On the Construction of Trust Metrics

Jonathan Barzilai
Dept. of Industrial Engineering, Dalhousie University
P.O. Box 15000, Halifax, N.S., Canada B3H 4R2
barzilai@dal.ca

Abstract

We do not share information, actions, strategy, or plans with agents (human or otherwise) we do not trust, because they may use it against us or pass such information voluntarily or unknowingly to others who may use it against us. We may also have doubts about the identity of the recipient of our trust, his motivation and relations with others, his reliability (hardware, software, personal, or organizational) and vulnerability to other agents who may harm us. Since *trust* is an important consideration in determining the degree of cooperation and collaboration among agents, it is one of the elements of *coalition* forming — a game-theoretic subject. In addition, since *trust* is not a physical variable, the problem of constructing metrics and measurement scales for non-physical variables must be taken into account.

On Trust and Value Scale Construction

Overview

Value and *trust,* e.g. trust in the stability of the financial system, are fundamental concepts in economic theory. While the literature on value (and its synonyms preference and utility) is vast there seems to be nothing written about the construction of trust scales. The reason for this is, very probably, that trust has different meanings in different contexts and is a multi-layered variable. We begin by outlining the difficulties involved in the simpler construction of value scales and sketch some of the additional difficulties with the measurement of trust.

Value is not a physical property of the objects being valued. Whether non-physical variables can be measured in order to apply mathematical operations on them was an open question as late as 1940 following a lengthy scientific debate (see Ferguson *et al*. 1940). The controversy appeared to have been settled in *Theory of Games and Economic Behavior* (von Neumann and Morgenstern 1944). However, rather than establish the applicability of mathematical operations on non-physical variables, they have addressed the unrelated problem of scale uniqueness which, following Stevens's extension of von Neumann and Morgenstern's work, has since been known as the problem of scale classification. As a result, the applicability of mathematical operations has not been further investigated in the literature and such operations have been applied incorrectly and without foundation throughout the literature of decision theory, the theory of games, microeconomics, measurement theory, and elsewhere.

Addition and multiplication, the operations of fields, vector, and affine spaces, are not applicable on ordinal, ratio, or difference scale values. The correct model where addition and multiplication are applicable for variables that have no absolute zero is a one-dimensional affine space. This is the case for physical variables such as time and potential energy and for all non-physical variables including *value* and *trust*.

Scale Definition — the Framework

An empirical system E is a set of empirical objects together with operations, and possibly the relation of order, which characterize a property under measurement. A mathematical model M of the empirical system E is a set with operations that reflect the operations in E as well as the order in E when E is ordered. A scale s is a homomorphism from E into M, i.e. a mapping of the objects in E into the objects in M that reflects the structure of E into M. The purpose of modelling E by M is to enable the application of mathematical operations on the elements of the mathematical system M and mathematical operations in M are applicable if and only if they reflect empirical operations in E.

The framework of mathematical modelling is essential because it is the only way by which mathematical operations can be introduced into decision theory, the theory of games, economic theory, or any other theory. To enable the application of mathematical operations, the empirical objects are mapped to mathematical objects on which these operations are performed. In mathematical terms, these mappings are functions from the set of empirical objects to the set of mathematical objects. Given two sets, a large number of mappings from one to the other can be constructed, most of which are not related to the characterization of the property under measurement: A given property must be characterized by empirical operations which are specific to this property and these property-specific empirical operations are then reflected to corresponding operations in the mathematical model. Measurement scales are those mappings that reflect the specific empirical operations which characterize the given property to corresponding operations in the mathematical model. Therefore, the construction of measurement scales requires that the property-specific empirical operations be identified and reflected in the mathematical model.

Scale construction for physical variables requires the specification of the set of objects and the property under measurement as well as the applicable operations on these objects. Since *value* is not a physical property of the objects being valued, the construction of scales for *value* and other non-physical properties (which are also referred to as personal, psychological, or subjective), requires also the specification of the evaluator. For example, the characteristic function of game theory is ill-defined for this reason.

Applicability of Operations: Mathematical Spaces

Mathematical spaces, e.g. vector or metric spaces, are sets of objects on which specific relations and operations (i.e. functions or mappings) are defined. They are distinguished by these relations and operations — unless explicitly specified, the objects are arbitrary.

Only those relations and operations that are defined in a given mathematical space are relevant and applicable when that space is considered — the application of undefined operations is an error. For example, although the operations of addition and multiplication are defined in the *field* of real numbers, multiplication is undefined in the *group* of real numbers under addition — multiplication is not applicable in this group.

When the conditions for applicability of addition and multiplication on non-physical variables are satisfied, these variables are represented by points in one-dimensional affine spaces. Although vector-space operations are not applicable in affine spaces, they are applied, incorrectly, throughout the literature of economics, theory of games, decision theory, and other disciplines. For example, potential energy, which does not have an absolute zero, is an affine — rather than a vector — variable and the sum of two potential energies is undefined. The same holds for "utility" or "value" scales: the sum of "utilities" $u(x) + u(y)$ is undefined not only for different persons but also for a single person using a single fixed scale, a fact that is not recognized in the literature of welfare economics. (The operation of addition is applicable on *differences* of potential energy, time, or position.) Another example of an undefined sum appears in von Neumann and Morgenstern's definition of the characteristic function of a game (von Neumann and Morgenstern 1944, 25:3:c, p. 241) which is an additional error in the definition of this ill-defined function.

Summary

The considerations concerning value scales apply to the construction of trust scales as well and the common problem of formation of groups of agents involves the additional difficulties of group decision making and game theory. Game theory is founded on mathematical errors that do not appear easily solvable and both disciplines are treated incorrectly in the literature. Whether the difficulties that are outlined here can be overcome is an open question. For details see (Barzilai 2010, 2011 and 2012).

References

Barzilai, J. 2012. On Microeconomics Errors and Ordinal Group Decision Making, AAAI Technical Report SS-12-01, pp. 4—7.

Barzilai, J. 2011. On Ordinal, Cardinal, and Expected Utility, Technical Report, Department of Industrial Engineering, Dalhousie University, Halifax, N.S. Canada.

Barzilai, J. 2010. Preference Function Modeling: The Mathematical Foundations of Decision Theory. In *Trends in Multiple Criteria Decision Analysis,* 57—86, Ehrgott M.; Figueira, J.R.; and Greco, S. eds. Springer.

Ferguson, A.; Myers, C.S.; Bartlett, R.J.; Banister, H.; Bartlett, F.C.; Brown, W.; Campbell, N.R.; Craik, K.J.W.; Drever, J.; Guild, J.; Houstoun, R.A.; Irwin, J.O.; Kaye, G.W.C.; Philpott, S.J.F.; Richardson, L.F.; Shaxby, J.H.; Smith, T; Thouless R.H.; and Tucker, W.S. 1940. Quantitative Estimates of Sensory Events. *Advancement of Science* 2:331—349.

Neumann, J. von and Oskar Morgenstern, O. 1944. *Theory of Games and Economic Behavior*. Princeton University Press.