To Cognize Is to Categorize Revisited: Category Theory Is Where Mathematics Meets Biology

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Abstract
This paper claims for a shift towards "the formal sciences" in the cognitive sciences. In order to explain the phenomenon of cognition, including aspects such as learning and intelligence, it is necessary to explore the concepts and methodologies offered by the formal sciences. In particular, category theory is proposed as the most fitting tool for the building of an unified theory of cognition.

This paper proposes a radically new view based in category theory. A cognitive model is informally defined as a mapping between two different structures, while a structure is the set of components of a system and their relationships.

Put formally in categorical terms, a model is a functor between categories that reflects the structural invariance between them.

In the paper, the theory of categories is presented as the best possible framework to deal with complex system modeling -i.e: biologically inspired systems that transcend and offer a much more powerful tool kit to deal with the phenomenon of cognition that other purely verbal tools like the psychological categories that Rosch or Harnad refer.

Overcoming the structure-function approach
Cognition extracts from the environment features on the basis of which a system reacts. Based upon this definition, the terms environment and feature are key and must be explored in more detail. A theory is needed for the measurement of the neural activity, which is able to identify the ecologically relevant stimuli and to interpret the neural activity, as independent as possible from the experimenter's suppositions and beliefs.

As Herbert Simon said, there is no principled way to see the brain and environment as separate systems, the brain and the environment form a nearly-decomposable system.

According to Duncan Luce (Luce 1995), as honest scientists, as far as we disentangle the structure of the physical objects we are studying, we must to begin to describe it in formal terms.

The formalisation of the structure of a system is not just an issue of honesty, but a requirement to be fulfilled in order to attain the character of a system -i.e: a pattern, conducting a behavior, defined in mathematical terms.

A mathematical toolkit
Admittedly, mathematical formalisms do not tell anything per se about the way physical systems behave. Rather, a mathematical theory of natural phenomena is, in the best case, a bunch of definitions, axioms and theorems that for some reason, happen to provide an appropriate framework able to describe, predict and explain better than any other conceptual framework, the way in which nature does behave.

We would like to define in the theory of categories, some key terms that are relevant in order to build such explanatory framework for cognition. (Pierce 1991)

Definition: category consists of the following:

i. a collection of objects A, B, C . . .
ii. a collection of arrows (or morphisms) f,g,h . . .
Cognitive function

Functor F

Cat-neuron C

Functor G

Cat-neuron B

Cat-neuron A

Figure 1: Cat-neurons are categories whose objects are neurons and their arrows synapses. The functors are structure-preserving maps between the categories. In this description, the reverse inference problem consists of determining the category of functors and natural transformations

Category theory brings a mathematical toolkit able to express with rigor, concepts such as the multi-realizability of one function by different structures, or the reducibility of one complex structure in some simpler pattern.

Structure is not just a 3-dimensional physical structure, this is a layman’s view of structure. Using the resource of mathematics, we can go deeper into this concept. Indeed, symmetry and continuity are structures that can be realised in both physical and non-physical systems.

It goes without saying that one system has many possible structures, the structure however, is contingent upon the behavior of the system that is considered.

As any partition of a system could be its structure, it is necessary to make a distinction between relevant and irrelevant structures(for some observed behavior). The relevant structures are those that are preserved under transformation. Consequently, the structure must reflect the invariance through transformations, which is embodied in the functor.

Figure 1

Conclusions

The time has come to set the agenda for a "hard cognitive science". To that end, the authors propose to translate into mathematical terms key concepts, that until now, have been used loosely. Category theory is presented as a suitable framework for complex system modeling, and as a sophisticated toolkit for mental theories.

Acknowledgements

We acknowledge the support of the European Commission through grant ICEA: Integrating Cognition, Emotion and Autonomy

References


