho - c_v}{\alpha c_v} \frac{\partial}{\partial t} \left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} \right) &= k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \\ c_\rho - c_v &= \frac{K_0}{\rho} \alpha^2 T_0 \end{aligned} \quad (1.3)$$

where ρ density, c_ρ, c_v specific thermal capacities.

First we neglect thermo-conductivity, $k = 0$, and obtain

$$T - T_0 = - \frac{c_\rho - c_v}{\alpha c_v} \left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} \right) \quad (1.4)$$

Placing (1.4) in equations of motion and (1.2) we can write system of equations:

$$\begin{aligned} a^2 \frac{\partial^2 U_x}{\partial x^2} + b^2 \frac{\partial^2 U_x}{\partial y^2} + (a^2 - b^2) \frac{\partial^2 U_y}{\partial x \partial y} + \\ + \bar{\delta} \frac{\partial}{\partial x} \left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} \right) &= \frac{\partial^2 U_x}{\partial t^2} \\ a^2 \frac{\partial^2 U_y}{\partial y^2} + b^2 \frac{\partial^2 U_y}{\partial x^2} + (a^2 - b^2) \frac{\partial^2 U_x}{\partial x \partial y} \\ + \bar{\delta} \frac{\partial}{\partial y} \left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} \right) &= \frac{\partial^2 U_y}{\partial t^2} + \end{aligned} \quad (1.5)$$

where $\bar{\delta} = \frac{K_0}{\rho} \frac{c_p - c_v}{c_v}$, $K_0 = \lambda + \frac{2}{3}\mu$ - volume elastic modulus, λ, μ -Lame constants, and for stresses

$$\begin{aligned} \frac{\sigma_{yy}}{\rho} &= (a^2 - 2b^2) \left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} \right) \\ &+ 2b^2 \frac{\partial u_y}{\partial y} + \bar{\delta} \left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} \right) + \\ \frac{\sigma_{xy}}{\rho} &= b^2 \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \end{aligned} \quad (1.6)$$

Let us denote $U_x = U$, $U_y = V$.

One can consider further simplified boundary value problem for half-plane: $y = 0$,

$$\begin{aligned} \frac{\partial U}{\partial t} + \frac{dV}{dx} &= 0, -\infty < x < \infty \quad V = 0, x < 0 \\ \frac{\partial V}{\partial t} &= - \left\{ \frac{K(\bar{a}^2 - b^2)\rho}{\rho_s} + \xi \right\} \frac{\partial U}{\partial x} - \\ &- \left(\frac{K\bar{a}^2\rho}{\rho_s} + \xi \right) \frac{\partial V}{\partial y} - \frac{i_0}{\rho_s} H(x)H(t), x > 0 \end{aligned} \quad (1.7)$$

$$\text{where } \xi = \frac{c_p - c_v}{c_v} \frac{vb_0\gamma\rho}{\alpha\rho_s}.$$

The solution is looked for by integral transformations $\bar{U}; \bar{V}$ of Laplace on t and $\bar{\bar{U}}; \bar{\bar{V}}$ Fourier on x . In plane x, y one can write solution as

$$\begin{aligned} \bar{U}; \bar{V} &= \sum_{n=1}^2 \int_{-\infty}^{\infty} e^{i\bar{\alpha}x + i\bar{\beta}_n y} \bar{\bar{U}}_n; \bar{\bar{V}}_n d\bar{\alpha}, \\ \bar{\beta}_1 &= \sqrt{\frac{\omega^2}{c_n^2} - \bar{\alpha}^2}, c_1 = \bar{a}, c_2 = b \end{aligned} \quad (1.8)$$

where $s = -i\omega$ is parameter of Laplace transformation. Placing (1.8) in (1.7) one can for functions Ω^+ and V^-

$$\Omega^+ = \frac{1}{2\pi} \int_{-\infty}^0 \left\{ \left(\frac{K(\bar{a}^2 - b^2)\rho}{\rho_s} + \xi \right) \frac{\partial \bar{U}}{\partial x} + \left(\frac{K\bar{a}^2\rho}{\rho_s} + \xi \right) \frac{\partial \bar{V}}{\partial y} \right\} e^{-i\bar{\alpha}x} d\bar{\alpha}$$

$$V^- = \frac{1}{2\pi} \int_0^{\infty} \bar{V} \Big|_{y=0} e^{-i\bar{\alpha}x} dx$$

Obtain Winner-Hopf equation

$$\Omega^+ - \frac{i_0}{2\pi i \alpha s \rho_s} = iR(\bar{\alpha})\beta_2 C_0 V^- \quad (1.9)$$

One can make factorization by Nobell(), Martirosyan (2007) and obtain solution for $\bar{\bar{V}}_1 + \bar{\bar{V}}_2 = V^-$

The inverse transformations Laplace and Fourier from $V^- = \bar{\bar{V}}$, gives solution in Smirnov-Sobolev form Martirosyan (2007), Bagdov et.al. (2010) and for $y = 0$ one obtains

$$\begin{aligned} \frac{t}{a} \frac{\partial^2}{\partial t^2} V &= \frac{t}{\alpha x} \operatorname{Re} \frac{\frac{i i_0}{\pi \rho_s a b} \bar{a}^{-2} \sqrt{1-\alpha} \left(\frac{\bar{a}}{b} - \alpha \right)}{\alpha \sqrt{\frac{K_2}{a} - 1} \sqrt{\frac{K_2 + K_3}{a}} (\alpha - \alpha_1) (\alpha - \alpha_2 i) G(\alpha)}, \alpha = \frac{\bar{a}}{x} \\ V &= x \operatorname{Re} \frac{\frac{i i_0}{\pi \rho_s a b} \bar{a}}{\sqrt{\frac{K_2}{a} - 1} \sqrt{\frac{K_2 + K_3}{a}}} \int_0^x \frac{\frac{\bar{a}}{x} \sqrt{1-\alpha'} \left(\frac{\bar{a}}{x} - \alpha' \right) \left(\frac{\bar{a}}{b} - \alpha' \right) d\alpha'}{(\alpha' - \alpha_1) (\alpha' - \alpha_2 i) \alpha' G(\alpha')} \quad (1.10) \\ G(\alpha) &= \operatorname{Exp} \left[\frac{1}{\pi_1} \int_{\pi_1}^{\frac{\bar{a}}{b}} \frac{\frac{1}{b^2 a} \sqrt{\zeta^2 - 1} + \frac{2}{a^4} (K_2 - K_3) \zeta^2 \sqrt{\zeta^2 - 1} \sqrt{\frac{\bar{a}^2}{b^2} - \zeta^2}}{\frac{1}{a} (K_3 (\zeta^2 - 1) - K_2 \zeta^2) \left(2\zeta^2 - \frac{\bar{a}^2}{b^2} \right)} d\zeta \right] \\ b &= b_0 + V \end{aligned}$$

$$\begin{aligned} K_2 &= \frac{K(\bar{a}^2 - 2b^2)\rho}{\rho_s} + \xi, \quad K_3 = \frac{K\bar{a}^2\rho}{\rho_s} + \xi, \\ \text{where} \quad C_0 &= \frac{(K_2 + K_3)b^2}{a^{-2}} \end{aligned}$$

The calculations of (2.9) for $\frac{i_0}{\rho_s a} = \frac{1}{2 \cdot 10^5}$,

$$\bar{a} = 10^5 \frac{\text{cm}}{\text{sec}}, \quad \frac{\bar{a}}{b} = \sqrt{3}, K_2 = 0,8, K_3 = 0,3$$

$\alpha_{1,3} = \pm 0.9814; \alpha_{2,4} = \pm 2.736i$ give graphs of fig. 1 and fig. 2

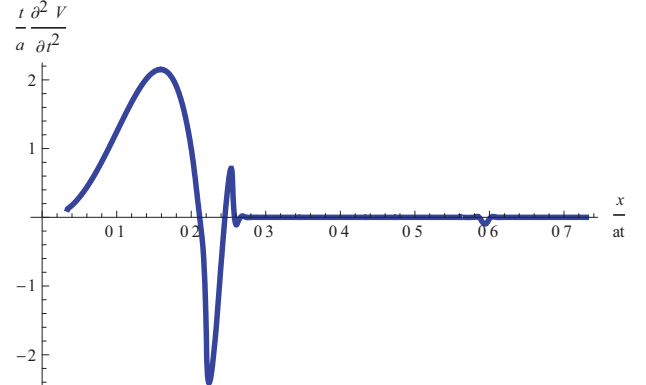


fig. 1 Dimensionless accelerations of vertical

displacements of crack's surfaces

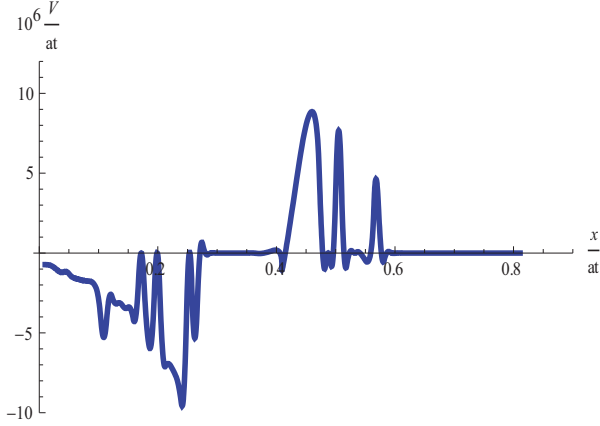


fig. 2

Dimensionless vertical displacements of crack's surfaces

Approximate taking into account of thermoconductivity by method of Gliko (2008) yields to additional term in (1.10)

minus $\frac{8\gamma C_1 k}{3\rho_s c_{pf} \sqrt{\pi a_1^2}} t^{\frac{3}{2}}$. Then Gliko (2008) condition of

healing of crack yields $b = 0$ and from (3.12) one obtains equation for $\bar{a}t$, which can be solved numerically for

given formerly constants $\frac{i_0}{\rho_s \bar{a}}$, $\frac{K_3}{\bar{a}}$, γ . Furthermore we

are used formula (3.11) in calculations of time t and coordinate x of crack's healing.

For example one can consider case of thermal cracks in soils Gliko (2008) where one has constants

$$\gamma = 10^{-5} K^{-1}, \rho_m = 3 \cdot 10^3 \frac{kg}{m^3}, \rho_s = 2,5 \cdot 10^3 \frac{kg}{m^3},$$

$$c_{pm} = 10^3 J / kgK,$$

$$c_{pf} = 4 \cdot 10^3 \frac{J}{kgK}, \bar{a} = 10^5 \frac{cm}{sec} \text{ for biological media}$$

and approximately for geothermal crack in thermoelastic media, J,K are Jowl and Kelvin units.

There are carried out calculations for following constants:

1. For water-like fluids, injecting into crack, thermo-conductive coefficient Ebert(1963) p 316.

$$k = \frac{1}{2} \frac{kkal}{m \cdot hour \cdot grad} = \frac{20}{36} \frac{J}{m \cdot sec \cdot grad},$$

$$\rho_m \cdot c_{pm} = 3 \cdot 10^6 \frac{J}{m^3 K}, K \approx 300 grad, \text{ then}$$

coefficient of temperature conductivity a_1 is

$$a_1^2 = \frac{k}{\rho_m c_{pm}} \approx \frac{1}{2} 10^{-4} \frac{m^2}{sec}, a_1 \approx \frac{2}{300} \frac{m}{sec^{1/2}}. \text{ One}$$

can for rate

of increasing of temperature in time on boundary fluid-elastic massive C_1 take two variants,

$$C_1 = 10^3 grad/sec, 10^5 grad/sec, ,$$

$$C_1 \cdot a_1 = 20/3 m \cdot grad/sec^{3/2},$$

i.e.

$$C_1 a_1 = 2000/3 m \cdot grad/sec^{3/2}.$$

These two variants relates to biological and geothermal cracks closing.

Using fig.2. one can for negative part of V values obtain

for any values of $\frac{x}{\bar{a}t}$ values of $\frac{10V}{t}$, and since

$$\bar{a} = 10^5 \text{ equation } b_0 + V = 0 \text{ yields}$$

$$b_0 \cdot \frac{10}{t} + V \frac{10}{t} = 0 \quad (1.10)$$

Then one, for example, can consider following variants in case $b_0 = 0, 1cm$, using fig.2

For example one can consider case of thermal fracture in soils [5] where one has constants

$$\gamma = 10^{-5} K^{-1}, \rho_m = 3 \cdot 10^3 \frac{kg}{m^3},$$

$$\rho_s = 2,5 \cdot 10^3 \frac{kg}{m^3}, c_{pm} = 10^3 J / kgK,$$

$$c_{pf} = 4 \cdot 10^3 \frac{J}{kgK}, \bar{a} = 10^5 \frac{cm}{sec}$$

for biological media and approximately for geothermal crack in thermo-elastic media, J,K are Jowl and Kelvin units.

There are carried out calculations for following constants:

1. For water-like fluids, injecting into crack, thermo-conductive coefficient[5]

$$k = \frac{1}{2} \frac{kkal}{m \cdot hour \cdot grad} = \frac{20}{36} \frac{J}{m \cdot sec \cdot grad},$$

$$\rho_m \cdot c_{pm} = 3 \cdot 10^6 \frac{J}{m^3 K}, K \approx 300 grad, \text{ then coefficient}$$

of temperature conductivity a_1 is

$$a_1^2 = \frac{k}{\rho_m c_{pm}} \approx \frac{1}{2} 10^{-4} \frac{m^2}{sec}, a_1 \approx \frac{2}{300} \frac{m}{sec^{1/2}}. \text{ One can}$$

for rate of increasing of temperature in time on boundary fluid-elastic massive C_1 take two variants,

$$C_1 = 10^3 grad/sec, 10^5 grad/sec, \text{ i.e.}$$

$$C_1 \cdot a_1 = 20/3 \text{ m} \cdot \text{grad} / \text{sec}^{3/2},$$

$C_1 a_1 = 2000/3 \text{ m} \cdot \text{grad} / \text{sec}^{3/2}$. These two variants relates to biological and geothermal cracks closing.

Using Fig.1. one can for negative part of V values obtain for any values of $\frac{x}{at}$ values of $\frac{V}{at}$, and since $\bar{a} = 10^5$ equation $b_0 + V = 0$ yields. Then one, for example, can consider following variants in case $b_0 = 0,1 \text{ cm}$, using Fig.1

$$\text{a) } \frac{x}{at} = 0.064, \quad \frac{V}{at} = -0.0108831,$$

$$t = 0.0000918856 \text{ sec}, \quad x = 0.588068 \text{ cm}$$

$$\text{b) } \frac{x}{at} = 0.136, \quad \frac{V}{at} = -0.00586324,$$

$$t = 0.000170554 \text{ sec}, \quad x = 2.31954 \text{ cm}$$

$$\text{c) } \frac{x}{at} = 0.262, \quad \frac{V}{at} = -0.0177721,$$

$$t = 0.000056268 \text{ sec}, \quad x = 1.47422 \text{ cm}$$

$$\text{e) } \frac{x}{at} = 0.397, \quad \frac{V}{at} = -0.0317016,$$

$$t = 0.000031544 \text{ sec}, \quad x = 1.2523 \text{ cm}$$

whence one can in plane x, t construct graph of process of healing. More detail solution of (3.5), where $b = 0$, give for case 1. following graphs x from t for different b_0 for

$$C_1 = 10^3, \quad b_0 = 10^{-5}$$

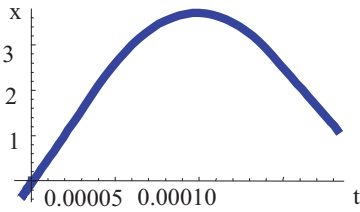


fig.3 Time t and coordinate x dependence curve of crack healing for $b_0 = 10^{-5} \text{ cm}$

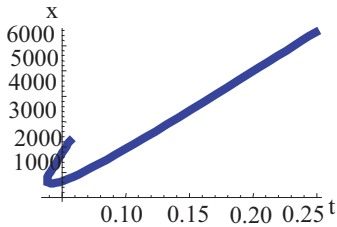


fig.4 Time t and coordinate x dependence curve of crack healing for $b_0 = 0.1 \text{ cm}$.

2. For metal-like fluids, as aluminum at high temperatures, $\bar{a} \approx 10^5 \frac{\text{cm}}{\text{sec}}$, and thermo-conductivity [5]

$$k = 200 \frac{\text{kcal}}{\text{m} \cdot \text{hour} \cdot \text{grad}}, \quad a_1 = \frac{4}{3} \frac{\text{m}}{\text{sec}^{1/2}}, \quad \text{and anew}$$

$C_1 = 10^3 \text{ grad/sec}$, 10^5 grad/sec , there are two variants for technological cracks.

For case 2. graphs for $b_0 = 10^{-5}; 0,1; 1 \text{ cm}$ are almost same as case 1. Using method of nonlinear wave dynamics [3] one can on mentioned graphs, representing mean curves of healing processes and in the same time shock waves of probabilities of stochastic processes, use the equation $\frac{dx}{dt} \approx \frac{\gamma}{2} P'$, where $P' = P - P_0$, P is probability and $P_0 \approx \frac{1}{2}$. Since for macroscopic cracks

considered in this paper one can assume processes $x(t)$ as almost deterministic one can assume on direct lines of microscopic sizes of width of crack of fig.3 and macroscopic sizes of width of crack of Fig.3

$$P = 1 \quad P' = \frac{1}{2} \quad \text{and from graphs one can obtain}$$

$$\frac{dx}{dt} = 0.397 \cdot 10^5, \quad \gamma P' / 2 = 0.397 \cdot 10^5,$$

$$\gamma = 1.588 \cdot 10^5 \frac{\text{cm}}{\text{sec}}.$$

The same results are true for all mentioned other graphs with direct lines. And for curve-linear part of line fig.3 approximately $\frac{dx}{dt} = 0.136 \cdot 10^5$

$$\text{and } P' = \frac{5}{16} \text{ i.e. for mentioned values of } x, t \text{ on Fig.2}$$

for microscopic crack curve where $b_0 = 10^{-5} \text{ cm}$, process of healing of crack is more chaotic. The used here method of shock waves of probability in examination of curves of process of healing of crack can be applied to inverse processes of generation of macroscopic crack by examination of curves of process of transition from small micro and mezo cracks to macro crack [6].

Let the edge of a semi infinite crack, which contains the fluid-crystalline inclusions, moves with arbitrary law in an isotropic thermo elastic medium. Consider the following problem when the boundary conditions are generalization of a little changed, written from left to right, conditions for case of moving crack ($y = 0$)

$$\sigma_{xy} = 0, -\infty < x < \infty$$

$$V = 0, x > l(t)$$

$$\frac{\partial V}{\partial t} + \left\{ \frac{K(\bar{a}^2 - b^2)\rho}{\rho_s} + \xi \right\} \frac{\partial U}{\partial x} + \left(\frac{K\bar{a}^2\rho}{\rho_s} + \xi \right) \frac{\partial V}{\partial y} = (1.11)$$

$$= -\frac{i_0}{\rho_s} H(x - \xi) H(t - \tau), x < l(t), \xi < l(\tau)$$

(1.11) where $\ell(t)$ —the law of motion of the crack, K —the coefficient of vertical wear. When $t = 0$ we have zero initial conditions $u = 0, v = 0, \frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0$. Solution of (1.11) is sought by the integral transformations of Laplace on t and of Fourier on x . Denoting \bar{U}, \bar{V} Laplace transforms on t from U, V and by $\bar{\bar{U}}, \bar{\bar{V}}$ the Fourier transforms from \bar{U}, \bar{V} on x , solution is sought in the form of

$$\bar{U}; \bar{V} = \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} e^{-i\bar{\alpha}x - i\bar{\beta}_n y} \bar{U}_n; \bar{V}_n d\bar{\alpha} \quad (1.12)$$

$s = -i\omega$ —there is a parameter of the Laplace transform on t and we have the relations

$$\bar{\beta}_n = \sqrt{\frac{\omega^2}{c_n^2} - \bar{\alpha}^2}, c_1 = \bar{a}, c_2 = b, \quad \bar{V}_1 = \frac{\bar{\beta}_1}{\alpha} \bar{U}_1, \quad \bar{V}_2 = -\frac{\bar{\alpha}}{\bar{\beta}_2} \bar{U}_2 \quad (1.13)$$

Substituting (1.12) in boundary conditions (1.11), we obtain

$$\bar{\bar{\Omega}} = i\bar{N}_0 R(\bar{\alpha}) \bar{\beta}_2 \bar{\bar{V}} \quad (1.14)$$

where we use the notation

$$\bar{\bar{\Omega}} = \int_{-\infty}^{\infty} \int_0^{\infty} e^{-st + i\bar{\alpha}x} \left(\frac{\partial V}{\partial t} + \left\{ \frac{K(\bar{a}^2 - b^2)\rho}{\rho_s} + \xi \right\} \frac{\partial U}{\partial x} + \left(\frac{K\bar{a}^2\rho}{\rho_s} + \xi \right) \frac{\partial V}{\partial y} \right) dt dx \quad (1.15)$$

$$R(\bar{\alpha}) = \frac{-\bar{\beta}_1 \frac{\omega^3}{b^2} + \bar{\alpha}^2 K_2 (\bar{\beta}_2^2 - \bar{\alpha}^2 - 2\bar{\beta}_1 \bar{\beta}_2) + K_3 (\bar{\beta}_1^2 (\bar{\beta}_2^2 - \bar{\alpha}^2) + 2\bar{\alpha}^2 \bar{\beta}_1 \bar{\beta}_2)}{\frac{\omega^2}{b^2} \bar{\beta}_1 \bar{\beta}_2 C_0},$$

$$C_0 = \frac{(K_2 + K_3)b^2}{\bar{a}^2}$$

Function of $R(\bar{\alpha})$ in the complex plane $\bar{\alpha}$ has two purely imaginary and two real roots:

$$\bar{\alpha}_{1,3} = \pm \frac{\omega}{a} \alpha_{1,3}, \bar{\alpha}_{2,4} = \pm \frac{\omega}{a} i \alpha_{2,4}; \alpha_1, \alpha_2 \in R \quad \text{and}$$

$R(\bar{\alpha}) \rightarrow 1$ at $\bar{\alpha} \rightarrow \infty$. After selecting the branches of the functions $\bar{\beta}_n$, we obtain factorization of the function $R(\bar{\alpha})$ as follows

$$R(\bar{\alpha}) = R^+(\bar{\alpha}) R^-(\bar{\alpha}) \quad ,$$

$$R \pm (\bar{\alpha}) = \frac{G_{\pm}(\bar{\alpha}) \left(\bar{\alpha} \pm \frac{\omega}{a} \alpha_1 \right) \left(\bar{\alpha} \pm \frac{\omega}{a} i \alpha_2 \right)}{\left(\frac{\omega}{a} \pm \bar{\alpha} \right)^{1/2} \left(\frac{\omega}{b} \pm \bar{\alpha} \right)^{3/2}} \quad (1.16)$$

where $R^+(\bar{\alpha})$ and $R^-(\bar{\alpha})$

are analytic and different from zero functions,

respectively, in the half planes $\text{Im}(\bar{\alpha}) > 0$ and

$\text{Im}(\bar{\alpha}) < 0$. Then equations (1.14), (1.15) yield

$$\bar{\bar{\Omega}} = \bar{P}_+ \bar{P}_- \bar{\bar{V}} \quad \text{or} \quad \bar{\bar{V}} = \bar{S}_+ \bar{S}_- \bar{\bar{\Omega}} \quad (1.17)$$

where

$$\bar{P}_- = iC_0 R^- \bar{\beta}_2 = \frac{\left(\bar{\alpha} - \frac{\omega}{a} \alpha_1 \right) \left(\bar{\alpha} - \frac{\omega}{a} i \alpha_2 \right)}{\left(\frac{\omega}{b} - \bar{\alpha} \right) \left(\frac{\omega}{a} - \bar{\alpha} \right)^{1/2}} G_-(\bar{\alpha})$$

$$\bar{P}_+ = R^+ \bar{\beta}_2 = \frac{\left(\bar{\alpha} + \frac{\omega}{a} \alpha_1 \right) \left(\bar{\alpha} + \frac{\omega}{a} i \alpha_2 \right)}{\left(\frac{\omega}{b} + \bar{\alpha} \right) \left(\frac{\omega}{a} + \bar{\alpha} \right)^{1/2}} G_+(\bar{\alpha})$$

$$\bar{S}_+ = \frac{\left(\frac{\omega}{b} + \bar{\alpha} \right) \left(\frac{\omega}{a} + \bar{\alpha} \right)^{1/2}}{\left(\bar{\alpha} + \frac{\omega}{a} \alpha_1 \right) \left(\bar{\alpha} + \frac{\omega}{a} i \alpha_2 \right)} G_+^{-1}(\bar{\alpha})$$

$$\bar{S}_- = \frac{1}{iC_0} \frac{\left(\frac{\omega}{b} - \bar{\alpha} \right) \left(\frac{\omega}{a} - \bar{\alpha} \right)^{1/2}}{\left(\bar{\alpha} - \frac{\omega}{a} \alpha_1 \right) \left(\bar{\alpha} - \frac{\omega}{a} i \alpha_2 \right)} G_-^{-1}(\bar{\alpha})$$

In the formula (4.13) on substitution $y = \frac{\sqrt{\alpha-1}}{\sqrt{t-\frac{\alpha x}{a}}}$,

and using Cauchy residue theorem, we can, after several transformations (integration by part and differentiation on p parameter under sign of the integral) to obtain

$$S_+(t, x) = \frac{2i\sqrt{t}H(x)}{\Gamma\left(\frac{1}{2}\right)\sqrt{x}} \frac{\partial}{\partial x} \left[H\left(\frac{a}{x}-1\right) \left[1 - \int_{\frac{a}{x}(u-\alpha_1)(u-\alpha_2 i)}^{\frac{a}{b}\left(\frac{a}{b}-u\right)\sqrt{u-1}F_2(u)du} H\left(\frac{1}{b}-\frac{t}{x}\right) \right] \right] \quad (1.25)$$

Similarly, we can obtain expression for original $P_+(t, x)$, and the functions $S_-(t, x); P_+(t, x)$ are obtained, respectively, from $S_+(t, x); P_-(t, x)$ replacing x by $-x$ and multiplying by the constants $\frac{1}{iC_0}$ and iC_0 . Since transforms of functions

$$\Omega(t, x); V(t, x) \text{ are } \bar{\Omega}(s, \alpha); \bar{V}(s, \alpha) \text{ one can write} \\ \bar{\Omega} = \bar{\Omega}_+ + \bar{\Omega}_-, \quad \bar{V} = \bar{V}_+ + \bar{V}_-, \quad \bar{V}_+ = 0 \quad (1.26)$$

Where $\bar{\Omega}_+(t, x); \bar{V}_-(t, x)$, are unknown and must be determined. For originals $V(t, x), \Omega(t, x)$ one can write:

$$\begin{aligned} V(t, x) &= v(t, x), \\ v(t, x) &= v_+(t, x)H(x - \ell(t)) + v_-(t, x)H(\ell(t) - x) \\ \Omega(t, x) &= \Omega_+(t, x)H(x - \ell(t)) + \Omega_-(t, x)H(\ell(t) - x) \end{aligned} \quad (1.27)$$

Here $\Omega(t, x) = \Omega_+(t, x)$, for

$x > \ell(t)$, $v = v_-(t, x)$ when $x < \ell(t)$ is unknown.

From formula (4.15) is easily seen that $\bar{S}(s, \alpha)$ is such that the above factorization leads to the functions \bar{S}_\pm, \bar{P}_\pm , originals of which satisfy the conditions

$$\begin{aligned} S_+(t, x) &= P_+(t, x) = 0 \text{ at} \\ x < bt, S_-(t, x) &= P_-(t, x) = 0 \text{ at} \\ x > -bt, -b < \dot{\ell}(t) &= d\ell/dt < b \end{aligned} \quad (1.28)$$

Substituting (1.25) (1.26) in (1.18) and using (1.28), we can as in Martirosyan(2007), to obtain a solution to the problem in the form of convolutions of x, t in the form

$$V_- = S_- ** \left[(S_+ ** \Omega_- - P_- ** V_+) H(1 - x + 0) \right],$$

$$\Omega_+ = -P_+ ** \left[(S_+ ** \Omega_- - P_- ** V_+) H(x - 1 + 0) \right] \quad (1.29)$$

From (4.1) we have that

$$\Omega_- = -\frac{i_0}{\rho_s} H(x - \xi) H(t - \tau), \quad x < l(t), \quad \text{and we}$$

can obtain using (4.15), represent $S_+ ** \Omega_-$ in the form

$$\begin{aligned} S_+ ** \Omega_- &= -\frac{i_0}{\rho_s} \int_{-\infty}^{\infty} \int_0^t S_+(t', x') H \\ (x - x' - \xi) H(t - t' - \tau) dt' dx' &= \\ &= \frac{2i_0}{\rho_s \sqrt{i\pi}} H(x - \xi) \sqrt{x - \xi} H(T - 1) \end{aligned} \quad (1.30)$$

$$\begin{aligned} &\left\{ 1 - \int_T^{\frac{a}{b}} \frac{\left(\frac{a}{b} - u\right) \sqrt{u-1} \sqrt{u-T} F_2(u) du}{(u - \alpha_1)(u - \alpha_2 i) u} H\left(\frac{a}{b} - T\right) \right\}, \\ T &= \frac{\bar{a}(t - \tau)}{x - \xi} \end{aligned}$$

Substituting $S_+ ** \Omega_-$ and $S_-(t, x)$ in (1.29), using (1.26), after lengthy calculations, we obtain

$$V_- = \frac{8i_0(x - \xi)}{\rho_s C_0} \text{Re} \left(A(I_{11} + I_{12}) + I_{21} + I_{22} + I_{23} + I_{24} \right) \quad (1.31)$$

Where

$$\begin{aligned}
I_{11} &= \left(\frac{1}{2} \ln \left| \frac{\varphi_0 - 1}{\varphi_0 + 1} \right| - \frac{(l(t_0^*) - x) \varphi_0}{x - \xi} \right) H(L_0 - 1) + \\
&\frac{1}{2} \left(\ln(T - \sqrt{T^2 - 1}) - \sqrt{T^2 - 1} \right) H(1 - L_0) \\
I_{12} &= \int_{L_0}^{\frac{\bar{a}}{b}} F_4(u) \left(\frac{\sqrt{u - L_0} (l(t_0^*) - \xi)}{(T - u)(x - \xi) \varphi_0} + \right. \\
&\left. \frac{1}{2\sqrt{u+1}} \ln \left| \frac{\varphi_0 - \sqrt{\frac{u+1}{u-L_0}}}{\varphi_0 + \sqrt{\frac{u+1}{u-L_0}}} \right| \right) H(L_0 - 1) du + \\
&+ \int_1^{\frac{\bar{a}}{b}} F_4(u) \left(\frac{\sqrt{u-1} \sqrt{T^2-1}}{T-u} + \right. \\
&\left. \frac{1}{2\sqrt{u+1}} \ln \left| \frac{\sqrt{\frac{T+1}{T-1}} - \sqrt{\frac{u+1}{u-1}}}{\sqrt{\frac{T+1}{T-1}} + \sqrt{\frac{u+1}{u-1}}} \right| \right) H(1 - L_0) du \\
I_{21} &= \int_1^{\frac{\bar{a}}{b}} F_3(h) \left\{ \left(\frac{1}{2} \ln \left| \frac{L_1 - \sqrt{h}}{L_1 + \sqrt{h}} \right| - \frac{(l(t_1^*) - x) \varphi_1}{x - \xi} \right) H(L_1 - 1) \right. \\
&\left. + \left(\frac{1}{2} \ln \left| \frac{\sqrt{\frac{T+h}{T-1}} - 1}{\sqrt{\frac{T+h}{T-1}} + 1} \right| - \frac{\sqrt{T^2-1}}{h+1} \right) H(1 - L_1) \right\} dh \\
I_{22} &= - \int_1^{\frac{\bar{a}}{b}} F_3(h) \int_{L_1}^{\frac{\bar{a}}{b}} F_4(u) \left(\frac{(l(t_1^*) - x) \varphi_1 \sqrt{u - L_1}}{u(x - \xi) + (l(t_1^*) - \xi) \left(L_1 + \frac{h}{\varphi_1^2} \right)} + \right. \\
&\left. \frac{1}{2\sqrt{u+h}} \ln \left| \frac{\varphi_1 - \sqrt{\frac{u+h}{u-L_1}}}{\varphi_1 + \sqrt{\frac{u+h}{u-L_1}}} \right| \right) H(L_1 - 1) du dh
\end{aligned}$$

$$\begin{aligned}
I_{23} &= \int_1^{\frac{\bar{a}}{b}} F_3(h) \int_{L_1}^{\frac{\bar{a}}{b}} F_4(u) \left(\frac{\sqrt{(bu - \bar{a})(bT - \bar{a})(T+h)}}{(u+h)(T-u)} + \right. \\
&\left. \frac{1}{2\sqrt{u+h}} \ln \left| \frac{\sqrt{\frac{T+h}{T-\bar{a}}} - \sqrt{\frac{u+h}{u-\bar{a}}}}{\sqrt{\frac{T+h}{T-\bar{a}}} + \sqrt{\frac{u+h}{u-\bar{a}}}} \right| \right) H(1 - L_1) \times \\
&\times \left(T - \frac{\bar{a}}{b} \right) du dh \\
I_{24} &= - \int_1^{\frac{\bar{a}}{b}} F_3(h) \int_{L_1}^{\frac{\bar{a}}{b}} F_4(u) \left(\frac{\sqrt{(u-1)(T+h)(T-1)}}{(h+1)(T-u)} + \right. \\
&\left. \frac{1}{2\sqrt{u+h}} \ln \left| \frac{\sqrt{\frac{T+h}{T-1}} - \sqrt{\frac{u+h}{u-1}}}{\sqrt{\frac{T+h}{T-1}} + \sqrt{\frac{u+h}{u-1}}} \right| \right) \times \\
&\times H(1 - L_1) du dh \\
F_3(h) &= \int_h^{\frac{\bar{a}}{b}} \frac{d}{du} \left(\frac{\left(\frac{\bar{a}}{b} - u \right) \sqrt{u-1} F_2(u)}{(u - \alpha_1)(u - \alpha_2 i)} \right) \frac{du}{\sqrt{u-h}}, \\
F_4(u) &= \frac{\left(\frac{\bar{a}}{b} - u \right) \sqrt{u-1} (u - T) F_2(u)}{(u - \alpha_1)(u - \alpha_2 i) u}, \\
T &= \frac{\bar{a}(t - \tau)}{x - \xi}, \\
l(t_n^*) - x - \frac{a(t - t_n^*)}{h_n} &= 0, \varphi_n = \sqrt{\frac{l(t_n^*) - \xi}{l(t_n^*) - x}}, L_n \\
&= \frac{a(t_n^* - \tau)}{l(t_n^*) - \xi}, h_0 = 1, h_1 = h \\
A &= 1 - \int_1^{\frac{\bar{a}}{b}} \frac{\left(\frac{\bar{a}}{b} - u \right) F_2(u) du}{(u - \alpha_1)(u - \alpha_2 i)}
\end{aligned}$$

As seen from this formula, in difference from absence of tribological effects of Goriacheva et al.(1988),when there is concentration of stresses near edge of crack in Martirosyan(2007), if $x \rightarrow l(t)$ has no singularities.

2. Conclusions. The study of healing of fractures is very actual and practical geophysical, technological and biological problem. Many of works on geophysics are devoted to investigation of this problem, where there are used some onedimensional models of description of healing processes of fractures by precipitation of silica, containing in hot fluid current, entering in fracture, mainly on account of thermodiffusion effects, in any of works are taken into account also contribution in solution from stresses. There are obtained quantitative and qualitative results, based on these onedimensional along line of fracture treatments, in any works using also onedimensional in transverse direction to fracture thermoconductivity equation, whose solution mainly used near top of fracture. Therefore the full investigation of healing problem using unsteady twodimensional equations of thermoelasticity in surrounding coupled by boundary conditions, where are used thermodiffusion effects, currents of mixtures with crystallines, transverse diffusion current of them and tribological effects of fracture surfaces is necessary. In present paper is done solution of this unsteady plane coupled mixed boundary value problem of thermoelasticity by application of effective method of integral transformations, solution of Wiener-Hopf equation and bringing of solution for vertical displacement of banks of fracture and temperature to analytical form of Smirnov- Sobolev due to method in Bagdoev et.al.(2010). The graphs are constructed and equating of width of fracture to zero are obtained dependences of coordinate from time for healing processes and are for various range of parameters constructed their graphs. For relatively small values of leading parameters taking influence on solution to obtaining formula of vertical displacement on fracture is added term, obtained by solution of only thermodiffusion onedimensional effects and also are constructed corresponding graphs. From obtained graphs for different, macroscopic and microscopic cracks, as well as values of main parameter determining influence on healing processes, are obtained physical conclusions of processes which are in qualitative accordance in investigations obtained by other treatments formerly.

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