Abstract
In this paper, economic principles and the paradigm of a game are used to create a signal control strategy. The game structure is not formal (as in game theory), but the idea of a game is used nonetheless. That is, instead of using the standard techniques of minimum greens, maximum greens, and gaps to control the signal indications, an economically based game structure is employed. The intersection’s space is viewed as a scarce commodity whose use is determined through a bidding process. Movement Managers manage the vehicle departures for specific turning movements. Arriving motorists pay the Movement Managers an initial fee, and make voluntary contributions as they perceive necessary to arrange times of entry for them. Movement Managers submit bids for use of the intersection’s space and the highest bidders win. Distributed processing and connected vehicle technology are seen as the mechanisms by which implementation would be feasible. The value in such an idea is that one can study and reach an understanding of the economics that underlie effective traffic control.

Introduction
In principle, economics plays an instrumental role in transportation decision making. The idea of using a market based approach to alleviate congestion in transportation networks was first suggested by (Pigou 1924). Later, (Beckmann, McGuire, and Winston 1956) suggested imposing tolls to minimize total network-wide travel times. However, only in recent years have researchers suggested adapting market-based ideas to traffic signal control. (Schepperle, Bohm, and Forster 2007) proposed an auction-based policy for intersection control. Here, the intersection control agent starts an auction for the earliest departure time slot among the vehicles that are approaching the intersection on each lane. Only the lead vehicle in each queue is allowed to participate in the auction.

(Vasirani and Ossowski 2011) (Vasirani and Ossowski 2012) propose a multiagent approach to design a competitive computational market for the distributed allocation of an urban road network. Specifically, they propose an intersection manager driver model in conjunction with cooperative learning techniques to coordinate the prices of individual intersections. This framework uses a Walrasian auction system for selling intersection schedule slots. Basically a Walrasian auction involves a set of buyers and a set of suppliers. At a given time, each buyer in the set of buyers notifies the suppliers of the quantity of resources he/she is going to buy at a preset published price. The Intersection manager in turn uses this information to compute total demand and excess demand. Using this information, intersection reservation prices are updated (reserve prices are upward adjusted in case of excess demand and downward adjusted in case of excess supply). Drivers choose routes based on their own preferences between time and cost, participating in intersection auctions as long as they are willing to meet the reserve price. So, in that sense, the intersections that drivers pass through are a function of their willingness to pay the preset price for using those intersections. Similarly, (Carlino, Boyles, and Stone 2013), propose a framework in which each driver in the queue bids for the lead vehicle in the queue to be discharged; the auction format is that of a second price, sealed bid auction. The winners will split the second-price cost proportionally to what they originally bid. The method for computing these proportional payments was originally suggested by (Clarke 1971) (Groves 1973).

In this paper, we consider a variant of this market-based model to intersection control. Our model has two key distinctions. First, traffic signal control is formulated as a shared decision process, where the drivers that pass through a given intersection collectively determine the behavior of the traffic signal. Rather than reacting to prices that are dynamically set by the intersection control system as a function of overall demand and supply as in (Vasirani and Ossowski 2012) drivers instead contribute to their queue’s bids as a function of their travel goals (e.g., whether they are in a hurry or not) without the presence of a target price. A nominal fee is imposed on all drivers (to cover marginal costs of operating the signal) and then drivers can individually make decisions about voluntary contributions to help expedite their travel times. As one consequence, drivers have freedom of route choice and are not excluded (or priced out) of taking specific routes. A second distinguishing characteristic of our model is that it incorporates the constraints imposed by traditional signal control theory and practice (i.e., use of minimum green times, temporal gaps required for detecting vehicles, yellow and all-red clearance intervals). The models of (Vasirani and Ossowski 2012) and others, in con-
trast, start from the premise that vehicles are reserving specific time slots for moving through an intersection, regardless of approach direction. Although such a model is reasonable for an envisioned future state where all vehicles are autonomously driven, it ignores the reality of basic safety and fairness constraints that must be enforced when shifting control from one approach to another in current day traffic signal systems. The framework proposed in this paper incorporates these movement phase transition constraints.

**Background on Actuated Signal Control**

Actuated control uses detector inputs to determine the green time durations. Movement sequences run in parallel, called rings, and they resolve the spatial conflicts. The ring structure ensures that movements end simultaneously to ensure that spatial conflicts do not arise. Green time durations are determined by minimum greens, gap timers, and maximum greens.

In fully-actuated control, detectors are placed on all of the movements, at the stop-bar. The detectors identify the passage or presence of vehicles. The detector inputs enable the signal controller to create phase sequences (movement combinations) and switching times that are response to the traffic streams. Typically, Once the intersection control is allocated to a specific approach, vehicles on that approach are serviced for a duration equivalent to minimum green. Starting at the end of minimum green, decisions are made concerning extending green on the subject approach; gap-timer settings on the detectors play an instrumental role in making those decisions. The purpose of setting a gap-timer is to measure the time interval between successive vehicle arrivals (vehicle headways). Green extensions on the subject approach continue until the vehicle headways are greater than the pre-specified passage times on the stop-bar detector (or until maximum green is reached). If a gap out occurs and there is a non-zero queue on other conflicting approach, then display of green is terminated on the subject approach and a pre-specified clearance interval is imposed before shifting control to the approach with non-zero queue.

**Intersection Control as a Game**

If one treats the intersection control problem as a game, there can be movement managers bidding each turn for use of the intersection space. Then the outcome is a sequence of winners which translates into decisions about which movements use the intersection at which points in time. That is, playing such a game creates the signal timings. The game-like structure presented in this paper has three types of players: movement managers, drivers, and a municipality. Details about each of these players and the nature of their interactions are described in this section.

**Movement Managers**

Movement managers are associated with approaching queues and are the players who bid against one another for use of the intersection. When a movement manager wins, he releases vehicles from his waiting queues. Furthermore, movement managers develop bidding strategies that increase their chances of winning. They collect initial fees from their drivers upon arrival and any additional voluntary monetary contributions that drivers make. Note that movement managers cannot force drivers to pay more than they are willing. Hence, movement managers are interested in providing a good quality of service to their drivers, subject to a constraint of financial solvency.

Obviously, the bidding strategies developed by the movement managers are heavily influenced by the information they have access to. In the realization presented here, it is assumed that the movement managers only have access to information about vehicle arrival patterns on their respective approaches. This means, they are unaware of vehicle arrival patterns in other approaches, as well as the bids submitted by other movement managers. They do know what they bid and whether or not the bid won. Hence, they do expect that higher bids increase the probability of winning. When they win, they pay what they bid (first-price bidding). The movement managers strive to maximize the chances of winning, subject to remaining solvent. Therefore, a movement manager submits bids that are as high as possible, while ensuring that they have sufficient funds to discharge the rest of the vehicles in his queue at a nominal fee. Consequently, the movement managers continue to learn how to negotiate on behalf of arriving drivers for the games full duration.

Movement managers make use of historical data (if available) in determining their bids. In the game realization presented here, they keep track of win/loss bids associated with every queue length that they see on their respective approaches. Based on this historical information, for each possible queue length they develop:

- The probability density function (PDFs) associated with the winning bids;
- The average winning bid;
- The odds ratio (i.e., the ratio of number of winning bids to the number of losing bids). If this ratio is greater than 1, they infer that they are doing a good job of managing their queue.

Movement managers compute three candidate bids and select the largest. The first candidate bid is the highest possible amount ($x\%$ above nominal fee) that the manager can bid, given a constraint that at least a nominal fee can still be paid to discharge each of the remaining vehicles in queue. The second candidate is the highest bid possible if the current funds are equally distributed among all drivers in queue. The third candidate bid is the average winning bid for this queue length, adjusted downward if necessary to ensure solvency. To determine this bid possibility, the manager makes use of knowledge about win/loss bids associated with the current queue $i$. The odds of winning are computed, as well as the average winning bid associated with it (of course one can chose other measures such as the median or 75th percentile bid as possible candidates instead of the average bid). If necessary, this bid is adjusted to ensure that the movement manager remains solvent.
Drivers

Drivers make payments to the movement managers: first a fixed fee, and then voluntary contributions. They learn about how much to pay as their short time in the game unfolds. They have information about their queue position, and the win/loss record of their movement manager from the time they join the queue. They make contribution decisions based on the movement managers performance, their value of time (the framework presented in this paper considers two classes of drivers: those with high value-of-time (VOT) and those with low VOT. The notion is that drivers with high VOT have less tolerance to higher delays; hence they make larger voluntary monetary contributions as opposed to those with low VOT, and the delay they have incurred. Since drivers are transient players, they learn all this while in queue. Drivers do not pass on their knowledge to other drivers.

A drivers main objective is to transit the intersection in minimum time. When drivers arrive at a given intersection, they have a desired delay . The value of the desired delay is a function of their initial perception on movement managers ability to submit a winning bid; the parameter captures this aspect of their behavior. If driver has an initial position of in the queue, and is the saturation headway, then

\[ d_j = x_j^0 \times \rho_j \times \frac{1}{h_s}, \]

where:

- \( d_j^{k+1} \) = estimated delay of queued vehicle \( j \) in \( (k+1)^{st} \) bidding cycle
- \( d_j^k \) = actual delay of queued vehicle \( j \) at the end of \( k^{th} \) bidding cycle
- \( x_j^k \) = position of vehicle \( j \) in queue at the end of \( k^{th} \) bidding cycle
- \( p_j^{k+1}(\{D_j^k\}) \) = probability that the movement manager loses in \( (k+1)^{st} \) turn

Drivers compute \( p_j^{k+1}(l) \) based on bidding outcome data since the time they joined until the end of the \( k^{th} \) turn \( (D_j^k) \). The rationale underlying the update procedure described in equation (1) is that as \( p_j^{k+1}(l) \rightarrow 1 \) the estimate of anticipated delay increases, and as \( p_j^{k+1}(l) \rightarrow 0 \) the estimate of \( d_j^{k+1} \) goes down. Subsequent material presented in this section provides more details on probabilistic reasoning used by the drivers.

Here the fundamental uncertainty lies in driver perceptions about movement managers ability to submit a winning bid. The drivers take different actions (voluntary monetary contributions, updating estimate of anticipated delay) after they update their perceptions of the movement managers performance. Drivers use Bayesian inference to update their belief about movement managers ability to submit a winning bid.

Every bidding cycle, the movement manager either submits a winning bid (outcome, \( W \)) or not (outcome, \( L \)). \( P(W) = \theta \) is the probability that the movement manager will win, and \( P(L) = 1 - \theta \) is the probability that the movement manager will lose. Here the distribution of \( \theta \) is modeled as beta distribution with parameters \( \alpha \) and \( \beta \) as shown in equation (2):

\[
\theta | (\alpha, \beta) \sim Beta(\alpha, \beta)
\]

\[
P(\theta | \alpha, \beta) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{Beta(\alpha, \beta)}
\]

Since arriving drivers do not have any historical knowledge regarding their movement managers ability to submit a winning bid, they assign equal probabilities to the movement managers success/failure in the next bid event; this is achieved by setting \( \alpha = 1 \) and \( beta = 1 \) in equation (2).

Let denote the outcome of \( i_{th} \) bidding cycle:

\[
w_i = \begin{cases} 
1 & \text{if the outcome is } W, \\
0 & \text{if the outcome is } L.
\end{cases}
\]

It is clear that there will be \( n + 1 \) states at the end of \( n_{th} \) bidding cycle and it is easy to see the update procedure for computing posterior probability. Suppose a driver waiting in queue observes \( n \) successive bidding cycles. Among these, \( k \) outcomes were winning bids and \( n - k \) outcomes were losing bids. The posterior distribution is then given by:

\[
\theta | D_j^n \sim Beta(\alpha + k, \beta + n - k)
\]

\[
P(\theta | D_j^n) \propto \theta^{\alpha+k-1} \times \theta^{\beta+n-k-1}
\]

(4)

Drivers use this new information from the current bidding cycle to update their assessment of whether their movement manager will win or lose subsequent bids. They compute \( P(H | D_j^n) \).

Here \( H \) is the hypothesis that the movement manager submits a winning bid and, using equation (2), we compute Bayes factor (which is the ratio of posterior odds to prior odds) using this information as shown in equation (5):

\[
\frac{O(H | D_j^n)}{O(H)} = \frac{P(D_j^n | H)}{P(D_j^n | H)} = K
\]
The left-hand side of equation (5) is the ratio of the posterior and prior odds, whereas the right-hand side is the likelihood ratio, also known as Bayes factor, $K$. For the model presented here, $K$ is the win to loss ratio at the end of a given bidding cycle. If $K > 1$, then $D^u_j$ is more likely under $H$ than under $\bar{H}$, if $K = 1$, then $D^u_j$ is equally likely under either hypothesis, and if $K < 1$, then $D^u_j$ is more likely under $\bar{H}$ than under $H$.

For every bidding cycle, drivers compute: 1) their Bayes Factor ($K$), that is, the right-hand side of equation (5), and 2) their expected delay until departure using equation (1). In every bidding cycle, the driver decides whether to make a voluntary contribution. Table 1 shows the logic and the possible contributions.

The driver first determines $K$ as described previously. Then, the outcome of Test is determined. The variable Test equals 1 if $d^s_j > \hat{d}_j$ and it equals 0 if $d^s_j \leq \hat{d}_j$. Depending on the values of $K$ and Test, various possible contribution amounts pertain. For example, if Test = 1 and $K \geq 1$, and the driver has a high VOT, then the amount is 0.50. If the drivers VOT is low, then it is 0.20. (The numerical values of the voluntary contributions are chosen for illustrative purposes. They are logical, but not based on any empirical data.)

Then, there is a probability that the driver will actually make the contribution indicated in Table 1. That probability is determined by the function:

$$p(\text{cont}) = \frac{1}{1 + \exp(10 \times (\frac{K}{\pi + 1} - 0.5))} \quad (6)$$

To illustrate, if $K$ is 1.0, i.e., the movement manager is as likely to win as to lose, then $p(\text{cont}) = 0.5$. In other words, the probability that the driver will make a contribution is 50%. As $K \to 0$, i.e., the movement manager becomes very unlikely to win, and $p(\text{cont}) \to 1$: the driver will almost always make a contribution. As $K \to \infty$, i.e., the movement is very likely to win, $p(\text{cont}) \to 0$: the driver is very unlikely to make a contribution.

**Municipality**

As with every game, there is a set of rules that govern how the game is played. The municipality defines these rules and makes decisions about how to assign control among the movement managers. The municipality also controls the times associated with green, yellow, and all-red durations, as well as pauses in the bidding process. The municipality acquires information on queue lengths for each approach from their respective movement managers. On the basis of this information, the municipality tells the movement managers when to submit bids.

**Control Structure of the Game**

As mentioned earlier, signal control strategies in use today achieve safety and efficiency by using clearance intervals, minimum and maximum greens, and gap values. Except for the maximum greens, all of these are considered. (The maximum greens are omitted so as to not cloud the analysis through their impacts).

To incorporate clearance intervals, minimum greens and gaps in the bid-based control, the following actions are taken. For the clearance intervals and minimum greens, if control shifts from one manager to another as a result of the bidding process, bid submission ceases for a clearance interval plus a minimum green. At the end of this time period, managers with non-zero queues again submit bids for use of the intersection space. For the gaps, bidding is suspended until the end of the gap time or the next vehicle arrival, whichever occurs first. The pseudo code for the bid-based control is presented in Figure 2.
During the bidding event, if only one movement manager has a service queue that manager is an automatic winner; it is allowed to use the intersection space for a duration equivalent to the minimum green (if control shifts) or the minimum of the 3-second gap or the headway to the next arriving vehicle. For every vehicle discharged in this manner, the manager pays a nominal fee to the municipality.

If both movement managers have service queues, then both movement managers submit bids (refer to the previous section for details about how the bids are computed); the winning bidder is selected by the municipality, and the winner is allowed to use the intersection space for the same duration described earlier; the winning movement manager pays the municipality what was bid (first-price bidding); and discharges the first-in-queue vehicle at saturation headway. All movement managers then update their win/loss PDFs using the results of the bid. Remaining drivers update their belief about their movement managers likelihood of winning, they re-compute their $p_{(i,j)}^{act}$, and decide whether or not they want to make voluntary contributions.

### Simulation Experiments

Figure 3 presents schematic of the test network considered for analysis. As one might see, the network consists of three nodes with main corridor along NB. The intersecting approaches at each node represent single-lane-one-way streets with the total intersecting volume held constant at 1500 veh/hr/lane. Three traffic flow combinations are examined (scenarios):

1. Scenario 1: Equal volumes $(v_N = 750, v_E = 750)$
2. Scenario 2: Slight imbalance $(v_N = 900, v_E = 600)$
3. Scenario 3: significant imbalance $(v_N = 1200, v_E = 300)$

The following two objectives are considered while designing the simulation experiment: 1) Viability of bid-based control strategy for network control; 2) the penetration of high value-of-time drivers on systems performance.

To address the first objective, the results of bid-based control outputs are compared to those obtained from actuated control. Actuated control uses detector inputs to determine the green time durations. Movement sequences run in parallel, called rings, and they resolve the spatial conflicts. The ring structure ensures that movements end simultaneously to ensure that spatial conflicts do not arise. Green time durations are determined by minimum greens, gap timers, and maximum greens. To address the second objective, nine cases were created by varying the percentage of high value-of-time drivers between 0 and 80 with increments of ten percent from one case to the next. For example, in case-1, all the drivers in the network have low VOT, whereas in case-2, 10% of drivers in the network have high VOT, and in case-3 20% of drivers have high VOT and so on.

An agent-based simulation model of the bid-based control was developed in Python consistent with the experimental design objectives. For the purposes of benchmarking the analysis, a Python-based model of actuated control is also developed. These two simulation models exist within the same analysis program.

For a given input volume on the facility, the program creates a sequence of arrival headways for each scenario for both approaches. A shifted negative exponential headway distribution is employed with a minimum headway of 1.5 seconds and an average headway consistent with the arrival flow rate. These arrival patterns are used both by the bid-based control model and the actuated control mode. The saturation headway is set to be uniform between 1.5 to 2.6 seconds with an average of 2.1 seconds (1,714 vehicles/hour) see below. The other simulation parameters are a nominal fee = $1 and an initial fee = $1. The results presented in this paper are based on the data obtained from 20 Monte Carlo simulations each 27,000 seconds long.

### Evaluation and Analysis

Two metrics are employed to compare and contrast the results from the various scenarios: 1) Total network delay; 2) Comparison of average statistics

### Total Network Delays

Figure 3 presents sum of delays of all vehicles that traversed the network. The figure contains three tables (one for each scenario). Each table consists of 12 columns: the first column describes the control strategy under study (actuated or bid-based); the second column presents percentage of high VOT drivers in the network (as one might recall, this pertains only to bid-based control); columns 3-6 represent sum of delays experienced by NB vehicles at Node-1, Node-2, Node-3 and for the entire path respectively; columns 7-9 represent delays experienced by EB drivers at Node1, Node2, and Node3 respectively; column 10 represents sum total of delays in the network (which is equivalent summing values in columns 7-9); columns 11 represents percentage change in total delay across various cases when compared to bid-based case 1 (i.e., no high VOT drivers in the network); column 12 represents similar results but results from actuated control are used as basis for computing percentage change.
There are two main inferences one can draw based on these results. First one is that the bid-based control does a better job of reducing total delays in the network when compared to actuated control (the values in columns 11 and 12 manifest this point). Furthermore, a close look at the table suggests that actuated control does a slightly better job of reducing NB delays at the expense of significant increase in side-street delays. The reason for this is that the actuated control logic never finds the opportune time to end the main street green and serve the side street. Second inference is that increasing the penetration of high VOT of drivers increases sum total delays in the network. Intuitively this makes sense.

### Comparison of Average Statistics

Figure 4 presents statistics on average delays, average driver payments, and average cost of service at each of the three nodes for various cases for scenario 1 flow condition. The three tables to the left present average delay statistics at three nodes, whereas the three tables to the right represent statistics on economics (average driver payments, average cost of service) at the corresponding nodes. Each of the delay statistics table contains eight columns: the first column describes the control strategy under study (actuated or bid-based); the second column presents percentage of high VOT drivers in the network (as one might recall, this pertains only to bid-based control); column 3-5 represent average delays experienced by all drivers, drivers with high VOT, and drivers with low VOT respectively on NB approach; columns 6-8 present similar results but for EB approach. Similarly, each of the tables on economics consists of 9 columns: column 1 presents percentage of high VOT drivers in the network; columns 2-4 represent average driver payments on NB approach made by all drivers, drivers with high VOT, and drivers with low VOT respectively; column 5 represents average amount paid by the movement manager to discharge vehicles on his approach; columns 6-9 represent similar results but for EB approach. Furthermore, the summary tables presented in Figures 5 and 6 represent similar results but for Scenario 2 and Scenario 3 conditions respectively.

### Figure 4: Summary of delays of all vehicles in the network

Following are some of the inferences one can draw based on the results presented in these tables: First is that the disparity in the average delays on NB and EB approaches is more pronounced in the case of actuated control; whereas bid-based control is able to serve side streets (EB) in a reasonable manner without compromising for delays on NB approach. For example, in scenario 1, in the case of bid-based control average delays of NB and EB vehicles at each node.
Figure 7: Summary tables for other statistics (Scenario 3) is about 23-25 seconds, whereas in the case of actuated control, these numbers range between 18-34. Similar trends can be observed in both scenario 2 and scenario 3 flow conditions. Second is that, in most cases the average delay of high VOT drivers is slightly less than that of low VOT drivers. Thirdly, drivers with a high value-of-time are contributing higher amounts than the average cost of service, whereas the drivers with a low value-of-time are contributing less than the average cost of service. These payments reflect the importance that the drivers place on being serviced expeditiously. This is again evidence that the drivers with high value-of-time are cross subsidize those with a low value-of-time and produce lower delays for both groups. While one could argue the marginal benefit for being a high VOT driver is insignificant as far as delay distributions are concerned, there is value in considering different classes of drivers in the traffic stream. First, in reality not all drivers in the network have same value associated with the trip (for example making a trip to airport vs. going home from work). Secondly, it is possible to experiment with the voluntary contribution matrix presented in Table 1 to achieve better delay distributions for high VOT drivers.

Conclusions

This paper has presented an auction-based model for intersection control. Furthermore, the framework presented in the paper proposes a bid-based control strategy that takes into consideration minimum green, clearance intervals, and gaps. Using a simulation-like setting, movement managers (computer applications) bid for green time on behalf of the vehicles for specific turning movements. Drivers try to manage their delays by monitoring their movement managers performance and making voluntary contributions to expedite service. Both of these are described in terms of what they do, how they interact, the influence of data availability on how they behave, and how their decisions influence the outcome of the simulation. Moreover, the results of several simulation analyses have been presented.

In the future, our work will focus on three directions.

- our current research is investigating adjustments to our model to better expedite drivers who are willing to contribute more.
- only first price closed bidding concepts are explored in this paper; we plan to investigate the impact of various auction systems on the simulation.
- movement managers access to other information such as win/loss bids of every other movement manager in the simulation will influence his own bid. We plan to study the impact of this information exchange.

References


