Nothing Is Absolute

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Abstract

This paper argues that approximate knowledge might be better for an AI system; because it is better in dealing with contradictory bases. This argument is based on the hypothesis that categorization is the basic means that we comprehend the world, and this is also the way we abstract specific instances into general rules. However, during abstracting process we might lack some information, which may cause our theories about the world to be incomplete. In this case, if our theories about the world are too certain, we would be unable to predict facts and relations with lower likelihood. And I will demonstrate that if we admit that our knowledge is approximate, despite the incompleteness we will be able to predict facts and relations that are of lower likelihood through Pattern Matching.

Some Philosophical Thinking

How is the world? And how do we know about the world? These problems have puzzled philosophers for thousands of years and may retain no answer. So, let's just simply assume that the world is a big set of Facts and Relations. Facts are what happened on Entities. Relations are connections of facts, which are used as rules by us in reasoning. For example, there is a causal relation between "rain" and "wet ground". When we see wet ground we guess it might have rained; and when we see rain, we guess the ground must be wet. Both of the reasoning processes are based on the connection between "rain" and "wet ground" and the facts. The conclusions are not facts, they are just our guesses. So, in this sense, only facts can be assigned truth values (true or false), while guesses should be assigned possibilities (necessary, possible or impossible). *In other words, facts cannot deny guesses, and guesses cannot deny facts.*

Intuitively, if we are very confident about a relation, and we have a fact as evidence, we may have definite conclusion; if we are not very sure about a relation, even though we have a fact as evidence, we cannot have definite conclusion. These can be represented as set of rules, as bellow. It is hypothesized that these rules are our axioms of reasoning.

Axioms:

$$\Box \mathrm{R}\{\omega' B, \omega'' A\}, \Diamond \omega' B \vDash \Diamond \omega'' A$$
$$\Box \mathrm{R}\{\omega' B, \omega'' A\}, \Box \omega' B \vDash \Box \omega'' A$$
$$\Diamond \mathrm{R}\{\omega' B, \omega'' A\}, \Box \omega' B \vDash \Diamond \omega'' A$$
$$\Diamond R\{\omega' B, \omega'' A\}, \Diamond \omega' B \vDash \Diamond \omega'' A$$

The squares mean "absolute", "true", or "necessary"; the diamond ds mean "relative", "uncertain" or "possible". The capitalized latters stand for facts and guesses. The omegas are labels of different worlds. They are meaningful in reasoning. For example, last night and a night of last month are of different worlds. When we see wet ground in the morning we guess it might have rained last night. But, we wouldn't infer that it rained in a night of last month. We can detect other people's plan, because we know other people's opinions about the world, which might be different from ours. Different opinions won't contradict in our heads, because they are kept in different "worlds".

Facts and entities are our cognitive objects[1]. A basic means to know about them is to classify them into types (Henri Cohen and Claire Lefebvre, 2005). For example, when we see any types of rain, we guess the ground must be wet. When any type of rain happens, we say "it's raining". Conversely, if none type of rains happens, we say "it's not raining", as illustrated by the first and second rule of Definition 1 in bellow. In a well-formed type system, the following rules hold. Here, they are to define type consistency; where Q is a subcategory of P, $\omega\emptyset_{\mathrm{P}}$ means "not known", or "cannot find an instance of P or Q".

Definition 1: Type Consistency

$$\Box \omega \mathrm{P} \Leftrightarrow \Box \exists \mathrm{Q} \omega(\mathrm{Q}) \nLeftrightarrow \Box \forall \mathrm{Q} \omega(\neg \mathrm{Q})$$

[1] Relations also could be our cognitive objects. For example, the relation between "rain" and "wet ground" is also applicable to subcategories of rain, such as shower, downpour, and sleet.

$\square\omega\neg P \Leftrightarrow \square\forall Q\omega(\neg Q) \nLeftrightarrow \square\exists Q\omega(Q)$
$\Diamond\omega P \Leftrightarrow \Diamond\exists Q\omega(Q) \vee \omega\emptyset_P \nLeftrightarrow \square\forall Q\omega(\neg Q)$
$\Diamond\omega\neg P \Leftrightarrow \Diamond\exists Q\omega(\neg Q) \vee \omega\emptyset_P \nLeftrightarrow \square\forall Q\omega(Q)$

$\Leftrightarrow$ means "logically equal"; $\nLeftrightarrow$ means "it cannot be the case"; $\exists$, $\forall$, $\neg$, $\vee$ literally mean "some", "all", "not" and "or" respectively; other symbols are the same as those in the "Axioms" above. As it is shown, Negation as Failure------ a very useful technique in traditional FOLs is not accepted, since we cannot deny something when we don't know anything about it. That's why $\omega\neg P$ cannot be defined through $\neg\omega P$.

Soundness and Completeness

This research has a different angle of seeing reasoning. It takes the real world as "semantics", and reasoning as "syntax"; since it is the real world that is to be known about. So, a theory about the world is sound if it reflects the actual relations among facts; a theory about the world is complete if all the actual relations among facts can be described by such theory.

Intuitively, if our reasoning is sound, we won't have contradictory conclusions. So, unsound reasoning is defined as Definition 2, where Ω^f and Ω^R represent sets of facts and sets of relations respectively, $\omega A^\tau = \omega A \cup \omega A^*$, A^* stands for subcategories of A.

Definition 2:
Unsound reasoning: $R \in \Omega^R$, $\omega' B \in \Omega^f \vDash \omega A^\tau$, and $R, \omega' B \vDash \omega\neg A^\tau$.

Theorem 1:
An inconsistent system may cause unsound reasoning.

***Proof*:**
It is easy to know that if both ωA and $\omega\neg A$ exist in Ω^f, then Ω^f is inconsistent. If Ω^R includes rules that are contradictory to each other, such as $R\{\omega' B, \omega'' A\}$ and $R\{\omega' B, \omega''\neg A\}$, then it is inconsistent. With the type consistency (Definition 1), we can obtain:

$$\begin{cases} \square R\{\omega' B, \omega'' A\}, \Diamond\omega' B \vDash \Diamond\omega'' A \Leftrightarrow \Diamond\exists A^*\omega''(A^*) \\ \square R\{\omega' B, \omega''\neg A\}, \Diamond\omega' B \vDash \Diamond\omega''\neg A \Leftrightarrow \Diamond\exists A^*\omega''(\neg A^*) \end{cases} \quad (1)$$

$$\begin{cases} \square R\{\omega' B, \omega'' A\}, \square\omega' B \vDash \square\omega'' A \Leftrightarrow \square\exists A^*\omega''(A^*) \\ \square R\{\omega' B, \omega''\neg A\}, \square\omega' B \vDash \square\omega''\neg A \nLeftrightarrow \boldsymbol{\square\exists A^*\omega''(A^*)} \end{cases} \quad (2)$$

$$\begin{cases} \Diamond R\{\omega' B, \omega'' A\}, \square\omega' B \vDash \Diamond\omega'' A \Leftrightarrow \Diamond\exists A^*\omega''(A^*) \\ \Diamond R\{\omega' B, \omega''\neg A\}, \square\omega' B \vDash \Diamond\omega''\neg A \Leftrightarrow \Diamond\exists A^*\omega''(\neg A^*) \end{cases} \quad (3)$$

$$\begin{cases} \Diamond R\{\omega' B, \omega'' A\}, \Diamond\omega' B \vDash \Diamond\omega'' A \Leftrightarrow \Diamond\exists A^*\omega''(A^*) \\ \Diamond R\{\omega' B, \omega''\neg A\}, \Diamond\omega' B \vDash \Diamond\omega''\neg A \Leftrightarrow \Diamond\exists A^*\omega''(\neg A^*) \end{cases} \quad (4)$$

Things get a little complicated when approximate-ness is taken into account. Note that mostly cases are okay except when guesses are definite. Apparently, rules in (1) are invalid though the conclusions are acceptable, because it cannot be the case that something definitely causes something to happen and definitely causes not to happen. So, definite contradictory relations cannot be accepted. Even if definite contradictory relations were acceptable, (2) obviously is unsound. Because, it cannot be the case that something did not happen and it did happen.

However, rules of (3) and (4) are okay, despite that they are contradictory. They can explain some situations in the actual world, for example when we try to comprehend some complex things. Sometimes, it seems that it is the case, sometimes it's not. *It should be noted that contradictory rules even relative are still potential factors to cause unsound reasoning.* But, situations are not that bad. Along with the accumulation of experiences, most of our reasoning processes are sound[2].

It is so good to see that such way of reasoning could be sound. A contrary voice might be that what's wrong with accurate knowledge? We can remove those definite contradictory rules, and then we can ensure all reasoning processes are sound. However, I will prove that this way of reasoning is **Incomplete**. For example, given a set of facts $\Omega^f = (\omega' B^*, \omega'' A)$, and a rule $\square R\{\omega' B, \omega''\neg A\}$, it will yield $\square\omega''\neg A$. But, in fact $\omega'' A$. In other words, it cannot predict unexpected relations. How come?

The reason that such way of reasoning is incomplete is that during abstracting specific instances into general rules, we may lose some other information, as shown in Figure 1. I call it insufficient observing.

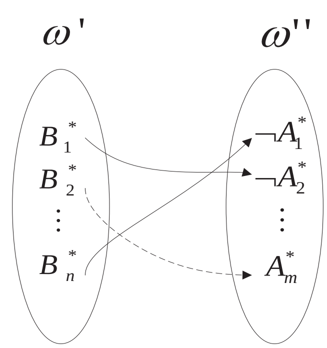

Figure 1. Insufficient observation of complex relations

[2] Even better, we will finally find correct answers with approximate knowledge, because if "A" is not correct, then "not A" might be. This is feasible because, approximate knowledge can provide us more choices.

Proof: $\Box\mathrm{R}\{\omega'(B_1^*),\omega''\neg A\}$ and $\exists B_1^*\omega'(B_1^*) \Leftrightarrow \Box\omega' B$, so $\Box\mathrm{R}\{\omega' B,\omega''\neg A\}$. However, there might be some other cases, such as $|\omega'(B_2^*),\omega'' A|$, that are not known.

A rule is completely correct only when all specific instances are observed, as shown in (5) and (6). R^* and $\mathrm{R}^*\{\neg\}$ are instances of R which can be further interpreted into $\left|\omega' B_i^*,\omega'' A_j^*\right|$, and $\left|\omega' B_i^*,\omega''\neg A_j^*\right|$ respectively, where $1 \le i \le n, 1 \le j \le m$:

$$\Box\mathrm{R} \Leftrightarrow \forall\mathrm{R}^* \not\Leftrightarrow \exists\mathrm{R}^*\{\neg\} \tag{5}$$

$$\Box\mathrm{R}\{\neg\} \Leftrightarrow \forall\mathrm{R}^*\{\neg\} \not\Leftrightarrow \exists\mathrm{R}^* \tag{6}$$

However, due to our sensing and cognition system, nobody can know everything. But, "there will arise situations in which it is necessary to act, to draw some inferences, despite the incompleteness of the knowledge base" (Reiter, 1980).

One choice might be to admit that our knowledge is approximate, as in (7) and (8), so as we won't be shocked at unexpected reasoning results.

$$\Diamond\mathrm{R}\begin{cases}\Leftrightarrow \exists\mathrm{R}^* \not\Leftrightarrow \forall\mathrm{R}^*\{\neg\} \\ \Rightarrow \exists\mathrm{R}^*\{\neg\}\end{cases} \tag{7}$$

$$\Diamond\mathrm{R}\{\neg\}\begin{cases}\Leftrightarrow \exists\mathrm{R}^*\{\neg\} \not\Leftrightarrow \forall\mathrm{R}^* \\ \Rightarrow \exists\mathrm{R}^*\end{cases} \tag{8}$$

An uncertain rule means that there might be some contrary examples. Contrary examples can be obtained either through observing or through guessing, like some new findings that have been predicted by theorists.

Though it still needs further research, I suppose that approximate-ness is the prerequisite for reasoning under incompleteness, and for predicting the relations between facts. In the followings, $|\cdot|$-s are patterns[3]. (10) is obtained through (8).

$$(\omega' B,\omega'' A) \Rightarrow \begin{cases}|\omega' B,\omega'' A| \ \ or \\ |\omega' B|,|\omega'' A|\end{cases} \tag{9}$$

$$\Diamond\mathrm{R}\{\omega' B,\omega''\neg A\} \Rightarrow \begin{cases}\exists R^*\omega R^*(\neg) \Leftrightarrow \left|\omega' B_i^*,\omega''\neg A_j^*\right| \\ \exists R^*\omega R^* \Leftrightarrow \left|\omega' B_i^*,\omega'' A_j^*\right|\end{cases} \tag{10}$$

When we saw two facts happen successively we guess they might be related. To test our hypothesis, we assume either they are related or nonrelated, as shown in (9). As we search our knowledge base, we coincidentally find that there is a rule that might be applicable to the facts. This rule could be interpreted into two sets of patterns, as shown by (10). If the assumed patterns are matched with any one of the patterns in knowledge base, then the assumption is testified. And probably a new rule will be generated. So, by such way of reasoning, it is not surprised to see $(\omega'\mathrm{B},\omega''\mathrm{A})$ being contrary to the rule. But these must be under the admission of approximateness. If we replace (10) with (6) above, then we cannot predict the relation between $\omega' B$ and $\omega'' A$ anymore.

[3] Patterns can be considered as intermediate states before specific instances are generalized into rules.

Discussion

It is quite interesting to ask if it's possible to achieve a goal like $\Diamond\omega'' A$, given a set of rules and a set of facts. The answer is "yes", as long as it does not violate the Type Consistency, and there is a rule that includes an item matches the goal.

Corollary 1: "Nothing is impossible".

Proof: Obviously, $\Box\mathrm{R}\{\omega'\mathrm{B},\omega''\mathrm{A}\},\omega'\emptyset_B \vDash \Diamond\omega''\mathrm{A}$ and $\Diamond\mathrm{R}\{\omega'\mathrm{B},\omega''\mathrm{A}\},\omega'\emptyset_B \vDash \Diamond\omega''\mathrm{A}$.

By this theorem, it's always hopeful to construct a plan, even when facts do not support; because reasoning can be carried on with conjectures. This describes some situations when we rack our brains dealing with some complex problems. We don't know what it is, and we cannot find an instance, but we are so sure that it exists and it shares some properties with its sibling categories.

For example, we may need to open a beer jar, but we don't have an opener. We know that opener is useful because it is hard, shown in (11). So we need to find some replacement which is hard and probably with a curve on the edge (12). And we know things made of metals usually are hard (13). So, we would probably try metal instruments first (14), if they don't work we'd probably try something else made of other materials (15). In the followings, J represents "jar", H represents "hard", M represents "metal", O represents "opener":

$$\Diamond\mathrm{R}_1\{|\omega J,\omega' O|,|\omega' O,\omega' H|\} \tag{11}$$

$$\Diamond\mathrm{R}_1\{|\omega J,\omega' X|,|\omega' X,\omega' H|\},\Box|\omega J,\omega' X| \vDash \Diamond|\omega' X,\omega' H| \tag{12}$$

$$\Diamond\mathrm{R}_2\{|\omega' X,\omega' M|,|\omega' X,\omega' H|\},\Diamond|\omega' X,\omega' H| \vDash \Diamond|\omega' X,\omega' M| \tag{13}$$

$$\Diamond\mathrm{R}_3\{\omega' X,\omega' M\} \Rightarrow |\omega' X,\omega' M^*|,\ \omega'\emptyset_X \vDash \omega' M^* \tag{14}$$

$$\Diamond\mathrm{R}_3\{\omega' X,\omega' M\} \Rightarrow |\omega' X,\omega'\neg M^*|,\omega'\emptyset_X \vDash \omega'\neg M^* \tag{15}$$

The above example shows another merit of approximate knowledge, i.e. when the hypotheses do not work, we can change our strategies. Even better, new strategies had been proposed right at the beginning of the reasoning. Suppose if we replaced the diamond in (14) with square then we wouldn't have (15). So, then if (14) does not work, we would be at our wits' end.

Note that the decision process to replace an opener with something else is beyond the scope of this paper. But I hope I could have an opportunity to talk about it in some other paper. Just a hint, it might be related to intelligence levels and personalities. For some people, finding a replacement is always an unresolvable problem; and for some people, they would never admit that their original methods are wrong even after all tries failed.

Conclusion

This paper argued that approximate knowledge might be better for an AI system; since it is the way we comprehend the world and since we want to build intelligent systems that can replace human intelligence labor as much as possible. I demonstrated that contradictory rules even relative are potential factors to cause unsound reasoning. However, approximate knowledge even sometimes it implies there might be contrary examples, it can provide us more choices which will finally lead us to correct answers. While a consistent system with accurate knowledge will not cause unsound reasoning, but it cannot describe the facts and relations with lower likelihood.

So, it seems that we need to choose between the two cases, i.e. either to reason with likelihood and give up the opportunity to predict what beyond our knowledge, or to reason with accurate knowledge and forego to correct our inefficient behaviors afterwards. Actually, human's wisdom is not to negotiate. On the one hand we try to accumulate knowledge as much as we can so as to promote the soundness of our reasoning; on the other hand we leave rooms for correcting our behaviors.

Considering this, I propose to interpret approximate rules, when necessary, into patterns------intermediate states from specific instances to general rules. Patterns of contrary examples can be obtained either through observing or through guessing. They can provide us alternatives when original strategies don't work. Patterns can make up the incompleteness of our theories because they can provide alternative choices. Patterns cannot avoid unsound reasoning, but the alternative choices are not arbitrary. They are either based on observation or on reasoning. So from another point of view, patterns can promote the soundness of reasoning. But, all of these are feasible only when they are under the admission of approximateness.

References

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