A Model-Theoretic Semantics for Two-Sided Argumentation

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Abstract

Argumentation is a natural meaning of reasoning in the daily life, and has also become a highly interested topic of knowledge representation in the past few years. In this paper, we will use the term "two-sided argumentation" for a type of formalization for our real world debate: an issue with a pro-side supports it and a con-side opposes it. Then, we will point out that, when we use the term "argumentation", we in fact mean a binary concept: a method of reasoning, and a type of negotiation. For both case, we will consider the semantics: argumentative models for the former, argumentation games for the latter. We will also give out some results about the relationship between them.

Keywords: Two-sided argumentation, Argumentation game, Argumentative models

Introduction

When two people do not agree with each other, they argue. So as we can see, argumentation is a quite natural method for handling inconsistency in our daily life. Informally, we can describe what is an argumentation procedure for a "two-sided argumentation game" like this: there is an issue, and two people raise arguments around the issue, each attacks the opponent. Finally, there shall be a winner, and his/her opinion will be considered more "justified" than the other.

In 1995, Dung(1995) began to use the term "argumentation framework" for an abstract framework for an argumentation. Some researchers(Vreeswijk and Prakken 2000; Modgil and Caminada 2009; Cayrol, Doutre, and Mengin 2003) who focus on a proof theory for Dung's semantics, point out that an argument game can be used to calculate Dung's extensions, which reasons around a single argument, and with "challenger" and "defender" as two sides. It could be easily noticed, that such argument game correspond to what we do in our daily debate. But still, it is only for an abstract framework, and cannot totally be considered as a formalization for real-world debate.

For some approaches of argumentation(Prakken and Sartor 1997; Simari and Loui 1992; García and Simari 2004), especially *defeasible logic programming*(García and Simari 2004), an argumentation procedure is shown as an argumentation line, each argument in the line attacks the former one, with a dialectical tree to present all possible argumentation lines.

Defeasible logic programming is successful and has many applications. But also for some reasons, we are not quite satisfied about the argumentation procedure. First, an argument in an argumentation line is only related to arguments which next to it, but in a real argumentation procedure, arguments raised by a same player should be somewhat related (at least, they should not disagree with each other). Second, the rules take on the form of logic program clauses, under which some reasoning rules cannot be used (for example, $p \leftarrow q$ is not equal to $\neg q \leftarrow \neg p$, while *reductio ad absurdum* is always used in a real debate). Third, the definition of argumentation line partly depends on an order among the arguments, but such order cannot always be given.

These call for a model-theoretic semantics for an argumentation procedure, which is what we wish to present in this paper. We will also use argumentation lines and a dialectical tree for an argumentation procedure, but the definition will be quite different.

In section 2, we define arguments and attacks under propositional logic. Next, in section 3, we give out a more concrete version of two-sided argumentation game under propositional logic. Finally, in section 4, we give out a semantics which is similar to Dung's abstract semantics, but also under propositional logic, and show the relationship between it and the gaming semantics.

Propositional Argumentation Framework

Now, before we have the model-theoretic semantics for an argument game, we first define the argumentation framework under propositional logic.

Definition 1. *Propositional Argumentation Framework*

An argumentation framework under a propositional language L is a pair $\langle \mathcal{R}, att \rangle$, where $\mathcal{R}$ is a set of formulas in L (called rules), and att is a relationship between formulas in L and rules in $\mathcal{R}$.

We can see that just the same as defaults, the relationship "att" is used to describe the real world in addition of the given rules. The rules do not hold for all cases, and the exceptions are marked out by att. But it should be noticed that the semantics are totally different. In a default logic, if the defaults do not hold, although the rule can no longer be used for reasoning, it is not overthrown. In argumentation, however, if a rule is attacked, it cannot hold in the same model with the attacker. We give out the semantics of a propositional argumentation framework by the definition below:

Definition 2. *A model M is called compatible under a given framework $\mathcal{AF}$ with respect to a set of rules $R \subseteq \mathcal{R}$, if for each $(d, r) \in att$ and $r \in R$, $M \models d$ and $M \models r$ do not hold at the same time. If $R = \mathcal{R}$, we simply say M is a compatible model under $\mathcal{AF}$.*

If all compatible models of Σ w.r.t R are models of p, we call Σ implies p under $\mathcal{AF}$ w.r.t R, written by $\Sigma \models_{\mathcal{AF}(R)} p$. If $R = \mathcal{R}$, we simply say Σ implies p under $\mathcal{AF}$ and written by $\Sigma \models_{\mathcal{AF}}$.

The relationship att only covers a small amount of attacks. It is natural that $\neg r$ attacks r, and if d_1 and d_2 all attack r, $d_1 \vee d_2$ attacks r as well. In fact, as we used the term *compatible model* to line out all conflicts, an attack can be defined by: the attacker and the attacked arguments share no common compatible models.

Definition 3. *(Attack)*

Under a given framework $\mathcal{AF}$, we say a formula p attacks a set of rules $\Sigma \subseteq \mathcal{R}$, if $\Sigma \cup \{p\} \models_{\mathcal{AF}(\Sigma)} \perp$.

Now, with the given framework, we can talk about an argument under propositional language.

Definition 4. *(Argument)*

An argument of a given framework $\mathcal{AF}$ is a pair $A = \langle \Sigma, p \rangle$, where:

1 $\Sigma \not\models_{\mathcal{AF}} \perp$.

2 $\Sigma \models p$.

3 Σ is a minimal subset of $\mathcal{R}$ satisfies 1 and 2.

p is called the issue *of the argument (noted by $I(A)$), and Σ is called the* support rules *(noted by $R(A)$).*

If p attacks a set of rules R', we say A attacks R', and if $R' = R(A')$, we say A attacks A'.

Although our relationship "attack" is an issue attacks one or some of the rules, but it can also be extended to attacks between arguments, since the issue can represents an argument, and the rules are those which support an argument. Thus, our framework can be viewed as an instantiation of abstract framework.

Example 1. *Let $A_1 = \langle \{a, a \to c\}, c \rangle$, and $A_2 = \langle \{b, b \to \neg c\}, \neg c \rangle$. Then A_1 and A_2 attack each other. Furthermore, if there is $A_3 = \langle \{d\}, d \rangle$, and $(d, a \to c) \in att$, then A_3 is an attack to A_1.*

Propositional Argumentation Games

The definition of argumentation procedure might be more complicated than what is under abstract framework. For example, if $R(C) \subseteq R(A) \cup R(B)$, and the pro-side has both A and B in its acceptable set, it must also have C in its acceptable set. Besnard and Hunter(2001) discussed such cases in the means of syntax, but we wish to consider it from a semantics way.

Definition 5. *(Argumentation Line)*

Given a sequence of arguments $\mathbb{A} = [A_1, ..., A_n]$, we use Pr_i to note the presenter of A_i. $Pr_i \in \{P, C\}$, where P for the pro-side and C for the con-side, $Pr_1 = P$, and $Pr_{i+1} \neq Pr_i$.

For each $i = 1, ..., n$, the state *of the (propositional) argumentation line is noted by $\langle P_i, C_i \rangle$, where $P_i = \bigcup_{j \leq i \wedge Pr_j = P} R(A_j)$ and $C_i = \bigcup_{j \leq i \wedge Pr_j = C} R(A_j)$.*

Then, $\mathbb{A}$ is a legal argumentation line, if for each i:

1 If $Pr_i = P$ and $i > 1$, $I(A_i)$ attacks C_i; if $Pr_i = C$, $I(A_i)$ attacks P_i;

2 For each argument A, if $Pr_i = P$, $R(A) \subseteq C_i$ and $I(A)$ attacks P_i, there is a formula f attacks $R(A)$ which $P_i \models f$; and if $Pr_i = C$, $R(A) \subseteq P_i$ and $I(A)$ attacks C_i, there is a formula f attacks $R(A)$ which $C_i \models f$;

3 $P_i \not\models_{\mathcal{AF}} \perp$ and $C_i \not\models_{\mathcal{AF}} \perp$;

4 For every i, $\langle P_i, C_i \rangle \neq \langle P_{i+1}, C_{i+1} \rangle$.

We note $I(\mathbb{A}) = I(A_1)$ as the issue *of the argumentation line.*

An argumentation line $[A_1, ..., A_n]$ is maximal, if there is no A which $[A_1, ..., A_n, A]$ is a legal argumentation line.

In the definition above, (1) means that, each argument must attack the opponent, so that all arguments in the line would be related to the main issue. (2) means that, every player must defends him/herself against the opponent. (3) means that, all rules holds by the same player must be conflict-free. (4) means that, each time the player must raise new arguments, so that when the rule base is finite, the argumentation line always ends.

We can see that, by the definition of states, we combine all arguments raised by a same player as a whole, and each new argument is related to the current state, not only one former argument.

A maximal argumentation line describes a full argumentation procedure. By two ways an argumentation will ends, one is falling into deadlock, the other is that one player be unable to attack its opponent. It is also necessary that we make difference between the two types of maximal argumentation lines.

Definition 6. *A maximal argumentation line $[A_1, ..., A_n]$ is $\mathbb{E}$-type if there is no A which $[A_1, ..., A_n, A]$ satisfies (1)-(3), otherwise it is $\mathbb{D}$-type.*

A $\mathbb{D}$-type argumentation line ends because if we allow a player to raise an argument twice, the line will be infinite. However, we still cannot determine the winner, so the argumentation ends with a draw. But in an $\mathbb{E}$-type line, the winner is the one who raised the last argument. To distinguish between the two types, we add a symbol $\mathbb{D}$ at the end of a $\mathbb{D}$-type line.

One may attach a partial order upon $\mathcal{A}$ to avoid cases of draw, such as in (Martínez, García, and Simari 2007). However, it makes the construction of argumentation lines extremely difficult, despite that such ordering cannot always be given.

Definition 7. *Dialectical Tree*

With a given formula I, a dialectical tree around I is a tree structure, its root node labeled by a special label $\mathbb{R}_I$, all other nodes labeled by an argument or $\mathbb{D}$, which:

1 The label of each path in the tree (from root to one of the leaves) except the root forms a maximal argumentation line around I ($\mathbb{D}$-type if the last label is $\mathbb{D}$, $\mathbb{E}$-type otherwise);

2 Each maximal argumentation line around I can be expressed by labels of a path in the tree.

We show the presenter of an argument by labeling the level of the tree from $\{P, C\}$: level 0 (the root level) is labeled by C, and level $i+1$ with a label different by level i.

In order to determine which side has a winning strategy, we can label nodes in the tree with an additional symbol P, C or D, which represent the pro-side wins, the con-side wins, and the game draws.

The additional labels are given from leaves to root by:

1 For each leaf node, if it is $\mathbb{D}$, it is labeled by D, otherwise, it is labeled by the label of its level.

2 Given an order $P > D > C$. If a node is on a level labeled by P, it is labeled by the minimal label of its sub-nodes. If a node is on a level labeled by C, it is labeled by the maximal label of its sub-nodes.

It is easy to see, if the root is labeled by P, the pro-side can choose each time on his/her turn a node labeled by P to form a winning argumentation line, and we say that the pro-side wins the game, or the issue is justified. If the label is C, the con-side wins, and the issue is overthrown. If the label is D, neither side can win, and the game draws.

We can see that the pro-side always hold the issue, and the con-side always oppose it. We will give an example of an argumentation game.

Example 2. *Back to the example above. Suppose we had list out all rules and attacks in the given framework. Consider an argumentation around I. We can see that $[A_1, A_2, \mathbb{D}]$ is a $\mathbb{D}$-type argumentation line, while $[A_1, A_3]$ is a $\mathbb{E}$-type argumentation line. It is easy to see that the con-side wins the game.*

If the argumentation is around $\neg I$, $[A_2, A_1, \mathbb{D}]$ and $[A_2, A_1, A_3]$ are both maximal argumentation lines, and the pro-side can win the argumentation.

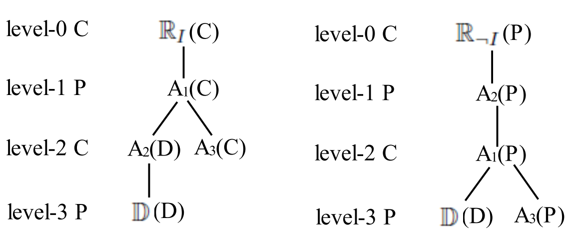

Figure 1: An example of proposition dialectical tree

Such idea of justifying an issue is quite fit for simulating a real world debate. However, a real world debate does not always get a more reliable result.

Suppose an issue is justified, so each path (also an argumentation line) in the tree which P is the winner corresponds to a model set $Mod(P_i)$. However, if there is no common model among all these model sets, we cannot be sure which model shows the truth, and whether the justified result is reasonable.

This enlightened us to restrict all arguments raised by the same player within a single model, so that there will be no conflicts between them. This is what we will discuss in the next section.

Argumentative Model

To restrict an argumentation game with a model, first, as the definition of legal argumentation lines, we shall define an argument induced from a model. To make it short, when we mention a model below, it means a model that is compatible under the given framework.

Definition 8. *An argument $\langle \Sigma, p \rangle$ is induced from a model M, if for every $r \in \Sigma$, $M \models r$.*

Now, we can talk about the idea of argumentative models.

Definition 9. *(Argumentative Models)*

If for any argument A, either $M \models I(A)$ or there is an argument A' induced from M which A' attacks A, we call that M is a positive model under the framework.

If for any argument A induced from M and an argument A' attacks A, there is an argument A'' induced from M which A'' attacks A', we call that M is a defensive model.

Of course, all positive models are defensive models, but not vice versa.

Theorem 1. *Let M be a compatible model under a framework $(\mathcal{R}, att)$, and $\mathcal{R}_1 = \{r | r \in \mathcal{R} \wedge M \models r\}$, $\mathcal{R}_2 = \{r | r \in \mathcal{R} \wedge M \not\models r\}$.*

Then M is a positive model iff for each $r \in \mathcal{R}_2$, $D = \{d | (d, r) \in att\}$, $\mathcal{R}_1 \models \bigvee D \vee \neg r$.

Referred to Dung's semantics for abstract framework(1993), if we view a model as the set of all arguments induced from the model, we can see that a positive model corresponds with a stable extension, and a defensive model corresponds with a preferred extension.

Argumentative models are partly related to argumentation games in the previous chapter, just as the theorem below:

Theorem 2. *Let p be true under a positive model M. The pro-side cannot win an argumentation game around $\neg p$.*

Let p be true under a defensive model M, and there is at least one argument A induced from M which $I(A) = p$. Then the con-side cannot win an argumentation around p.

Theorem 3. *Let p be an arbitrary formula. If p is justified, then for all positive models M, p is true under M.*

Note that although we haven't mentioned the condition "two-player" in our discussion of argumentative model, it still cannot be ignored. If an argumentation has more players, there might be difference. Give an example.

Example 3. *The framework is $\langle \{a, b, c\}, \{(b, a), (c, b), (a, c)\} \rangle$. It is clear that there are neither positive models nor defensive models under the framework. However, when there are three players, named A, B, C, who raise only arguments $\langle \{a\}, a \rangle, \langle \{b\}, b \rangle, \langle \{c\}, c \rangle$ each, the argumentation game will comes into a deadlock. So models $\{a, \neg b, \neg c\}, \{\neg a, b, \neg c\}, \{\neg a, \neg b, c\}$ are all "positive" under the condition of three players.*

This is the problem of "odd-length cycles"(Baroni and Giacomin 2003) in Dung's abstract framework. We can see, why such problem occurs, is that such semantics are only suitable for an argumentation with two players.

Related Works

We use argumentation lines and dialectical trees to define and calculate whether an issue can be justified. Similar definitions can also be found in (Simari and Loui 1992; García and Simari 2004; Besnard and Hunter 2001). However, our definition of argumentation line is totally different from others. In other definitions of argumentation line, an argument in the line is only related to the one right next to it. Although the difference between pro-side arguments and con-side arguments was mentioned, all pro-side arguments or all con-side arguments are not considered as a whole. When we take it back into real-world debates, where an argument is always used to defeat some of the opponent's arguments combined, such approaches are not-so-acceptable. By using the concept *argumentation states* in our definition of argumentation lines, we handle such cases more correctly. Also, with the definition of states, we can define draws more precisely, not by arbitrarily cutting down the argumentation line to prevent infinity.

We have mentioned above, that our framework is a instantiation of Dung's framework(1993), and our argumentative models correspond with Dung's extensions. But it should be noticed that since we define our semantics under propositional logic, we avoided some trouble when using Dung's framework. We give an example in our framework which is similar to one presented by (Caminada and Amgoud 2007):

Example 4. *Let* $\mathcal{A} = \langle\{a, d, c, a \to b, d \to e\}, \{(c, b \wedge e)\}\rangle$, *and* $A_1 = \langle\{a, a \to b\}, b\rangle, A_2 = \langle\{d, d \to e\}, e\rangle, A_3 = \langle\{c\}, c\rangle$.

If we consider A_1, A_2, A_3 *as abstract arguments in Dung's abstract framework, they all have no defeaters, thus* b, e, c *are all justified. However,* c *attacks* $b \wedge e$.

Our formalization also shares much similarity with *dialogue games for argumentation* discussed by Prakken(2005). Multi-replies and non-immediate replies had also been included under his framework. However, as it is a framework with variable degrees of flexibility, it is not totally logical and without semantics.

Conclusion and Future Work

In this paper, we define an argumentation framework under propositional logic, and use it on the special case of two-sided argumentation. We then describe the argumentation procedure in two different ways, as argumentation games and as (positive or defensive) models. Although these are different approaches, they are related to each other, and the goal is the same: formalize real world argumentation, and use it for reasoning.

Because our definition of arguments is far from syntax, the problem of generating proper arguments in an dialectical tree is rather complicated. We are still working on to find an efficient algorithm for such problem.

The two-sided argumentation we discuss about has its limits also. There are lots of discussions around the "odd loop problem" of Dung's framework, and lots of researchers seek to improve. In this paper, however, we showed that the problem occurs because Dung's semantics only fits two-player argumentation. The semantics of arbitrary n-player argumentation has still a lot to learn about.

Argumentation has been discussed for thousands of years, and there are many useful results. To completely formalize argumentation in logic, there is still a long way to go.

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