On the Complexity of Bribery and Manipulation in Tournaments with Uncertain Information

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Abstract

We study the computational complexity of optimal bribery and manipulation schemes for sports tournaments with uncertain information: cup; challenge or caterpillar; and round robin. Our results carry over to the equivalent voting rules: sequential pair-wise elections, cup, and Copeland, when the set of candidates is exactly the set of voters. This restriction creates new difficulties for most existing algorithms. The complexity of bribery and manipulation are well studied, almost always assuming deterministic information about votes and results. We assume that for candidates i and j the probability that i beats j and the costs of lowering each probability by fixed increments are known to the manipulators. We provide complexity analyses for cup, challenge, and round robin competitions ranging from polynomial time to NP^{PP}. This shows that the introduction of uncertainty into the reasoning process drastically increases the complexity of bribery problems in some instances.

Introduction

Sports competitions are common forms of entertainment and recreation. In most sports contests both observers and players have a sense of which competitors are favored over others. Many individuals, including some players, wager on the outcomes of particular games and tournaments. Google reveals dozens of players, coaches, referees, and judges convicted of manipulating the outcome of sports competitions through match fixing, point shaving, and cheating. Many websites (such as `www.kenpom.com`) produce and publish in depth statistics for overall team win/loss predations and predictions for individual player stats on a per game basis. It is a world of probabilities and manipulation.

We use sports tournaments as a motivating example of other domains in which bribery (Faliszewski, Hemaspaandra, and Hemaspaandra 2009) and coalitional manipulation (Conitzer, Sandholm, and Lang 2007) can undermine the integrity of competition. Tournaments and single winner elections, when the set of candidates and the set of voters are equivalent, are used in many domains including self organization of ad-hoc wireless sensor networks, where leaders are elected to delegate work or act as central routing

nodes (Sohrabi et al. 2000), and multi-criteria decision making, where page rankings are sometimes determined by links from the set of pages under consideration (Bouyssou et al. 2006). In addition to these important applications of tournaments, there has been recent empirical research in political science and sociology revealing that, in the United States, voter preferences in political elections can be significantly affected by apparently irrelevant events, specifically sports tournaments (Healy, Malhotra, and Mo 2010).

By *cup tournament* we mean a single-elimination competition (or knockout tournament (Vu, Altman, and Shoham 2009)) over a complete binary tree where each entrant plays a sequence of matches head-to-head; the winner is the entrant left undefeated. NCAA Basketball Tournaments and most tennis majors fall into this category. By a *round robin tournament* we mean competitions where each entrant competes against every other entrant and earns a point for each victory; the winner is the entrant with the most points. The group play round of the FIFA World Cup falls into this category. By a *challenge* or *caterpillar tournament* we mean a series of matches where the winner of each match plays the next entrant in increasing order of rank; the winner is the entrant who wins the final match. Boxing titles and some PBA bowling competitions use this type of tournament. Figure 1 illustrates the difference between cup and challenge tournaments. These types of sporting events correspond to the voting rules *cup*, *Copeland*, and *linear balloting* respectively.

While this paper focuses on cash bribery, any resource that can be distributed unevenly among the entrants can be used to change probabilities. These resources could be referees or home fields in sports; volunteers and canvassers in political elections; or money spent on targeted advertising and product placement. The ComSoc community uses the term bribery when unequal distribution of resources is a more general viewpoint. We suggest the term "influence" to encompass the more general view.

Related Work

The traditional notation of manipulation, introduced by Bartholdi et al. (1989), has been extensively studied in the deterministic case. Conitzer et al. (2007) studied some manipulation problems in stochastic settings. However, many of their NP-completeness results break down in the setting studied here because the reductions require the ability to add
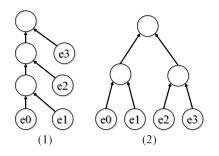
Figure 1: (1) A challenge tournament and (2) a cup tournament. The winner is the entrant who reaches the top node.

voters that are not in the candidate set. There are also results relating to manipulation under deterministic information for cup (Conitzer, Sandholm, and Lang 2007) and Copeland (Faliszewski et al. 2009). Likewise, the bribery problem for elections, introduced by Faliszewski et al. (2009), has been well studied for voting rules under deterministic information. Again, many of these results do not transfer; the hardness reductions require the ability to introduce an unrestricted number of non-candidate voters.

There has been significant work on the schedule control problem for cup or knockout tournaments: in the deterministic case by Lang et al. (2007) (for the similar problem of sequential majority voting), Vu et al. (2009), and Williams (2010); in the stochastic case by Hazon et al. (2008b). In addition, there is recent work by Russell and Walsh on the complexity of coalitional manipulation in sports tournaments (2009). In Russell and Walsh's model, entrants may be in a coalition of manipulators that can choose to lose matches. Given this model, there are natural questions which arise, such as: "What are the odds my preferred entrant wins?" and "Does my preferred entrant have a chance of winning?" There is also an extensive body of work on the elimination problem for sports tournaments. In the elimination problem, the probability that team i beats team j is known, and the problem is to find the probability that a team is eliminated given a sports season or playoff schedule (Kern and Paulusma 2004; Gusfield and Martel 2002).

There has been little work on stochastic models of elections or tournaments; some researchers have begun to address this gap (Erdélyi et al. 2009; Hazon et al. 2008a). Perhaps the closest notion is that of a *possible winner* — a notion intrinsic to reasoning under uncertainty introduced by Konczak and Lang (2005), with further study by Lang et al. (2007), and others. These papers address complexity questions when voters are defined by their (possibly incomplete) preference profiles over sets of outcomes.

Definitions

We define a model for reasoning about sports tournaments and other head-to-head competitions. The details closely follow the model proposed by Erdélyi et al. for lobbying in plurality elections (2009). Consider a tournament with n entrants $\{e_1, \ldots, e_n\}$. What distinguishes a tournament from

a voting rule here is the requirement that, for a given election with separate sets of candidates and voters, we require the set of candidates to be exactly the set of voters. Let $\mathbb{Q}_{[0,1]}^{n \times n}$ be the set of $n \times n$ matrices over $[0,1] \cap \mathbb{Q}$. Let $p_{i,j}$ denote the probability that entrant i will defeat entrant j. Let $P = [p_{i,j}] \in \mathbb{Q}_{[0,1]}^{n \times n}$. We require that $p_{i,j} + p_{j,i} = 1$. We choose $k \in \mathbb{N}^+$ to discretize the interval $[0,1]$ (we call this the *discretization level* of the problem). That is, we let each $p_{i,j}$ be in $\{0, 1/(k+1), 2/(k+1), \ldots, k/(k+1), 1\}; i,j \in \{1, \ldots, n\}$. Notice that taking $k = 0$ gives us the deterministic case where each game is either won or lost with certainty.

The *discrete price table*, C_P, has n^2 rows (indexed by pairs i, j) and $k + 2$ columns. For each entry in the table we have the integer value representing the *cost* to lower $p_{i,j}$ to a given probability. We assume that all entrants will compete to the best of their abilities and we cannot increase an entrant's probability of winning a particular match. Therefore we designate these entries as $--$. We also require that $c_{i,j}(p_{i,j}) = 0$. That is, it does costs nothing to have an entrant compete at its highest level. The entries for $c_{i,j}$ must also be monotone decreasing; payment of $c_{i,j}(l)$ changes the probability that i beats j to $l/(k+1)$. Bribery must be performed in one fell swoop, before the first game is played. Outside actors may not have access to the entrants after the start of the competition (i.e., the tournament is taking place in a remote or tightly controlled location) so all bribery must be done beforehand.

We are given a threshold t and an integer-valued budget B. For cup and challenge tournaments we are given a tree T with leaves labeled by entrants giving the order of the matches. Let $Pr[e_i|P,T]$ denote the probability e_i wins the tournament defined by schedule T and probability matrix P.

Example 1 *Let $k = 3$ and $n = 4$. We might have probability matrix P:*

	e_1	e_2	e_3	e_4
e_1	$--$	0.75	0.50	0.25
e_2	0.25	$--$	0.25	0.25
e_3	0.50	0.75	$--$	0.25
e_4	0.75	0.75	0.75	$--$

and cost matrix C_P:

$c_{i,j}$	0.00	0.25	0.50	0.75	1.00
$c_{1,2}$	100	40	20	0	$--$
$c_{1,3}$	30	25	0	$--$	$--$
$c_{1,4}$	200	0	$--$	$--$	$--$
$c_{2,1}$	10	0	$--$	$--$	$--$
$c_{2,3}$	10	0	$--$	$--$	$--$
$c_{2,4}$	10	0	$--$	$--$	$--$
$c_{3,1}$	120	75	0	$--$	$--$
$c_{3,2}$	200	100	50	0	$--$
$c_{3,4}$	300	0	$--$	$--$	$--$
$c_{4,1}$	300	200	100	0	$--$
$c_{4,2}$	300	200	100	0	$--$
$c_{4,3}$	400	300	200	0	$--$

In this paper we focus on the following three types of tournaments. In each case we assume that no game ends in a tie and every tournament has a unique winner. In the deterministic case a scoring model which includes ties or does not

normalize to certain forms for round robin tournaments and Copeland elections can have significant effects on the complexity of evaluation, manipulation, and bribery (Kern and Paulusma 2004; Faliszewski et al. 2009).

Challenge Tournament: In a challenge tournament the entrants are ordered $e_1, e_2, \ldots, e_n$. In the first match, e_1 plays e_2. In the second match, the winner plays e_3, and so on. The winner of the last match wins the tournament.

Cup: In a cup tournament we are given a complete binary tree T with entrants labeled on the leaves. Each internal node is decided by the competition between the two entrants on the level below. The winner is the entrant who reaches the top node of the tree.

Round Robin: In a round robin tournament (Copeland) each entrant plays each other entrant exactly once. In each match each entrant receives 1 point for a win and 0 points for a loss. The winner of the tournament is the entrant with the maximum number of points. If more than one entrant has the same maximum score, we employ a lexicographic tie-breaking scheme.

With this model we can define our problem instances and a set of related decision problems for $Y \in \{$Challenge Tournament (CT), Cup, Round-Robin (RR)$\}$.

Name: Y - PROBABILISTIC TOURNAMENT BRIBERY PROBLEM (Y-PTBP)

Given: A probability matrix $P \in \mathbb{Q}_{[0,1]}^{n \times n}$ describing the entrants e_i in the tournament with some preferred entrant e^* and (where necessary) cost matrix C_P, threshold t, budget B, set of manipulators $M \subseteq E$, and ordering T.

Evaluation: Is the probability that e^* wins Y (the sum of the probabilities of the futures with e^* a winner) above t ?

Pos-Win-\$: Can we raise e^*'s probability of winning Y above 0 by bribing specific entrants according to C_P and not exceeding the budget B?

Constructive Coalitional Manipulation (CCM): Can we raise e^*'s probability of winning Y above t by strategically setting all the probabilities associated with a subset M (a *coalition*) of the entrants?

Constructive Bribery: Can we raise e^*'s probability of winning Y above t by bribing specific entrants according to C_P and not exceeding the budget B?

Exact: Can we raise e^*'s probability of winning Y above t by bribing specific entrants according to C_P and spending exactly B?

We assume that the reader is familiar with the complexity classes P and NP and the idea of completeness for complexity classes (Garey and Johnson 1979). In this paper we consider several problems which may be in the functional class #P (equivalently PP) introduced by Valiant (Valiant 1979). An intuitive explanation is that #P is the counting analog of the class NP. For a given decision problem, the functional class #P asks, "How many solutions exist?"

If a problem is NP-complete, there are no algorithms that are always correct and always run in time polynomial in the

	Challenge	Cup	RR
Evaluation	P	P	$\in$ #P
Pos-Win-\$	P	P	P
CCM	P	$\in$ NP	$\in$ NP$^{\text{PP}}$
C. Bribery	$\in$ NP	$\in$ NP	$\in$ NP$^{\text{PP}}$
Exact	NP-complete	NP-complete	$\in$ NP$^{\text{PP}}$

Table 1: Complexity results for PROBABILISTIC TOURNAMENT BRIBERY PROBLEM. In cases where we have been unable to provide lower bounds, we note our upper bound results ($\in$).

size of the input, unless P=NP (a long-standing open problem). However, there are now many heuristic algorithms that are usually correct and usually fast. We do not yet have a suite of such heuristic algorithms for problems that are complete for #P.

Results

Challenge Tournament

We omit the proof of Theorem 2 for space considerations. We refer the reader to the proofs of Theorems 6 and 7, respectively, as they are similar in style and apply to the more general case of cups.

Theorem 2 *Evaluation and Pos-Win-\$ for* CT-PTBP *are in* P.

Recall that in the coalitional manipulation problem we are given a subset $M \subseteq E$ of the manipulating coalition. The coalitional manipulation problem is a special case of the bribery problem in our model. In a coalitional manipulation instance, $B = |M|$ and the cost of bribing members of M is also 1 (unit priced). The entrants $E \setminus M$ have bribery prices equal to ∞ (so they cannot be changed in this scenario).

Theorem 3 *Constructive Coalitional Manipulation for* CT-PTBP *is in* P.

Proof. Since there are no budget-related resource bounds in this problem we can assume our problem is to maximize e^*'s probability of winning by setting the entries for each $m \in M$ in the probability matrix P. We refer to the setting of M as a strategy. In this instance, C_P gives the available options of probability values for each $m \in M$ even though there are no prices.

There are $|M| \cdot (n - |M|)$ potential contests between coalition and non-coalition members with 2 possible strategies for each, and there are $|M| \cdot \frac{|M|-1}{2}$ contests between coalition members, with 3 possible strategies for each (*a* loses, *b* loses, both try to win), for a total of $2^{|M| \cdot (n-|M|)} 3^{|M| \cdot \frac{|M|-1}{2}}$ strategies.

Any manipulators who enter the tournament after e^* need to deterministically lose to e^* in order for e^* to win the championship. Therefore, for the rest of the proof, we focus on strategies for manipulators who enter the tournament before e^*. We construct a strategy in a step by step fashion

starting with the first game that e_{n-1} participates in. We proceed in reverse match order, one game at a time, setting the strategies for each $m \in M$ against each entrant as we go.

Step 0: For each $m \in M$, if m reaches e^* then m chooses to lose. This sets the strategy against e^*.

Repeat for: $j = 1$ to $n - 2$

   Step i: For each $m \in M$ that enters before e_{n-j}, suppose m reaches e_{n-j}. Set m's strategy against e_{n-j} to maximize the probability that e^* wins by evaluating the 2 or 3 possibilities. This is possible because strategies for all contests above e_{n-j} have been set. Also, if $e_{n-j} \in M$, then for each $e_i \notin M$, $i < n - j$, assuming e_i reaches e_{n-j}, pick e_{n-j}'s strategy against e_i to maximize e^*'s probability of winning.

After step $j = n - 2$, all strategies have been set for all entrants $\in M$. Whatever the strategy is at lower levels, $Pr(e^*$ wins $)$ is maximized by independently maximizing each $q_{ij} = Pr(e^*$ wins $|e_i$ reaches $e_{n-j})$. Since the events "e_i reaches e_{n-j}" for different i are disjoint, the q_{ij} can be maximized independently.

We do 2 or 3 evaluations for each pair of contestants. For each we require a call to the $\mathcal{O}(n^2)$ evaluation procedure mentioned in Theorem 2. Therefore we compute an optimal manipulation strategy in $\mathcal{O}(n^4)$. ❏

Theorem 2 gives us membership in NP for the constructive bribery problem since we can verify a bribery plan in polynomial time. Theorem 3 shows that in a situation of CCM we can achieve optimal manipulations in polynomial time. However, our results for CCM do not extend in a straightforward manner to the situation of priced bribery.

The hope would be that we can apply a *bang-for-the-buck* greedy algorithm to iteratively choose the cheapest available single bribe which results in the largest change in $Pr[e^*|P,T]/\text{COST OF BRIBE}$. However, this will not always reach an optimal solution. This can be illustrated by an example of CT-PTBP with 4 entrants, $e_0, \ldots, e_3$. Let $e_0 = e^*$ and let the entrants be situated on T in increasing order from left to right (e_0 in the left most node). We set the cheapest bribe of entrant e_1 to change the probability of winning by 0.25 with cost 3. We set the bribes for entrants e_2 and e_3 to change the probability of winning by 0.15 each with cost 2. If the budget is set to 4 then the best bang-for-the-buck bribe is the bribe for entrant e_1. However, the best action would be to bribe e_2 and then e_3. A simple greedy algorithm would make a non-optimal decision in this case.

In order to eliminate as many complications of the tournament graph as possible we turn our attention to the simplest case of a challenge tournament. This is a version where e^* is potentially involved in every game. This version, which we call CHALLENGE-THE-CHAMP-PTBP because e^* is taking on all challengers, is a common sub-problem for both the general CT-PTBP and CUP-PTBP problems.

Theorem 4 *There is a Constructive Bribery algorithm for* CHALLENGE-THE-CHAMP-PTBP *that runs in polynomial time with respect to B and n.*

Proof. With $e^* = e_1$ let $F_i = \Pi_{1 < i \leq n} p_{1,i}$ be the probability that e^* wins the first i games. Then the probability that e^* will be the champion after n rounds is F_n. Without loss of generality, we assume that B is bounded by the sum of all possible bribes.

Our goal is to maximize F_n (within budget). We use dynamic programming to build a $(n+1) \times B$ table D. Filling in each entry in the table takes polynomial time, so the problem is in P if the budget, B, is polynomial in the size of the input (e.g., B is input in unary). Otherwise, the algorithm is pseudo-polynomial.

The table entry $D[i,b]$ is the maximum we can make F_i via bribes to entrants $e_1, \ldots, e_i$, within budget b. Initialize $D[1,b] = 1$ for all b.

When we allow bribes to the first $i+1$ entrants, the maximum probability (of e^* winning) attainable either involves a bribe to e_{i+1}, and whatever bribes are needed with the remaining money, or does not require a bribe to e_{i+1}. This gives us the dynamic programming update.

A bribe to entrant e_{i+1} (to lose to e^*) changes $p_{1,i+1}$ to some new value $p'_{1,i+1}$. We can compute F_{i+1} for the given bribe by $F_{i+1} = F_i \cdot p'_{1,i+1}$.

Let $h = \max\{D[i, b-d] \cdot p'_{1,i+1}\}$ where d is the cost of increasing $p_{1,i+1}$ to $p'_{1,i+1}$. We set $D[i+1,b] = \max\{D[i,b] \cdot p_{1,i+1}, h\}$. Since there are at most $k+1$ bribes for each e_i, computing $\{D[i+1,b] : 0 \leq b \leq B\}$ takes $\mathcal{O}(k \cdot n \cdot B)$ operations. This is polynomial in k, n, and B. The algorithm runs in polynomial time with respect to $|B|$ and n. ❏

While we have been unable to show NP-hardness for any version of the constructive bribery problem, Theorem 4, with the insights listed in this section, leads to the conjecture that the problem is hard.

If we require our outside manipulator to spend exactly the allocated budget then we can show NP-completeness for challenge tournaments. Situations which require an organization to spend a budget exactly occur implicitly in many large organizations and governments. For resources other than money that could be used, such as referees in sports tournaments or canvassers in political elections, there is almost always the requirement of exact allocations.

Consider the SUBSET SUM PROBLEM, which is NP-complete (Garey and Johnson 1979) : **Given:** $w_1, \ldots, w_m, S \in \mathbb{N}_+$, does there exist a subset $I \subseteq \{1, \ldots, m\}$ such that $\sum_{i \in I} w_i = S$?

Theorem 5 *Exact Bribery for* CT-PTBP *is NP-complete.*

Proof. Membership is an immediate consequence of Theorem 2 and a guess and check algorithm.

To show NP-hardness we provide a reduction from SUBSET SUM: For a given SUBSET SUM instance we set up a challenge tournament with S players such that e^* always wins (i.e., e^* has a 100% chance of winning all games against all players) and we let $t = 0$, $B = S$, and $k = 0$. We then create the bribery cost matrix C_P with prices equal to the weights of $w_1, \ldots, w_m$. In this situation bribery will have no effect and we must distribute the money exactly. We have now established a polynomial time mapping such that there will be an exact bribery if and only if there is a subset such that $\sum_{i \in I} w_i = S$. ❏

Cup Tournament

Many of the results for CUP-PTBP use reasoning similar to that used for CT-PTBP. We assume that each CUP-PTBP instance is a complete tree. Note that this does not limit our results; we can always pad an incomplete tree with entrants who lose to all other entrants and cannot be bribed.

Theorem 6 *Evaluation for* CUP-PTBP *is in* P.

Proof. This is a proof by construction of a polynomial time algorithm to compute $Pr[e^*|P,T]$ for all entrants. We construct a table of size $n \cdot \lceil \log_2(n) \rceil$ and use dynamic programming to compute for each i: $L_{r,i}$ where $L_{r,i}$ is the probability that entrant i advances to round r (with $r = 1$ being the first match e_i competes in). When we get to the last round, our table will contain the probability that each entrant won.

Let $G(r,i)$ be the set of entrants e_i may face in round r.
for $i = 1$ to n **do**
  $L_{1,i} = 1$
end for
for $r = 2$ to $\lceil \log_2(n) \rceil$ **do**
  for $i = 1$ to n **do**
    $L_{r+1,i} = L_{r,i} \cdot \sum_{x \in G(r,i)} p_{i,x} \cdot L_{r,x}$
  end for
end for

Each e_i potentially competes against each x in exactly one round, so there are exactly $n(n-1) \in \mathscr{O}(n^2)$ multiplications in total, so the total time is $O(n^4 \log(k)^2)$. ❑

Theorem 7 *Pos-Win-$ for* CUP-PTBP *is in* P.

Proof. We provide a polynomial time dynamic programming algorithm to compute the minimum cost such that e^* has a non-zero probability of winning the tournament.

Let the tournament graph T be a complete binary tree. Let l be the level of the binary tree starting with 0 at the bottom level. We then create a vector for each entrant, V_{e_i}, of size n with all entries initialized to 0 (the cost to get to the $0th$ level). Let $V_{e_i}(l)$ be the minimum cost for e_i to have a non-zero probability of winning at level l. We construct the *minimum cost matrix*, $min\$$, an integer matrix of size n^2, as follows: $min\$_{ij} = 0$ if i starts with a non-zero probability to beat j and the minimum bribe cost $min\$_{ij} = c_{ij}(1/(k+1))$ otherwise.

for $l = 1$ to $\log(n)$ **do**
  for all $e \in T$ **do**
    Let $G(e,l)$ be the set of entrants in the sub-tree at level l containing e.
    $V_e(l) = V_e(l-1) + \min_{x \in G(e,l)-G(e,l-1)}(min\$_{e,x} + V_x(l-1))$
  end for
end for

At the end of execution $V_{e_i}(\log_2(n))$ will contain the minimum cost to promote e_i to the top level of T with a non-zero winning probability. We can compare this cost with B and either accept if $V_{e^*}(\log_2 n) \le B$, otherwise we reject. ❑

Theorem 6 immediately provides us with the following corollaries through standard guess and check algorithms.

Corollary 8 *Constructive Coalitional Manipulation and Constructive Bribery are in* NP *for* CUP-PTBP.
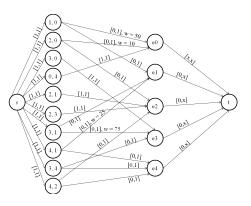

Figure 2: An example of a minimal cost winner determination such as described in the proof of Theorem 12.

We have not been able to show lower bounds results for CCM or constructive bribery for CUP-PTBP. The result does not hold for cup tournaments because the winner within separate sub-trees can affect the winning probability of all entrants in a neighboring sub-tree. We can, however, straightforwardly transfer our results to the exact case.

Theorem 9 *Exact Bribery for* CUP-PTBP *is* NP-*complete.*

Round Robin Tournament

Round Robin tournaments are of particular interest in the social choice community because of their close correspondence to the Copeland election system (Faliszewski et al. 2009). In addition, many papers on sports elimination, including those by Gusfield and Martel (2002) and Kerns and Paulusma (2004), study sports round robin tournaments for particular competitions (FIFA, Major League Baseball etc.). Our model is more general: we do not restrict our model to deterministic settings or specific tournament formats.

We have been unable to show #P-hardness for the evaluation problem for round robin tournaments, although we strongly believe this problem to be #P-hard. Hazon et al. showed that the problem is #P-hard for the voting system Copeland with an imperfect information model (2008a). However, their results do not transfer in a straightforward manner, as their reduction requires the ability to add voters who are not candidates. Likewise Gusfield and Martel showed a similar problem of determining the probability that a team is eliminated from a round robin sports competition is #P-hard (2002). Again, their result relies on the ability of their model to construct single round tournaments. We omit the proofs of Theorems 10 and 11 for space considerations.

Theorem 10 *Evaluation for* RR-PTBP *is in* #P.

Theorem 11 *CCM, Constructive Bribery, and Exact Bribery for* RR-PTBP *are in* NP^{PP}.

Consider the problem MINIMUM COST FEASIBLE FLOW (Ahuja, Magnanti, and Orlin 1993): **Given**: A graph $G(V,E)$ with vertices V, edges E, source node s, and sink node t. Each $e \in E$ has flow, capacity, and weight. An edge labeled "$[x,y],w$" has capacity y and requires flow of at least

x with weight *w*. Weight is accumulated on a per unit flow basis (i.e., if $x = 2$ and $w = 3$ then the total weight of the edge is 6). FIND: a flow from *s* to *t* such that the flow into each node is the same as the flow out, all minimum edge flows are satisfied, and weight is minimal. There is a polynomial time algorithm and its solution is integral (Ahuja, Magnanti, and Orlin 1993).

Using minimum cost feasible flows, Russell and Walsh provided an algorithm to compute minimal constructive manipulations (but not bribery) for deterministic round robin tournaments when the number of games that e^* can win is fixed (Russell and Walsh 2009). Likewise, Faliszewski et al. showed manipulation results for Copeland under deterministic information elections using feasible flows (Faliszewski et al. 2009). While our construction is similar to theirs (and to an earlier construction from Gusfield and Martel (2002)), there are significant differences between the constructions. Our results are for a more flexible model, which is able to encapsulate bribery and manipulation, and works for even or odd numbers of entrants in the tournament.

Theorem 12 *Pos-Win-$ for* RR-PTBP *is in* P.

Proof. We sketch the proof for space considerations. We construct an instance of a minimum cost feasible flow for our problem where the solution selects the cheapest bribes that will allow e^* to win. In the graph, units of flow correspond to points (wins) and edge weights correspond to bribery costs. When we construct the graph correctly, there will be a feasible flow if and only if e^* can be made a winner. An example of a constructed graph is shown in Figure 2. If the cost of this graph is $\leq B$ then we accept, otherwise we reject. ❏

Conclusion

We have studied the complexity of bribery in sports tournaments when the results of individual matches are represented by probabilities. This is a departure from other work in the field, which only considers bribery in situations where results of games are known and can be manipulated in definite terms. We have introduced a novel and more general model that encapsulates both coalitional manipulation and bribery. Our model captures specific tournament types which are also used as voting rules in many domains (web page ranks, senate ballots etc.). The problems, when considered in this probabilistic environment, have an apparent range of complexity from P to NP^{PP}. The previously studied deterministic cases of evaluation, constructive manipulation, and possible winner, are all polynomial time decision problems.

We continue to study the open problems mentioned here regarding bribery and manipulation in stochastic voting/scoring models. Additionally, it would be interesting to investigate these problems through the lens of parameterized complexity (Downey and Fellows 1999).

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