

# Towards Optimization-Based Multi-Agent Collision-Avoidance under Continuous Stochastic Dynamics

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## 1 Introduction

In our ongoing work, we aim to control a team of agents so as to achieve a prescribed goal state while being confident that collisions with other agents are avoided. Each agent is associated with a feedback controlled plant, whose continuous state trajectories follow some stochastic differential dynamics. To this end we describe a collision-detection module based on a distribution-independent probabilistic bound and employ a fixed priority method to resolve collisions. Due to their practical importance, multi-agent collision avoidance and control have been extensively studied across different communities including AI, robotics and control. However, these works typically assume linear and discrete dynamic models; by contrast, our work intends to overcome these limitations and to present solutions for continuous state space. While our current experiments were conducted with linear stochastic differential equation (SDE) models with state-independent noise (yielding Gaussian processes) we believe that our approach could also be applicable to non-Gaussian cases with state-dependent uncertainties.

## 2 Method

**Model.** We assume the system contains a set  $\mathcal{A}$  of agents indexed by  $\alpha \in \{1, \dots, |\mathcal{A}|\}$ . Each agent  $\alpha$ 's associated plant has a probabilistic state trajectory following a stochastic controlled  $D$ -dimensional state dynamics (we consider the case  $D = 2$ ) in the continuous interval of (future) time  $I = (t_0, t_f]$ . For our method to work all we need to require is that the trajectory's mean function  $m : I \rightarrow \mathbb{R}^D$  and covariance matrix function  $\Sigma : I \rightarrow \mathbb{R}^{D \times D}$  are evaluable for all times  $t \in I$ . A prominent class for which closed-form moments can be easily derived are linear Ito-SDEs. For instance, we consider the SDE

$$dx^\alpha(t) = -K(x^\alpha(t) - \xi^\alpha(t))dt + B dW \quad (1)$$

where  $K, B \in \mathbb{R}^{D \times D}$  are matrices  $x^\alpha : I \rightarrow \mathbb{R}^D$  is the state trajectory and  $W$  is a vector-valued Wiener process. Here,  $-K(x^\alpha - \xi^\alpha)$  will be interpreted as the control output of a linear feedback-controller regulating the state to converge to a desired signal  $\xi^\alpha(t) = \zeta_0^\alpha \chi_{\{0\}}(t) + \sum_{i=1}^{H^\alpha} \zeta_i^\alpha \chi_{\tau_i^\alpha}(t)$  where

$\chi_{\tau_i} : \mathbb{R} \rightarrow \{0, 1\}$  denotes the indicator function of the half-open interval  $\tau_i^\alpha = (t_{i-1}^\alpha, t_i^\alpha] \subset [0, T^\alpha]$  and each  $\zeta_i^\alpha \in \mathbb{R}^D$  is a *setpoint*. If  $K > 0$  the agent's state trajectory is determined by setpoint sequence  $p^\alpha = (t_i^\alpha, \zeta_i^\alpha)_{i=0}^{H^\alpha}$  (aside from the random disturbances) which we will refer to as the agent's *plan*. For example, plan  $p^\alpha := ((t_0, x_0^\alpha), (t_f, x_f^\alpha))$  could be used to regulate agent  $\alpha$ 's *start state*  $x_0^\alpha$  to a given *goal state*  $x_f^\alpha$  between times  $t_0$  and  $t_f$ . For simplicity, we assume the agents are always initialized with plans of this form before coordination commences.

**Task.** Each agent  $\alpha$  desires to find a sequence of setpoints ( $p^\alpha$ ) such that (i) it moves from its start state  $x_0^\alpha$  to its goal state  $x_f^\alpha$  along a low-cost trajectory and (ii) such that along the trajectory its plant (with diameter  $\Delta$ ) does not collide with any other agents' plant in state space with at least a given probability  $1 - \delta \in (0, 1)$ .

**Approach.** As the focus of our work is on collision detection and avoidance in continuous domains, we opt for a simple *fixed-priority* (FP) coordination scheme. While the rigidity of the approach typically leads to sub-optimality in terms of a social cost function, the simplicity and computational tractability of the method may be the reason why it has been deployed most widely in robotics (e.g. (Bennewitz, Burgard, and Thrun 2001)).

Specifically, each agent has a unique ranking (or priority) according to its index  $\alpha$  (i.e. agent 1 has highest priority, agent  $|\mathcal{A}|$  lowest). When all higher-priority agents are done planning, agent  $\alpha$  is informed of their planned trajectories which it has to avoid with a probability greater than  $1 - \delta$ . To this end, the two main mechanisms to develop are (I) an algorithm capable of detecting collisions on a *continuous* time interval  $I$  with a sufficiently high probability and (II) to use this information to augment the existing plan by new setpoints in order to avoid the collision with the given confidence bound  $1 - \delta$ . Next, we briefly sketch the underlying ideas of our approach to address these two problems.

(I) Based on a generalized Chebychev-bound (Whittle 1958), (Lyons, Calliess, and Hanebeck 2012) derived  $D$  distribution-independent constraints of the form  $C_j(t) \leq 0$  ( $j = 1, \dots, D$ ) depending on the components of the means and the covariance matrix functions of two stochastic trajectories, as well as on  $\delta$ . For any given time  $t$ , it was shown that satisfaction of at least one of the constraints is a sufficient condition for collision avoidance (with prob.  $> 1 - \delta$ ).

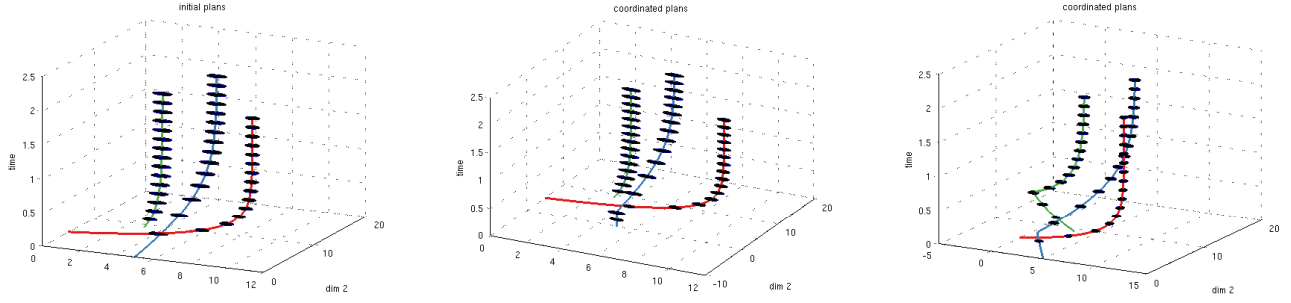


Table 1: Left to right: Uncoordinated Agents’ plans (l), after conflict resolution with methods WAIT (c) and FREE (r).

For conservative collision detection between two agents’ trajectories  $x^a, x^v$ , we propose to construct a (continuous) *criterion function*  $\gamma^{a,v}(t) := \max_j \{C_j(t)\}$ . By construction, a collision between the trajectories with probability above  $\delta$  can be ruled-out if  $\gamma^{a,v}$  attains only positive values. For collision detection at time  $t$  all an agent needs to do is to minimize the continuous function  $\Gamma^a(t) := \min_{v \neq a} \{\gamma^{a,v}(t)\}$  and (conservatively) assume a collision unless  $\Gamma^a(t) > 0$ .

(II) As long as agent  $a$  detects a collision at some time  $t \in I$  we will invoke a conflict resolution method. As a first method we could insert a time-setpoint pair  $(t, x_0^a)$  into the previous plan  $p^a$ . Since this aims to cause the agent to wait at its start location  $x_0^a$  we will call the method WAIT. It is possible that multiple such insertions are necessary until collisions are avoided. Of course, if a higher-priority agent decides to traverse through  $x_0^a$ , this method is too rigid to resolve a conflict. Alternatively, we could optimize for the time and location of the new setpoint. Let  $p_{\uparrow(t,s)}^a$  denote the plan updated by insertion of time-setpoint pair  $(t, s) \in I \times \mathbb{R}^D$ . We propose to choose the candidate setpoint  $(t, s)$  that minimizes a function being a weighted sum of the expected cost entailed by executing updated plan  $p_{\uparrow(t,s)}^a$  and a collision penalty  $-\lambda \min_t \Gamma^a(t) H(-\min_t \Gamma^a(t))$ . Here,  $\lambda$  is a large number,  $H$  is the Heaviside function and  $\Gamma^a$  is computed based on the assumption we were to execute  $p_{\uparrow(t,s)}^a$ . Since the new setpoint can be chosen freely in time and state-space we refer to the method as FREE.

### 3 Simulations

As a first test, we simulated a simple three-agent scenario. Each agent’s dynamics were instantiations of an SDE of the form of Eq. 1. The initial plans of the agents are depicted in Tab. 1 (left). The curves represent the predicted trajectories of Agents 1 (red), 2 (blue) and 3 (green) for the given initial plans and are depicted for a time interval of  $I = [0s, 2s]$ . The uncertainties are indicated by covariance ellipses plotted at discrete points in time every .13 s. Drawing 1000 sample paths from each of the SDEs with the Euler-Maruyama method we recorded a collision in 56% of the draws from the SDEs under the initial plan. The collision source is mainly the predicted collision found at the intersection between the first two trajectories around time .13s. The altered plans due to invoking collision avoidance method WAIT are shown in

the centre plot. Here, Agent 2 waited at its start location (5,0) for about .26s to let the first agent pass by until resuming towards its goal thereby avoiding the collision with the higher priority agent 1. Simulation of the resulting plans yielded collisions in 5% of the draws – mainly due to conflicts between 2 and 3 which could not be resolved by 3 waiting at its start location. Finally, the FREE method generated plans that caused agent 2 to circumvent 1 by arcing to the left. This brought it closer to the 3 which in turn, decided to leave for a while (until 2 had passed by) before returning to its goal. None of the draws from the simulated trajectories collided. In contrast, the WAIT method was computed faster (.41s vs 20s, on a standard Laptop running MATLAB) but could not prevent all collisions.

### 4 Ongoing Work

Our current work focuses on the following questions: How can we quantify the probabilities that all required minimizations succeed given a fixed computational budget? How can we bring to bear more advanced coordination mechanisms, such as auctions, in order to improve the social cost of the solutions? In addition we intend to assess our method in systems with non-linear dynamics and multiplicative noise. We anticipate being able to speed up our method significantly by means of improved implementation, hardware and parallelization. Finally, we are investigating the impact of tighter bounds for collision detection on social cost in settings where such bounds are available.

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