Analogy Engines in Classroom Teaching: Modeling the String Circuit Analogy

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Abstract

The importance of analogy-making and analogy-based reasoning for human cognition and learning by now has widely been recognized, and analogy-based methods are slowly also being explicitly integrated into the canon of approved education and teaching techniques. Still, the actual level of knowledge about analogy as instructional means and device as of today is rather low and subject to scientific study and investigation. In this paper, we propose the fruitful use of computational analogy-engines as methodological tool in this domain of research, motivating our claim by a short case study showing how Heuristic-Driven Theory Projection can be used to model the mode of operation of an analogy taken from a science class for 8 to 9 year old children.

Introduction

Analogical reasoning (i.e., the human ability of perceiving – and operating on – dissimilar domains as similar with respect to certain aspects based on shared commonalities in relational structure or appearance) has been proposed as an essential part of the human ability to learn abstract concepts (Gentner, Holyoak, and Kokinov 2001) or procedures (Ross 1987). Also, analogy seems to be constitutive for the ability to transfer representations across contexts (Novick 1988) or to adapt to novel contexts (Holyoak and Thagard 1995), and allegedly also plays a crucial role in children's cognitive repertoire for learning about the world (Goswami 2001).

Taking all this into account, it should not come by surprise that analogy also receives growing attention in the field of science education: For example Duit (1991) argues that analogies are powerful tools to facilitate learners' construction process of new ideas and conceptions on the grounds of already available concepts, and Arnold and Millar (1996) assert that analogies can foster understanding by abstracting the important ideas from the mass of new information, helping to clarify the system boundaries and internal dynamics, and providing an appropriate language for framing a scientific explanation. On the other hand, as already advocated in (Halasz and Moran 1982), analogy clearly is not the universal remedy for challenges and difficulties in teaching and explanation that it sometimes is proposed to be. Nonetheless, despite these insights and early proposals, the current level of knowledge about analogy as an instructional device in everyday practice has to be considered quite low, and the pedagogical use of analogies as means for triggering, framing and guiding creative insight processes still needs to be widely recognized as part of teaching expertise and incorporated to innovative teacher education schemes (Akgul 2006).

Working towards changing this rather unsatisfactory state of affairs, on the scientific side, there is a growing body of work specifically treating with analogies in an educational context, as for instance studies on the use of analogies in mathematics classrooms (Richland, Holyoak, and Stigler 2004) or in elementary science education (Guerra-Ramos 2011). Both studies support the initially cited assumption that analogy can be used for facilitating the understanding of concepts and procedures in often rather abstract and formal domains as mathematics, physics or science.

In this paper, we want to contribute to a deeper understanding of the role and the mode of operation of analogy in an educational context by showing how a computational analogy-making framework as Heuristic-Driven Theory Projection (HDTP) can be used to provide a formal computational reconstruction of an example of analogy-use taken from a real-life teaching situation: the string circuit analogy for gaining a basic understanding of electric current (Asoko 1996; Guerra-Ramos 2011). By doing so, we want to show one way (amongst several) of how analogy-engines and their corresponding background theories can fruitfully be applied to modeling and analysis tasks from the field of psychology of learning, education, and didactics.

The paper is structured as follows: After a compact introduction to Heuristic-Driven Theory Projection as framework for the formal modeling and computational reconstruction of analogy-making processes, a detailed description and corresponding HDTP-based reconstruction of the string circuit analogy for electric current is provided. Following this motivating case study, thoughts and recommendations for further applications of computational analogy-making frameworks to analogy-related questions arising in the domain of science education are given and shortly elaborated on. The conclusion provides a compact overview of conceptually and topically related work and projects.

Heuristic-Driven Theory Projection (HDTP)

Since the advent of computer systems, researchers in cognitive science and artificial intelligence have been trying to create computational models of analogy-making. One of the outcomes of this line of work is Heuristic-Driven Theory Projection (HDTP) (Schwering et al. 2009), a formal framework (and corresponding software implementation) conceived as a mathematically sound framework for analogy-making.

HDTP has been created for computing analogical relations and inferences for domains which are given in form of a many-sorted first-order logic representation (Guhe et al. 2011). Source and target of the analogy-making process are defined in terms of axiomatizations, i.e., given by a finite set of formulae. HDTP tries to produce a generalization of both domains by aligning pairs of formulae from the two domains by means of anti-unification: Anti-unification tries to solve the problem of generalizing terms in a meaningful way, yielding for each term an anti-instance, in which distinct subterms have been replaced by variables (which in turn would allow for a retrieval of the original terms by a substitution of the variables by appropriate subterms).

HDTP in its present version uses a restricted form of higher-order anti-unification (Krumnack et al. 2007), significantly expanding Plotkin's original theory of first-order anti-unification (Plotkin 1970). In higher-order anti-unification, classical first-order terms are extended by the introduction of variables which may take arguments (where classical first-order variables correspond to variables with arity 0), making a term either a first-order or a higher-order term. Then, anti-unification can be applied analogously to the original first-order case, yielding a generalization subsuming the specific terms. As already indicated by the naming, the class of substitutions which are applicable in HDTP is restricted to (compositions of) the following four cases: renamings (replacing a variable by another variable of the same argument structure), fixations (replacing a variable by a function symbol of the same argument structure), argument insertions, and permutations (an operation rearranging the arguments of a term). This formalism has proven to be capable of detecting structural commonalities not accessible to first-order anti-unification, but unfortunately does not guarantee to produce a unique least general generalization. Therefore, the current implementation of HDTP ranks generalizations according to a complexity measure on generalizations and chooses the least complex generalizations as preferred ones (Schwering et al. 2009; Schmidt et al. 2011). Once the generalization has been computed, the alignments of formulae together with the respective generalizations can be read as proposals of analogical relations between source and target domain, and can be used for guiding an analogy-based process of transferring knowledge between both domains (cf. Fig. 1). Analogical transfer results in structure enrichment on the target side, which usually corresponds to the addition of new axioms to the target theory, but may also involve the addition of new first-order symbols.

As of today, the HDTP framework has successfully been tested in different application scenarios, and its use in several others has been proposed and theoretically grounded.

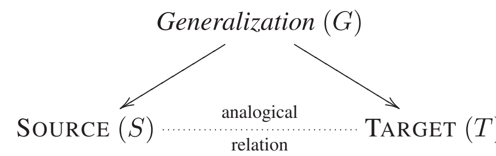

Figure 1: Analogy via generalization in HDTP

(Guhe et al. 2010) shows a way how HDTP can be applied to model analogical reasoning in mathematics by a case study on the inductive analogy-making process involved in establishing the fundamental concepts of arithmetic, (Guhe et al. 2011) applies HDTP to conceptual blending in the mathematics domain by providing an account of a process by which different conceptualizations of number can be blended together to form new conceptualizations via recognition of common features, and judicious combination of their distinctive features. In (Schwering, Gust, and Kühnberger 2009), HDTP has been used in the context of solving geometric analogies. On the more theoretical side, (Besold et al. 2011) considers how the HDTP framework could fruitfully be applied to modeling human decision-making and rational behavior, (Martinez et al. 2011) elaborates on how HDTP could be expanded into a domain-independent framework for conceptual blending, and (Martinez et al. 2012) provides considerations on the applicability of HDTP to tasks and problems in computational creativity.

The String Circuit Analogy For Electric Current

In the following, we reconstruct the string circuit analogy for electric current as used for teaching students aged 8 to 9 a basic understanding of energy transfer and current flow in simple electric circuits. We follow the account of the analogy given by Guerra-Ramos (2011), who analyzed the analogy and its mode of operation based on (Asoko 1996), trying to closely cover and reconstruct the provided descriptions.

The analogy uses a representation which places participants in a circle, making them loosely support with their hands a continuous string loop that one person subsequently makes circulate (for an analysis of the analogical correspondences between the electric circuit and the string circuit, cf. Fig. 2). Before introducing the analogy (as part of a bigger teaching sequence consisting of several successive sessions), in previous teaching sessions the students had from observation of simple electric circuits (battery, wires, light bulb) acquired basic ideas about some of the functions of simple parts of an electric circuit, e.g. the insight that "energy", "electricity" or "power" seems to leave a battery. When working with the analogy, the teacher set up the string circuit involving all the students and himself as the battery, and also actively encouraged students to conjecture about how their observations from the string circuit's workings might translate to the non-observable processes and mechanisms within the electric circuit (also emphasizing some crucial main ideas as, e.g., the provision of energy by the battery, and that energy is carried by current). Subsequent teaching

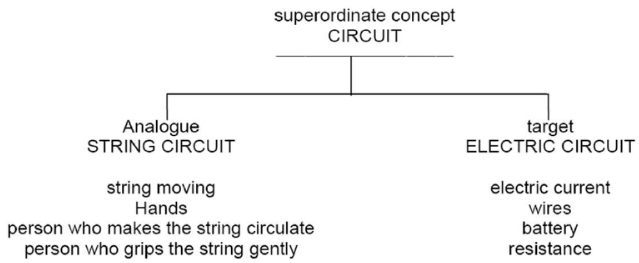

Figure 2: Representation of the "string circuit" analogy as given in (Guerra-Ramos 2011).

sessions built upon the insights gained from the analogy, expanding and deepening students' understanding of the topic.

From a formal perspective, in the described case the (better understood, and thus richer) string circuit domain serves as source domain for the analogy, whilst the more abstract setting in the electric circuit domain is the target domain of the analogy. A possible formalization of the students' initial ideas about setup and workings of an electric circuit can be conceptualized as in Table 1, whilst the string circuit domain can be represented as shown in Table 2.

Sorts:
agent, medium, phenomenon.
Entities:
battery, resistance : *agent.*
wires : *medium.*
electric_current : *phenomenon.*
Predicates:
part_of_circuit : *agent.*
drives : *agent* $\times$ *phenomenon.*
parts_connected_by : *medium.*
Facts:
(e_1) *part_of_circuit*(*battery*).
(e_2) *part_of_circuit*(*resistance*).
(e_3) *drives*(*battery*, *electric_current*).
(e_4) *parts_connected_by*(*wires*).

Table 1: Formalization of the electric circuit domain (target domain of the later analogy).

Given these formalizations, HDTP can be used for computing a common generalization of both, yielding a generalized theory like given in Table 3. For doing so, corresponding domain elements have been identified, aligned and generalized according to HDTP's heuristics-driven anti-unification mechanism. Consider the following examples:

- (e_1) *part_of_circuit*(*battery*) and (s_1) *part_of_circuit*(*person_boosting*) get generalized into (g_1) *part_of_circuit*(A_1), with A_1 being a variable of sort *agent*. The alignment of (e_1) with (s_1), instead of e.g. the pairing (e_1) with (s_2), is motivated by HDTP's heuristics, as an alignment of (e_1) with (s_1) allows for a reuse of the corresponding generalization (of *battery* and *person_boosting* to A_1) when later aligning and generalizing (e_3) *drives(battery, electric_current)* with (s_3) *drives(person_boosting, moving_string)*, in total minimizing the heuristic costs attached to the generalization.
- (e_4) *parts_connected_by*(*wires*) and (s_4) *circuit_supported_by*(*hands*) get generalized into (g_4) *C*(*M*), with *C* being a unary function variable and *M* being a variable of sort *medium*. During the alignment process, the heuristics-based selection of a counterpart for generalization with (s_4) *circuit_supported_by*(*hands*) is based on the similarity in structure (unary predicate, sort of argument) with (e_4) *parts_connected_by*(*wires*).

Sorts:
agent, medium, phenomenon, influence, real.
Entities:
person_boosting, people_gripping : *agent.*
hands : *medium.*
moving_string : *phenomenon.*
reinforcing, hindering, accelerating, decelerating : *influence.*
Functions:
exerts_influence: *agent* $\times$ *influence* $\times$ *phenomenon* $\rightarrow$ *real.*
Predicates:
part_of_circuit : *agent.*
drives : *agent* $\times$ *phenomenon.*
circuit_supported_by : *medium.*
conducted_via : *phenomenon* $\times$ *medium.*
change : *phenomenon* $\times$ *influence.*
Facts:
(s_1) *part_of_circuit*(*person_boosting*).
(s_2) *part_of_circuit*(*people_hindering*).
(s_3) *drives*(*person_boosting*, *moving_string*).
(s_4) *circuit_supported_by*(*hands*).
(s_5) *conducted_via*(*moving_string*, *hands*).
(s_6) *exerts_influence*(*person_boosting*, *reinforcing*, *moving_string*) ≥ 0.
(s_7) *exerts_influence*(*people_gripping*, *hindering*, *moving_string*) ≥ 0.
Laws:
(s_8) *exerts_influence*(*person_boosting*, *reinforcing*, *moving_string*) $>$ *exerts_influence*(*people_gripping*, *hindering*, *moving_string*) $\rightarrow$ *change*(*moving_string*, *accelerating*).
(s_9) *exerts_influence*(*people_gripping*, *hindering*, *moving_string*) $>$ *exerts_influence*(*person_boosting*, *reinforcing*, *moving_string*) $\rightarrow$ *change*(*moving_string*, *decelerating*).

Table 2: Formalization of the string circuit domain (source domain of the later analogy).

In cases where no corresponding counterpart for elements from one domain can be found in the other domain (as for instance for facts (s_5), (s_6), and (s_7), as well as laws (s_8) and (s_9) from the string circuit domain), formulae have been anti-unified as far as possible (reusing substitutions applied earlier in the generalization process) and then have been added to the shared core of the generalized theory (as exemplified by (g_5*),..., (g_9*)).

Concludingly, the generalized theory can be used to transfer knowledge in an analogy-based way from the (originally richer) string circuit domain to the electric circuit domain, resulting in a expanded theory for the electric circuit as given in Table 4. The newly introduced domain el-

Sorts:
agent, medium, phenomenon, influence, real.
Entities:
A_1, A_2 : *agent.*
M : *medium.*
P : *phenomenon.*
$(*)$ *reinforcing, hindering, accelerating,decelerating* : *influence.*
Functions:
$(*)$ *exerts_influence*: *agent* $\times$ *influence* $\times$ *phenomenon* $\rightarrow$ *real.*
Predicates:
part_of_circuit : *agent.*
drives : *agent* $\times$ *phenomenon.*
C : *medium.*
$(*)$ *conducted_via* : *phenomenon* $\times$ *medium.*
$(*)$ *change* : *phenomenon* $\times$ *influence.*
Facts:
(g_1) *part_of_circuit*(A_1).
(g_2) *part_of_circuit*(A_2).
(g_3) *drives*(A_1, P).
(g_4) *C*(M).
(g_5*) *conducted_via*(P, M).
(g_6*) *exerts_influence*$(A_1$, *reinforcing*, $P) \geq 0$.
(g_7*) *exerts_influence*$(A_2$, *hindering*, $P) \geq 0$.
Laws:
(g_8*) *exerts_influence*$(A_1$, *reinforcing*, $P)$ $>$ *exerts_influence*$(A_2$, *hindering*, $P)$ $\rightarrow$ *change*$(P$, *accelerating*$)$.
(g_9*) *exerts_influence*$(A_2$, *hindering*, $P)$ $>$ *exerts_influence*$(A_1$, *reinforcing*, $P)$ $\rightarrow$ *change*$(P$, *decelerating*$)$.

Table 3: Generalized theory of the electric circuit and the string circuit, already expanded by the generalized forms of the candidate elements for analogical transfer (marked with $*$) from the source domain to the target domain.

Sorts:
agent, medium, phenomenon, influence, real.
Entities:
battery, resistance : *agent.*
wires : *medium.*
electric_current : *phenomenon.*
$(*)$ *reinforcing, hindering, accelerating,decelerating* : *influence.*
Functions:
$(*)$ *exerts_influence*: *agent* $\times$ *influence* $\times$ *phenomenon* $\rightarrow$ *real.*
Predicates:
part_of_circuit : *agent.*
drives : *agent* $\times$ *phenomenon.*
parts_connected_by : *medium.*
$(*)$ *conducted_via* : *phenomenon* $\times$ *medium.*
$(*)$ *change* : *phenomenon* $\times$ *influence.*
Facts:
(e_1) *part_of_circuit*(*battery*).
(e_2) *part_of_circuit*(*resistance*).
(e_3) *drives*(*battery*, *electric_current*).
(e_4) *parts_connected_by*(*wires*).
(e_5*) *conducted_via*(*electric_current*, *wires*).
(e_6*) *exerts_influence*(*battery*, *reinforcing*, *electric_current*) ≥ 0.
(e_7*) *exerts_influence*(*resistance*, *hindering*, *electric_current*) ≥ 0.
Laws:
(e_8*) *exerts_influence*(*battery, reinforcing, electric_current*) $>$ *exerts_influence*(*resistance, hindering, electric_current*) $\rightarrow$ *change*(*electric_current, accelerating*).
(e_9*) *exerts_influence*(*resistance, hindering, electric_current*) $>$ *exerts_influence*(*battery, reinforcing, electric_current*) $\rightarrow$ *change*(*electric_current, decelerating*).

Table 4: Analogically enriched formalization of the electric circuit domain.

ements are (e_5*),. . .,(e_9*), directly corresponding to elements (g_5*),. . ., (g_9*) from the generalized theory (i.e., going back to elements from the string circuit domain that – due to lacking counterparts in the electric circuit domain – could not be aligned during the anti-unification process). In transferring (g_5*),. . ., (g_9*), the respective variables are reinstantiated with subterms in accordance with the anti-unifications used in generating the generalization, for instance inverting and reusing the anti-unification from *battery* to A_1, applied in generalizing (e_1) *part_of_circuit*(*battery*) into (g_1) *part_of_circuit*(A_1), when instantiating (g_6*) *exerts_influence*$(A_1$, *reinforcing*, $P) \geq 0$ into (e_6*) *exerts_influence*(*battery*, *reinforcing*, *electric_current*) ≥ 0.

On the interpretation side, we claim that the model generated by HDTP, consisting of the alignments between the domains, the generalized theory and finally the enriched electric circuit domain, computed on basis of the provided formalizations of the electric circuit and the string circuit domain (which in turn simply described the students' initially present knowledge about electric circuits, and the information about the string circuit analogy accessible to them via observation and experience), gives a remarkably realistic account of what plausibly is happening in the students' minds: After identifying corresponding elements between the two original domains (i.e., alignment), underlying basic principles and general relations are discovered (i.e., generalization) and used for forming conjectures about the inner workings of one domain based on knowledge about the other one (i.e. analogical transfer of knowledge), which later can then be tested and confirmed or retracted again, possibly resulting in an overall reconceptualization of the entire analogy (the latter steps corresponding to teaching sessions following the session introducing the analogy, involving hands-on experiments with simple electric circuits built by the students, cf. (Asoko 1996)).

Analogy-Engines Meet Education And Teaching

Of course we are not the first ones to consider the use of computational analogy-making systems in a context of education and teaching-related topics. In (Thagard, Cohen, and Holyoak 1989), the authors present a theory and implementation of analogical mapping that applies to explanations of unfamiliar phenomena as for instance used by chemistry teachers (both, explanations providing systematic clarification as well as explanations giving a causal account of why something happened). (Forbus et al. 1997) amongst others shows how an information-level model of analogical inferences (supporting reasoning about correspondences and mappings), together with techniques for the strucutral evaluation of analogical inferences, can be incorporated in a case-based coach that is being added to an intelligent learning environment, whilst (Lulis, Evens, and Michael 2004) analyzes the use of analogies in human tutoring sessions for medical students in order to build an interactive electronic tutoring system capable of also applying analogy. Siegler (1989) shortly conjectures how the Structure-Mapping Engine (SME) (Falkenhainer, Forbus, and Gentner 1989) as prototypical analogy-engine could be used to gain insights about developmental aspects of analogy use.

Still, our claim and vision is a stronger one, not only seeing analogy-making engines as a tool for integration in technical systems trying to imitate human tutors or providing intelligent learning environments, but taking up intuitions traces of which are already recognizable in (Thagard, Cohen, and Holyoak 1989) and (Siegler 1989) and expanding them into a proper program opening up a new application domain to computational analogy-making systems:

Modeling and analysis: Symbol-based analogy-engines, as for example the SME or HDTP, can be put to use for modeling and understanding the conceptual mode of operation of analogies in a teaching context, addressing a level of detail situated between the level of computational theory and the level of representation and algorithm in Marr's Tri-Level hierarchy (Marr 1982): Although there will most likely not be a perfect match on algorithmic level between the mental processes triggered in the student's mind and the inner workings of the analogy model, the correspondence can be deeper rooted than residing exclusively on the computational level, not only covering input/output behavior in a purely functionalist way but also addressing basic properties and principles of the mechanisms at work. Symbol-based approaches, as opposed to connectionist or hybrid ones, seem especially suitable for such an endeavor due to their explicit representation of domain elements and the resulting explicated conceptual structure of the analogies under consideration, allowing for a direct evaluation against the background of theories and results from the learning sciences and experimental classroom data.

Also, as proposed in (Siegler 1989), developmental aspects of analogy use may be addressed by taking into account results form cognitive and developmental psychology about children's cognitive development and cognitive capacities, accordingly constraining the mechanisms used by the analogy engine (as for example the types of substitutions applied by HDTP during anti-unification) and observing how the behavior and outcomes of the computational analogy-making system changes, possibly allowing for predictions about and projections back into the developmental context.

Exploration and testing: Heuristics-based analogy-making frameworks, or frameworks allowing for an incorporation of heuristics-like aspects to the guiding and steering mechanisms of the engine, may be used for exploring, developing and testing analogies for a teaching context. Provided that, based on reliable results from corresponding psychological research, the heuristics applied by the computational system are adapted as to mirror children's cognitive capacities and limitations in adequate ways, the analogy-engine can be used for testing the suitability of analogies considered for the use in a specific teaching situation. Also, in an exploration-type mode, the system could be put to use in discovering possible analogies between two domains that are given by the teacher for application in a specific educational context, thus simulating children's analogy generation and understanding in the respective situation.

Discovery and guidance: Given two domains by the teacher, the analogy-engine can be used for discovering what analogies possibly arise between these domains and how the analogy-making process might have to be guided (e.g., what framing facts have to be included in the initial domain theories) to obtain one specific, previously planned analogy as result of the process. Assuming similarity between the mechanisms implemented in the analogy-making system and the principles and constraints applying to analogy-making in students, the guiding constraints obtained during the run of the system may then be used for re-designing the initial domain theories, reflecting back to the knowledge and previous experience students would have to be equipped with in order to assure the rising of the intended target analogy in the concrete teaching situation.

Naturally, the just given list of three basic possibilities for using computational analogy-making systems in the domain of teaching and education is not exhaustive, but many more application scenarios are imaginable. Still, we are convinced that already the sketched use cases should be sufficient to motivate future work and effort.

Conclusion

Given the growing interest in analogy and analogy-based methods in the educational and teaching sector, together with the still early stage of understanding and study of the mode of operation of these means, applying already existing formal frameworks and computational systems for analogy-making to some of the arising tasks and challenges only seems logical and desirable. Here, the range of applications ranges from the reconstruction and analysis of single examples from the classroom (as exemplified by the string circuit analogy for electric current) to more ambitious endeavors as for instance the ones sketched in the previous section.

Concerning starting points for future work, each one of the three scenarios sketched in the previous section offers various possibilities, where from our perspective one of the most interesting – and at the same time probably most challenging – open questions is the adaption of the mechanisms of an existing analogy-engine to results and data from psychological studies. In the concrete case of HDTP this would mean to adapt the heuristic costs attached to the different types of substitution applied during anti-unification, and the different discounts e.g. for reuse of already applied substitutions, as to fit human experimental data, possibly resulting in a (to a certain extent) more "cognitively adequate" model of analogy-making than currently exposed by HDTP.

And also the reconstruction of concrete examples of analogy-use seem to offer significant potential for further study and research: Provided that the domain formalizations are suitably modeled and the theory underlying the framework adequately depicts some mechanisms of children's analogy-making on a computational level, different scenarios of applying analogy-engines for simulating how students will cope with and utilize an analogy, possibly detecting points where problems might arise or where additional input could significantly improve the learning outcome, would merit attention and investigation.

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