# Estimating the Impact of Public and Private Strategies for Controlling an Epidemic: A Multi-Agent Approach

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**Abstract**

This paper describes a novel approach based on a combination of techniques in AI, parallel computing, and network science to address an important problem in social sciences and public health: planning and responding in the event of epidemics. Spread of infectious disease is an important societal problem – human behavior, social networks, and the civil infrastructures all play a crucial role in initiating and controlling such epidemic processes. We specifically consider the economic and social effects of realistic interventions proposed and adopted by public health officials and behavioral changes of private citizens in the event of a "flu-like" epidemic. Our results provide new insights for developing robust public policies that can prove useful for epidemic planning.

## Introduction and Contributions

The threat of global disease outbreak such as pandemic Influenza is an important public health problem facing the world. It is estimated that a pandemic influenza of the 1918 size that killed 40 million people would today result in 150 million deaths and estimated $4.4 trillion in global economic output (Dobriansky 2006). In order to appropriately plan and respond to such pandemics, public health officials need to have a systematic assessment of the socio-economic impact of interventions and other mitigation efforts (Philipson 2000).

It is well known that people adapt their behavior in response to a threat posed by a potential epidemic, but a systematic study of how changes in individual behavior are likely to affect the eventual spread of the disease has not been undertaken to date. Our previous work shows that certain behavioral changes that might be viewed as beneficial have potentially harmful side effects in case of infectious disease outbreaks (Atkins et al. 2008). Personal behavior during an epidemic depends on people's socio-economic status as well as their perception of the epidemic in the community. People maximize their well being by choosing levels of prevention and strategies with respect to their own constraints.



In this paper, using a combination of tools from AI, network science, parallel computing and social sciences, we study the economic and social effect of various interventions adopted by the public health officials and behavioral adaptations of private citizens in the event of a "flu-like" epidemic. We investigate the fairness of different intervention strategies by examining its economic impact within specific demographic classes. The results identify population strata by demographics that are likely to win or lose from the implementation of these kinds of policies. A number of recent reports and Hurricane Katrina have underscored the importance of this kind of work (Personick and Patterson 2007).

The use of agent-based models is crucial here; such controlled experiments are virtually impossible to carry out in real life. Agent based models provide a realistic yet synthetic environment to carry out analysis of detailed what-if scenarios. Such an analysis is crucial for informing public policies.

We illustrate our ideas by studying the economic impacts of various public policies and individual behavior adaptation using a simulated outbreak of flu like disease in the New River Valley (NRV) region of Southwest Virginia, NRV contains about 150,000 people. We estimate the cost of preventive behavior aimed at limiting the occurrence of the disease. To understand the full impact of a disease, it is important to calculate not only the cost of the disease but also the cost of disease avoidance. Economic analysis has the power of separating the impact of public mitigation policies on health from the private preventive efforts. Traditional epidemiological models focus on the cost of the disease, the prevalence of the disease, and assume a big role of the public sector in controlling the infectious diseases. They *simply do not consider* the price paid by private citizens as they try and avoid getting infected. This represents a significant portion of the disease burden to the society. Individuals possess strong private incentives to avoid the disease. These incentives and the resulting dynamic agent modifications to personal social behavior is clearly apparent in case of infectious diseases. Our models try to capture this modified behavior and use it to calculate the cost of the disease avoidance (Philipson and Posner 1993).

**Related Work:** Earlier studies have emphasized the need to measure the comprehensive economic impact by incorporating both direct and indirect economic costs of interven-

tions; and the private and public cost of disease prevention. Indirect costs include costs incurred due to the sickness of dependent family members e.g. the cost of staying home for a working parent when a child is sick (Philipson 2000; Epstein et al. 2008). It also includes preventive and avoidance costs which are incurred by the private individuals who change their behavior e.g. get vaccinated, stay home from work etc. and by public interventions such as school closures. This study uses an individual based simulation system with detailed information on the individuals and their household structures to estimate the socio-economic impact of various private and public intervention strategies on the population by demographic strata.

Aggregate computational epidemiology models often assume that the population is partitioned into a few subpopulations (e.g. by age) with a regular interaction structure within and between sub-populations. The resulting, uniform mixing model can typically be expressed as a set of coupled ordinary differential equations. Such models focus on estimating the number of infected individuals as a function of time, and have been useful in understanding population-wide interventions. For example, they can be used to determine the level of immunization required to create herd immunity. See (Barrett, Smith, and Eubank 2005; Hethcote 2000) for more discussion on this class of models.

Disaggregate or individual-based models such as the one used here, in contrast, represent each interaction between individuals and can thus be used to study critical pathways of the diseases. Disaggregate models require neither partitions of the population nor assumptions about large scale regularity of interactions; instead, they require detailed estimates of transmissibility between individuals. For more than a few individuals, the state space of possible configurations of the dynamical system is so large that they are best studied using computer simulation. See (Eubank et al. 2004; Ferguson et al. 2006) for recent results and discussion on this topic.

## Simdemics: a Multi-Agent Simulation-Based Approach

We use a novel agent based modeling framework called **Simdemics** (Barrett et al. 2008a) for simulating epidemic outbreaks in large urban regions. **Simdemics** belongs to a new emerging class of models called network based epidemiological models that use a detailed representation of social contact networks; such a representation is crucial for studying the questions related to role of individual behavior and public policies. The computational model can be used to study the effect of public policies and individual behavior on the dynamics of infectious diseases. Policy planning has been one of the central focus of epidemiological research over the years. The recent SARS epidemic served as an excellent example of how individual behavior as well as public policies played an important role in changing the social network.

The mathematical model underlying **Simdemics** consists of two parts: (i) a co-evolving graphical discrete dynamical system (CGDDS) framework that captures the co-evolution of disease dynamics, social network and individual behavior, and (ii) a partially observable Markov decision process that captures various control and optimization problems formulated on the phase space of this dynamical system. CGDDS provides a novel mathematical formalism for the underlying agent-based model, which captures the interaction between policies, social contact network and disease dynamics.

**Simdemics** details the demographic and geographic distributions of disease and provides decision makers with information about (1) the consequences of a biological attack or natural outbreak, (2) the resulting demand for health services, and (3) the feasibility and effectiveness of response options (Eubank et al. 2004). The overall approach followed by disaggregate models consists of the following steps: **Step 1.** Creating a set of (synthetic interactors), **Step 2.** Generating (time varying) basic interaction networks, **Step 3.** Detailed simulation of the epidemic process, **Step 4.** Simulating the coevolution of disease dynamics, behavioral adaptation and social contact networks. We describe these steps below; more detailed discussion about **Simdemics** can be found in (Barrett, Smith, and Eubank 2005; Eubank et al. 2004).

**Step 1: Creation of Synthetic Interactors**: This step creates a synthetic population by integrating a variety of databases from commercial and public sources into a common architecture for data exchange. The process preserves the confidentiality of the original data sets, yet produces realistic attributes and demographics for the synthetic individuals. The synthetic population is a set of synthetic people and households, located geographically, each associated with a set of demographic variables drawn from the census. It is a collection of synthetic objects, each associated with a set of attributes. Synthetic populations are thus statistically indistinguishable from the census data (Beckman, Baggerly, and McKay 1996). Since they are synthetic, the privacy of individuals within the population is protected.

**Step 2: Generate Time Varying Interaction Network**: This step creates a set of activity templates for households are determined, based on several thousand responses to an activity or time-use survey. The modeling methodology is called *activity based travel demand models* and is now accepted as the *de facto* standard in transportation science, see (Bowman and Ben-Akiva 2001) for recent overviews. Our early work in this area (Barrett et al. 2001; Beckman, Baggerly, and McKay 1996) played an important role in the development of this methodology.

The social contact network from the above population is constructed as follows. We have a *labeled dynamic bipartite graph* $G_{PL}$, where $P$ is the set of people and $L$ is the set of locations. If a person $p \in P$ visits a location $\ell \in L$, there is an edge $(p, \ell, label) \in E(G_{PL})$ between them, where $label$ is a record of the type of activity of the visit and its start and end times. Each vertex (person and location) can also have labels. The person labels correspond to demographic attributes such as age, income, etc. The labels attached to locations specify the location's attributes such as its $x$ and $y$ coordinates, maximum capacity, etc. Note that, there can be multiple edges between a person and a location record-

ing different visits. This produces synthetic individuals that just like real individuals can now carry out other activities like eating, socializing, shopping, etc. An important point to note here is that such data is impossible to collect by measurements or surveys (Barrett et al. 2001).

**Step 3: Epidemic Simulation.** This step consists of developing a computational model for representing the disease within individuals and its transmission between them. The model can be viewed as a *networked probabilistic timed finite state machine*. Each individual is associated with a timed probabilistic finite state machine. Furthermore, the automata are connected to other automata – this coupling is derived from the social contact network. The state transition is probabilistic and is timed (i.e. depends on the duration of contact). It may also depend on the attributes of the people involved (age, profession, health status, etc.) as well as the type of contact (intimate, casual, etc.) states of individual automata as they update their states in responses to changes in internal state and state of its neighbors.

**Step 4: Simulating Effect of Policies and Behavioral Adaptations.** The final step consists of representing and analyzing various public policies and interventions using partially observable Markov decision process (POMDP). It allows us to capture sequential decision making process related to studying the efficacy of various interventions. The POMDP is exponentially larger than the problem specification and is intractable to solve optimally in general. We thus resort to efficient simulations. A key concept is that of *implementable policies* — policies or interventions that are implementable in the real world.

## Model Validation

Extensive efforts have been made to validate the overall approach and specific components of the model. This includes structural validity of models, matching the data produced to field data, and formal specifications of these models for software verification (Beckman, Baggerly, and McKay 1996; Barrett et al. 2001; Halloran et al. 2008; Eubank et al. 2004). Results on population mobility and social network construction were presented and reviewed annually at (TRBC 1995 2003).Epidemiological simulations were also reviewed and discussed as a part of a letter report by the National Academies and published in (Halloran et al. 2008). **Simdemics** has been used in more than half a dozen user defined case studies; these case studies have further improved and served to validate the various models. The economic analysis reported in this paper is new and was motivated by our recent work as a part of a DHHS requested analysis undertaken by the NIH sponsored MIDAS program. The study illustrates what we believe is the first use of high resolution simulation based microeconomic analysis in the context of public health epidemiology.

## AI Technologies

Infectious disease is an important societal problem and computational public health epidemiology can benefit from advances in several AI topical areas. The development of **Simdemics** uses a number of methods and technologies developed in AI in conjunction with methods in high performance computing and network science. The combination of ideas from various disciplines that formed the basis of **Simdemics** is an innovative aspect of the work. Examples of such technologies include: (i) high resolution multi-agent models, (ii) synthesis and analysis of large urban social and relational networks, (iii) light weight yet realistic behavioral modeling and (iv) development of a theory of graphical dynamical systems and games. We give an example to illustrate these ideas.

Developing *scalable* multi-agent systems for studying real world problem remains an active area of research. We first note that the notion of agency as used here should be distinguished from the notion of *entity* and *actor*. Agents are actors that have intent/motive and thus require a behavioral representation that on one hand is rich enough for the problem at hand and lightweight so that it can be scaled. We needed three new ideas to achieve this: (i) parametric representation of individual behaviors and local actions wherein, a single basic algorithm is used for each agent and the behavioral variation is obtained by randomization and agent specific attributes, (ii) behavioral decomposition: using automata theoretic techniques to represent each kind of local function associated with an agent by a separate automata (algorithm) and then using generalized cross product like construction to obtain a composite behavior, and (iii) unencapsulated agents: where in the notion of agency is distributed and does not reside within a single software object. For example, we have one within host disease progression model (represented using probabilistic timed transition system); the specific manifestation of disease within host is a function of demographic variable associated with the individual and affect the state transition. Similarly behavioral models that are used for individual decision making in the event of epidemics are parameterized. An individual's overall representation comprises of within host disease model and the individualized behavioral model. The description of the agent, or what an agent does is not confined within these local functions. Its interaction with other agents defines its overall behavioral description. In this sense the idea bears certain resemblance to the notion of (non)-modularity of functions in cognitive science (Fodor 1983) and in neuroscience under the concept of population coding (McIlwain 2001). So while local functions and the state attributes associated with agent determine its local dynamical evolution, the phase space encodes the system behavior and is necessary to understand the behavior of an individual agent.

In the course of developing **Simdemics**, we identified a number of potential ways in which computational epidemiology further motivates research in AI. This includes: (i) blending models of individual behavior in social sciences with cognitive theories for more realistic and yet light weight behavioral representations, (ii) further research in relational networks and machine learning for refining the social contact networks; the availability of new sensor data has raised the possibility of refining these social contact networks, at least in special circumstances (such as an office or school environment) and (iii) further research into theoretical aspects of graphical games and dynamical systems; it is

clear that we need to exploit the special structure of social contact networks to compute important invariants in such systems that shed light on the outcome of the collective individual behaviors; e.g. computing the fixed points in such large $n$-way minority games (Bauch and Earn 2004).

## Methodology

### Demographic Classes and Intervention Strategies

This study estimates the differences in economic impact on demographic classes caused by various public antiviral distribution and social distancing strategies as well as private behavioral strategies. Demographic classes were formed by dividing the synthetic population of the New River Valley region of Virginia into twenty-seven classes based on the household income, family size, and age. Classifications of the family's wealth were formed by defining three segments of families who make less than $25,000, $25,000-$75,000, and more than $75,000 per year. Household size classes were defined as those containing a single-member, two to three members, and more than three members. Age groups were defined as juveniles (0-18), working adults (19-64), and retirees (65+). Demographic classes were developed from all twenty-seven possible combinations of each factor. A single digit label is assigned to each of the three aspects of the classification. Table 1 displays all the categories as well as the proportion of people belonging to each class. Strategies include individual and public interventions that aim to reduce personal and public risk respectively. The strategies incorporated in this study are supported by public health officials and researchers (Blendon et al. 2008).

**Individual Strategies** People with different socio-economic constraints follow different preventive strategies to avoid getting infected. These strategies are based on how people perceive the society is doing as well as the kinds of actions their own peer group/demographic class is taking. We project that the change in individual behavior is triggered by the prevalence level of the virus in the overall society as well as within one's own demographic class. Thresholds for these two factors were set for each class as shown in Table 2. Once the number of infected people in the population or in a person's class reaches the personal threshold value, the individual decides to modify its behavior. For the affluent household members, the modified behavior is reflected through the purchase of antiviral medication kits available for private use. The members of the middle income class decide to modify their daily activity schedule in order to reduce their potential contact with infected individuals. This social distancing technique eliminated visits for unnecessary shopping trips and recreational activities. Finally the individuals from the poorest income class find it too expensive to purchase antiviral kits, or reduce contacts and hence decide to just rely on the herd immunity. Table 3 tabulates the private strategies implemented by the different income classes and the measurements used for global and local thresholds.

**Public Strategies** The strategies available to the public health officials include the distribution of antiviral kits and school closures. The trigger threshold for the public intervention is set at 1% of the total population becoming infected. The public stockpile of AV kits is limited to 10,000.[1] These kits are distributed to the individuals based on the following four selection techniques; randomly selected individuals, poorest individuals, first sick individuals, and the most vulnerable individuals. *Vulnerability of an individual is defined as the probability with the individual gets infected when the disease starts from a random person in the population* (Barrett et al. 2008b). We empirically estimate the vulnerability of all individuals in the population by running hundred simulation runs of the epidemic where each run starts from a different random individual. To calculate the vulnerability, we determine the number of times an individual gets sick and divide it by the total number of runs. For example, if an individual got infected 5 times during the hundred runs, his vulnerability was 0.05. Under the most vulnerable individual strategy, the AV kits were distributed to those who had the highest vulnerability value as calculated by the above procedure.

| Num | HHInc(frac) | HHSize(frac) | Age(frac) |
|---|---|---|---|
| 0 | 0-25K (0.32) | 1 (0.11) | 0-18 (0.20) |
| 1 | 25-75K (0.52) | 2-3 (0.54) | 19-64 (0.69) |
| 2 | 75K+ (0.16) | 4+ (0.35) | 65+ (0.11) |

Table 1: Demographic classes based on age, household income and size with an assigned number label. Numbers in the parenthesis show the fraction of population in each class.

For the "close school" strategy, the trigger threshold is set at 1% of the total population becoming infected. Each time the "close school" strategy is used, the schools are closed for a period of two weeks. We try various combination strategies that involve closing schools as well as the distribution of antiviral kits according to the four strategies mentioned above. The combination of AV distribution strategies and closing schools led to nine distinct governmental strategies.

**Strategy Label Description** We now describe the strategy labels that are being used in all the upcoming tables. The strategies described below are followed by the public health officials while distributing the public stockpile of AVs. In addition to distributed AV kits, the public authorities close down schools based on some trigger thresholds. Under all these scenarios, the private citizens continue to follow their respective strategies whenever their local and global thresholds are met. In the base case no interventions or strategies are used.

In the case of *Random* strategy the entire public stockpile of AVs is distributed randomly by the government to the people. Under the *poor* strategy, the public stockpile is distributed to the poorest individuals. Under the *Vulnerable* strategy, the entire stock of AVs is distributed to the most

[1]To test the sensitivity of the number of AV kits in the public stockpile, we repeated the entire experiment in which the public AV kits stockpile was limited to 1000. The results of that experiment were not statistically significantly different from the results of 10000 kits experiment.

| Classes | Global Threshold (% sick in the society) | Local Threshold (% sick in class) |
|---|---|---|
| 100 | 5 | None |
| 101 | 5 | 2 |
| 102 | 2 | 1 |
| 110 | 5 | None |
| 111 | 5 | 4 |
| 112 | 3 | 1 |
| 120 | 5 | None |
| 121 | 5 | 4 |
| 122 | 3 | 1 |
| 200 | .5 | None |
| 201 | 2 | 5 |
| 202 | 1 | 2 |
| 210 | .5 | None |
| 211 | 2 | 3 |
| 212 | 1.5 | 3 |
| 220 | .5 | None |
| 221 | 2 | 1 |
| 222 | 1.5 | 2 |
| Public | 1 | None |

Table 2: Demographic class labels and respective thresholds to trigger private intervention strategies. Note that the low income class (0−−) does not take any action and hopes to benefit from the herd immunity.

| Class | Individual Strategy by Income Class |
|---|---|
| LowInc | Herd immunity/No Action |
| MedInc | Stop non-essential activities |
| HighInc | Buy AV kits |
| **Threshold** | **Measurement** |
| Global | Total sick in the society |
| Local | People not reporting to work in class |

Table 3: Private intervention strategies by income class

vulnerable people in the society. Under the *sick* strategy, the AVs are distributed to the first sick individuals in the society. Strategy CS closes schools for 14 consecutive days. Under CS, no AVs are distributed. However, CS+R refers to closing schools as well as randomly distributing the public stockpile of AVs. Similarly, CS+P refers to closing schools plus giving AVs to the poorest people. CS+V refers to closing schools plus giving AVs to the most vulnerable people. CS+S is for closing schools as well as giving AVs to the first sick people. *NoGovt* strategy implies that there is no intervention by the government. No AVs are distributed and schools remain open. However, under all the above scenarios the private strategies are being implemented.

## Scenarios and Simulation

In order to assess the economic impact of various intervention strategies by demographic class, we develop eleven distinctive scenarios based on individual and governmental actions. Nine of these scenarios come from the public interventions. A base case or control simulations are conducted to determine the size of the epidemic in the absence of any interventions. Finally, a scenario is developed where no government intervention takes place and only private strategies are implemented.

For each epidemic run, five vulnerable individuals were used as the index cases or the set of initially infected individuals. One hundred simulations were conducted to ascertain the average spread of an epidemic for each scenario. In each simulation, the number of infected members of each demographic class was tracked for each day in the simulation. It was also assumed that infected individuals would not go to work or school. This resulted in a schedule change for infected workers, infected children, and the working parent of an infected child.

### Simulation Results

The data produced by the simulation provides some insight into the average epidemic under each scenario. The shape of the disease prevalence for the average epidemic under each scenario can be directly compared to each of the other scenario's results. The potential effects of an intervention strategy on the diffusion of the virus can thus be established. In all of the scenarios, any intervention by a government strategy or an individual's course of action greatly reduced the number of individuals infected during the course of the epidemic. Even the least effective intervention diminished the total size of the epidemic to less than half of the base epidemic. The prevalence of the disease at its height was reduced by two thirds. It is important to note that the interventions not only caused the peaks to drop significantly, but also delayed the outbreak and reduced the duration of the epidemic.

### Impact of Preventive Behavior

The results show that the most significant reduction in the size of the epidemic was caused by the preventive behavior undertaken by the individuals. These actions were driven by agent decisions to modify personal behaviors and activities within the social network by observing the simulation environment. This is indicated as the 'NoGovt' strategy performs only slightly worse than the random, poor, vulnerable and sick strategies. In all of these strategies, individuals modified their behavior once alerted that a number of peers in their demographic class have been infected. However, in the NoGovt strategy there was no distribution of AV kits whereas random, poor, vulnerable and sick strategies used AV kits provided by the government. This implies that the private behavior modification was the main factor in causing the reduction in the size of the epidemic in all these strategies. The second best reduction in the size of the epidemic came from the government strategy of closing schools which resulted in a drop in attack rate by another 10%. Note that attack rate is defined as the size of the epidemic divided by the size of the population. Under the CS (close schools) strategy, the attack rate was 15%, i.e. more than 10% lower than the attack rate of 26% under the NoGovt strategy. Among the public AV distribution strategies, the distribution of the AV kits to the most vulnerable individuals proved to be the most effective. The governmental

strategy of closing schools and distributing medications to the first sick individuals reached its peak prevalence much faster than the others, but the number of infected individuals also tailed off earlier compared to the strategies without school closures.

## Conclusions

This study uses an individual based modeling approach to obtain a detailed understanding of the economic and social impact of various mitigation strategies for a "flu-like" epidemic. The results show that the most important factor responsible for preventing income loss is the modification of individual behavior which alone drops the total income loss by 62% compared to the base case. The next most significant strategy is the closure of schools which further reduced the total income loss by 40% and the size of the epidemic by half. Compared to the base case, the school closure strategy helped drop the income loss due to care-taking by 93% and due to illness by 75%. This strategy works well across all demographic classes, but it is especially favorable to children and large families for whom the total costs could go down by more than 80%.

The best and most effective distribution strategy requires school closures, public distribution of AVs to the most vulnerable individuals in the society, and behavior modification by the private citizens. This strategy drops the attack rate by 87% and income loss by 82% compared to the base case. The cost of disease avoidance, as measured by the economic loss run close of $20 million whenever schools need to be closed for 2 weeks. In summary, both private citizens and public officials play an important role in successfully mitigating the disease.

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