Minimal Change in AGM Revision for Non-Classical Logics

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Abstract

In this paper, we address the problem of applying AGM-style belief revision to non-classical logics. We discuss the idea of minimal change in revision and show that for non-classical logics, some sort of minimality postulate has to be explicitly introduced. We also present two constructions for revision which satisfy the AGM postulates and prove the representation theorems including minimality postulates.

Introduction

Belief revision is a sub-field of knowledge representation that studies the dynamics of epistemic states. Alchourrón, Gärdenfors and Makinson (1985) set the basis of a tradition in this field which became known as the *AGM paradigm* due to the authors initials.

Following the AGM tradition, the *epistemic state* of a rational agent is represented as a set of sentences closed under a consequence operator ($K = Cn(K)$) called *belief set*. Three main operations over belief sets are studied: *expansion*, *revision* and *contraction*. The former is the simplest and is defined as $K + \alpha = Cn(K \cup \{\alpha\})$. We can extend it to deal with expansion by a set of formulas in a natural way by defining $K + A = Cn(K \cup A)$.

Contraction ($-$) is the operation used when one intends to open his mind over some proposition and *revision* ($*$) is the operation used when one intends to consistently incorporate a new piece of knowledge to his belief set. Both are typically defined via sets of rationality postulates. Furthermore, revision can be defined in terms of the two other operations using the following equation, known as the *Levi identity*:

$$K * \alpha = (K - \neg\alpha) + \alpha$$

This identitiy assumes the underlying logic to be closed under negation. However, this is not the case for many interesting logics which are used in practice. In the last decade, there was an effort to adapt the AGM paradigm to non-classical logics of interest in Artificial Intelligence, notably Description Logics (Flouris, Plexousakis, and Antoniou 2005) and Horn logics (Delgrande 2008). The use of the Levi identity in order to define constructions for revision has induced the common practice of concentrating on contraction operators and not revision. It should be noted, however, that on early AI work, the focus was on revision, and not on contraction:

> "To choose their actions, reasoning programs must be able to make assumptions and subsequently revise their beliefs when discoveries contradict these assumptions."(Doyle 1979)

The main goal of this work is to generalize AGM revision for a wide class of logics. Our first attempt in this direction, presented in (Ribeiro and Wassermann 2009; 2010), was applicable only for a certain class of logics, which does not include most Description Logics (DLs) and Horn logic.

After presenting the motivation and formal preliminaries, we investigate the role of minimality postulates for AGM revision in non-classical logics, the main result being that, for non-classical logics, AGM postulates for revision fail to guarantee even the weakest form of minimality. We argue that some minimality postulates such as relevance or core-retainment must be added to the list of rationality postulates for AGM revision in non-classical logics.

Before presenting the main results, we introduce some new postulates and argue that these are the adequate postulates for non-classical AGM belief revision.

Two general approaches for belief revision in non-classical logics that satisfy some minimality postulate are presented. They are general in the sense that they are applicable to a wide class of logics, but also in the sense that they do not take advantage of any singularity of a specific logic. We provide representation results for both approaches.

Finally, we briefly comment on related work, present some conclusions and point towards future work.

Formal Preliminaries

For the purposes of this article, a logic is a pair $\langle \mathscr{L}, Cn \rangle$ such that $\mathscr{L}$ is any enumerable set called *language* whose elements are called *sentences* and $Cn : 2^{\mathscr{L}} \rightarrow 2^{\mathscr{L}}$ is a function called *consequence operator* that is considered to be Tarskian i.e. for every $A, B \in 2^{\mathscr{L}}$ if $A \subseteq B$ then $Cn(A) \subseteq Cn(B)$ (*monotonicity*), $A \subseteq Cn(A)$ (*inclusion*) and $Cn(A) = Cn(Cn(A))$ (*idempotence*).

Throughout this paper, sentences will be denoted by lower case Greek letters (α, β etc), sets of sentences will be denoted by upper case letters (A, B etc) and the letter K will be reserved for *belief sets*.

A logic is called *supra-classical* if it contains every valid inference of Classical Propositional Logic (CPL); it is called *compact* if for every $A \cup \{\alpha\} \in 2^{\mathscr{L}}$ it follows that if $\alpha \in Cn(A)$ then there is a finite subset A_F of A such that $\alpha \in Cn(A_F)$. Most work in belief revision literature assumes the underling logic $\langle \mathscr{L}, Cn \rangle$ to satisfy certain properties: the language $\mathscr{L}$ is supposed to be closed under the standard connectives $\wedge$, $\vee$, $\rightarrow$ and $\neg$; the consequence operator is supposed to be compact, supra-classical and to satisfy the deduction theorem.[1] This set of properties are called the *AGM assumptions*.

Minimal Change in AGM Revision

In this section, we discuss AGM revision and the concept of minimal change.

In their seminal work, Alchourrón, Gärdenfors and Makinson (1985) proposed the following set of postulates for revision:

closure: $K * \alpha = Cn(K * \alpha)$.

success: $\alpha \in K * \alpha$.

inclusion: $K * \alpha \subseteq K + \alpha$

vacuity: If $K + \alpha \neq \mathscr{L}$ then $K + \alpha \subseteq K * \alpha$.

consistency: If $K * \alpha = \mathscr{L}$ then $Cn(\alpha) = \mathscr{L}$.

extensionality: If $Cn(\alpha) = Cn(\beta)$ then $K * \alpha = K * \beta$.

Examining these postulates one may notice the lack of a *minimality postulate* i.e. a postulate that states the principle of informational economy: "one should not give up beyond necessity" (Harman 1986). This principle, certainly desirable for revision, seems to be missing in the AGM paradigm.

Consider the following trivial construction for revision, where in case of inconsistency, one throws away everything but the new input:

$$K * \alpha = \begin{cases} Cn(\alpha) \textit{ if } K + \{\alpha\} = \mathscr{L} \\ K + \alpha \textit{ otherwise} \end{cases}$$

Alchourrón and Makinson (1982) have already argued that the AGM postulates for revision are not enough to prevent this trivial revision. Here, we get back to the original AGM notion of minimal change. In AGM contraction, minimality is given by the *recovery* postulate:

(recovery) $K \subseteq (K - \alpha) + \alpha$

This postulate does not say anything about the size of the resulting set, or that it should be maximal with respect to inclusion. It only states that whatever is given up in a contraction should be recoverable by expanding with the same formula. The AGM postulates for contraction (including recovery) allow for the (full) meet contraction (Alchourrón and Makinson 1982), which by using the Levi identity, yields the trivial revision.

On the context of contraction operations, Makinson and Hansson staged a significant debate on the importance of a minimality postulate. Makinson (Makinson 1987) argued that the postulate of recovery has several odd properties and should be removed from the list of AGM postulates for contraction. He proposed an alternative operation called *withdrawal* that satisfies all AGM postulates for contraction but recovery. Hansson (Hansson 1991), on the other hand, argued that withdrawal suffers from the lack of a minimality postulate. That is, giving up recovery altogether would mean to abandon the desirable principle of informational economy for contraction. Agreeing with the disadvantages of recovery, Hansson proposed to exchange it with some new postulate that stated minimality directly. Two postulates were proposed, relevance and core-retainment, and both were proved to be equivalent to recovery in classical propositional logic. In (Ribeiro et al. 2013) the authors reassembled this controversy and showed that this equivalence does not hold in general (e.g. it fails for Horn logic and most DLs). Hence, in general, minimality of change has to be guaranteed by either relevance or core-retainment, but not recovery. Actually, Flouris (2006) has shown that several logics are not *AGM-compliant*, i.e., they do not admit any contraction operation satisfying recovery together with the other AGM contraction postulates.

In the belief bases literature (Hansson 1991), one can find two postulates for minimality in revision analogous to the ones for contraction. Because of the strong analogy they have the same names as the their contraction counterpart:

(core-retainment) If $\beta \in K \setminus K * \alpha$ then there is $X \subseteq K$ such that $X + \alpha \neq \mathscr{L}$ and $X + \{\alpha, \beta\} = \mathscr{L}$.

(relevance) If $\beta \in K \setminus K * \alpha$ then there is X such that $K \cap (K * \alpha) \subseteq X \subseteq K$ and $X + \alpha \neq \mathscr{L}$, but $X + \{\alpha, \beta\} = \mathscr{L}$.

It is trivial to see that core-retainment follows from relevance, but the converse is not necessarily the case.

Although these minimality postulates can be found in the belief base literature, in AGM revision they are not mentioned. In fact, AGM revision does not state any postulate similar to any of these postulates of minimality explicitly. However, we can show that for logics satisfying the AGM assumptions, both relevance and core-retainment follow from the other AGM postulates.

Theorem 1. *If the underlying logic satisfies the AGM assumptions (i.e. be supra-classical, compact and satisfy the deduction theorem), relevance and core-retainment follow from the AGM postulates for revision.*

Corollary 2. *For logics satisfying the AGM assumptions, relevance does not prevent the trivial revision.*

Since most of the belief revision literature deals with supra-classical logics, formulating relevance explicitly does not bring anything new. However, for many non-classical logics this is not the case:

Theorem 3. *In general neither core-retainment nor relevance follow from the AGM postulates for revision.*

[1] $\beta \in Cn(A \cup \{\alpha\})$ if and only if $\alpha \rightarrow \beta \in Cn(A)$.

Following Hansson (Hansson 1991) and Harmann (Harman 1986) we argue that the principle of informational economy is desirable for a revision.

Surprisingly enough, the proof above shows that adding core-retainment to the AGM postulates blocks the trivial revision for some non-classical logics.

Besides introducing some postulate for minimality, we will introduce two postulates:

(strong inclusion) $K * \alpha \subseteq (K \cap K * \alpha) + \alpha$

(uniformity) If for all $X \subseteq K$, $X + \alpha = \mathscr{L}$ iff $X + \beta = \mathscr{L}$ then $K \cap K * \alpha = K \cap K * \beta$

The former is a stronger version of inclusion related to Harper identity and the latter is for syntactic independence. In the presence of these postulates vacuity, extensionality and inclusion became superfluous. Hence, we suggest the following set of postulates for revision in non-classical logics: *success*, *closure*, *strong inclusion*, *consistency*, *uniformity* and either *relevance* or *core-retainment*. Results from next section corroborate this choice of postulates.

Revision Constructions in Non-classical Logics

Following the AGM tradition, we will present in this section two constructions for revision. The first one is based on remainder sets, maximal subsets of the original belief set that together with the input are inconsistent. The second construction is based on the idea of kernels, minimal subsets of the original belief set that are inconsistent with the input. We will prove representation theorems for both constructions.

Partial Meet Revision

This section presents a construction for revision called *negation free partial meet revision*. It is an adaptation of partial meet revision (Alchourrón, Gärdenfors, and Makinson 1985) for logics that are not closed under negation and it was first introduced in (Ribeiro and Wassermann 2009), but with severe limitation on applicability. It will be reintroduced here together with a new, more general, representation theorem.

A *negation free remainder set* $K \downarrow \alpha$ is defined as the set of all maximal subsets of K that are consistent with the input α.[2] Formally:

Definition 4 (negation free remainder set). $X \in K \downarrow \alpha$ *iff:*

1. $X \subseteq K$
2. $X + \alpha \neq \mathscr{L}$
3. *If* $X \subset X' \subseteq K$ *then* $X' + \alpha = \mathscr{L}$

A *selection function* γ selects a non-empty subset of $K \downarrow \alpha$ if possible i.e. if $K \downarrow \alpha \neq \emptyset$. Otherwise it returns $\{K\}$.

Any selection function induces the following revision operation called *negation free partial meet revision*:

$$K *_\gamma \alpha = \bigcap \gamma(K \downarrow \alpha) + \alpha$$

For any compact logic that satisfies inconsistent explosion (i.e. that contains a formula $\bot$ such that $Cn(\bot) = \mathscr{L}$) this construction is fully characterized by a certain set of postulates.

Representation Theorem 5 (negation free partial meet revision). *For any compact logic that satisfies inconsistent explosion, $*$ is a negation free partial meet revision if and only if it satisfies* closure*,* success*,* strong inclusion*,* consistency*,* relevance *and* uniformity.

Kernel Revision

In the literature of belief base revision, we can find an alternative approach for constructing revision operation based on what is called kernels. Instead of finding the maximal subsets of the belief set that are consistent with the input, the kernel approach depends on finding the minimal subsets that are inconsistent with the input.

Formally, the *negation free kernel* $K \Downarrow \alpha$ is defined as follows:

Definition 6 (negation free kernel). $X \in K \Downarrow \alpha$ *iff:*

1. $X \subseteq K$.
2. $X + \alpha = \mathscr{L}$.
3. *If* $X' \subset X$ *then* $X' + \alpha \notin \mathscr{L}$.

Each element of $K \Downarrow \alpha$ will be called here an α*-kernel*. An *incision function* σ is a function that selects at least one element of each non-empty α-kernel. Formally it is any function such that $\sigma(K \Downarrow \alpha) \subseteq \bigcup K \Downarrow \alpha$ and if $\emptyset \neq X \in K \Downarrow \alpha$ then $X \cap \sigma(K \Downarrow \alpha) \neq \emptyset$.

Any incision function induces the construction of a revision operation:

$$K *_\sigma \alpha = (K \setminus \sigma(K \Downarrow \alpha)) + \alpha$$

In words, the incision function selects at least one element of each α-kernel in $K \Downarrow \alpha$. The elements selected by this function are removed from K and the result is expanded by α. This construction is the revision induced by σ.

One of the main differences between this and the previous representation theorem is the underlying minimality postulate. While relevance is the minimality postulate associated with negation free partial meet revision, core-retainment is the minimality postulate associated with negation free kernel revision.

Representation Theorem 7 (negation free kernel revision). *For any compact logic that satisfies inconsistent explosion, $*$ is a negation free kernel revision if and only if it satisfies* closure*,* success*,* strong inclusion*,* consistency*,* core-retainment *and* uniformity.

Related Work

There have been several proposals in the literature to adapt the AGM framework to non-classical logics, almost all of them dealing with very specific logics. Most of the recent work was concentrated around DLs and Horn Logics.

[2] This was called an *inconsistency-based remainder set* in (Delgrande 2008).

Delgrande and Peppas (2011) have proposed a construction for model-based revision of Horn theories and proved that it can be characterized by the eight AGM postulates (they include the supplementary postulates) together with an extra postulate for acyclicity of derivations, needed in the Horn case. It is interesting to note that they do not propose any minimality postulate. Zhuang, Pagnucco and Zhang (2013) show that they can obtain a revision characterized by the same postulates using a variation of the Levi identity and an operation for contraction of Horn theories. Their construction uses the idea of Horn strengthening and is thus specific for Horn Logics.

Qi, Liu and Bell (2006) have worked on revision operators for DLs based on the idea of weakening axioms to avoid inconsistencies. Again, this is a very specific method, tailored for specific logics. Qi et al. (2008) have proposed a kernel construction for revision in DLs, but they work with bases and not theories. Their operation satisfies a variant of core-retainment. There is, however, no proof that the postulates fully characterize the operation. More recently, Wang, Wang, and Topor (2010) have also proposed constructions for revision which satisfy most of the AGM postulates, but they focus on a very restricted logic of the DL-Lite family.

Conclusion

Traditional constructions for revision are based on contraction and the Levi identity. These constructions are not useful for logics that are not closed under negation. In this paper, we have shown two alternative constructions that do not depend on negation: negation free kernel revision (NFK) and negation free partial meet revision (NFPM).

Each of these constructions is associated to a certain minimality postulate: core-retainment and relevance respectively. For classical logics, both postulates follow from the other AGM revision postulates and, hence, are typically omitted. In general, however, they must be explicitly introduced.

Besides the postulates for minimality, we introduced two other postulates: uniformity (for syntactic independence) and a stronger version of inclusion.

NFPM revision had been already considered in a previous paper, but the representation theorem for NFPM proved in (Ribeiro and Wassermann 2009) depended on the logic being distributive. The version of the representation theorem for NFPM in this paper, on the other hand, is applicable to a wide class of logics, namely, any compact logic that satisfies inconsistent explosion. This is a much wider class of logics that includes, for example, Horn Logic and most DLs.

Besides fixing the representation theorem for NFPM, a construction based on kernel revision (NFK revision) was presented and the representation theorem was proved for the same class of logics.

Differently from related work, our approach solves the revision problem in non-classical logic from a general perspective. Our constructions and characterizations are applicable to a wide class of logics and do not rely on particularities of any specific one.

Future work includes a deeper study of the relation between contraction and revision in non-classical logics, as well as applying our results to Horn and DLs and comparing them with the specific proposals found in the literature for these logics.

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