Virtual Structure Reduction for Distributed Constraint Problem Solving

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Abstract
Distributed Constraint Problem solving represents a fundamental research area in distributed artificial intelligence and multi-agent systems. The constraint density, or the ratio of the number of constraints to the number of variables, determines the difficulty of either finding a solution or minimizing the set of variable assignment conflicts. Reducing density typically reduces difficulty. We present a fully distributed technique for reducing the effective density of constraint graphs, called Virtual Structure Reduction (VSR). The VSR technique leverages the occurrence of variables that must be assigned the same value based on shared constraints and can improve solver performance using existing algorithms. We discuss our Distributed Constraint Optimization Problem (DCOP) solver, integrated with the Distributed Stochastic Algorithm (DSA), called VSR-DSA. The VSR-DSA algorithm demonstrates performance gains vs DSA in both solution quality and time on 3-coloring problems.

Introduction
Constraint density, or the ratio of number of constraints to number of variables, has a significant effect on the difficulty of distributed problem solving. As the density of problem instances increases, a rapid progression from under-constrainedness to over-constrainedness occurs. As density increases Distributed Constraint Problems tend to become more difficult to solve. This paper presents a fully distributed method for reducing constraint density via the concept of frozen pairs (Culberson and Gent 2001).

This work draws upon theoretical results in Constraint Satisfaction Problem (CSP) analysis. In (Cheeseman, Kanefsky, and Taylor 1991) and (Culberson and Gent 2001), methods for reducing the average density of a CSP are suggested but not algorithmically defined. The operations provided, or discussed, are limited to a centralized approach, or CSP. We know of no distributed approach performing density reduction for constraint graphs. Our work codifies the ideas from (Cheeseman, Kanefsky, and Taylor 1991) and captures the phenomenon of frozen pairs from (Culberson and Gent 2001) to provide a fully distributed density reduction capability. Low constraint density problems are more likely to yield a consistent solution from an initial random assignment as opposed to high constraint densities where it becomes more likely that an algorithm will find portions of the problem space that are unsatisfiable given the number of consistent values available to the variables. For both Distributed Constraint Satisfaction Problems (DisCSP) and Optimization (DCOP), lower densities typically lead to problem instances that are easier to solve.

Virtual Structure Reduction
The Virtual Structure Reduction (VSR) technique relies on the detection of frozen pairs. We define frozen pairs in terms of a constraint graph with: a set of variables \( V = \{x_1, \ldots, x_n\} \), a set of domains \( D = \{D_1, \ldots, D_n\} \) where each \( D_i \) has exactly \( k \) values, and a set of \( m \) binary constraints \( C = \{C_{1}, C_{2}, \ldots, C_{m}\} \) where each \( C_{j}(d_p, d_q) \) is a predicate on the Cartesian product \( D_p \times D_q \) that returns 0 if the values of the variables satisfy the constraint, otherwise returns 1. Let \( N \) be the neighbor function, such that \( N(x_i) = \{x_p | \exists C(d_i, d_p) \in C\} \) and \( S \) is the shared neighbor function, such that \( S(x_i, x_j) = N(x_i) \cap N(x_j) \). We define \( x_i \) to be frozen with \( x_j \) iff \( S(x_i, x_j) \) contains a maximal (k-1)-clique.

When frozen, \( x_i \) and \( x_j \) must have the same variable assignment. Variables that are frozen to one another have a value that can be induced by the assignment of the core. This property ensures that any agent who is frozen can negotiate a local assignment with the members of the core and share that assignment with the other frozen agent. We term this negotiating agent as the surrogate. Surrogate agent’s hold all agents whom they are negotiating for in a spine list. Once the surrogate finds a suitable variable assignment, it informs its spine list.

The VSR technique is an integrated solution with the Distributed Stochastic Algorithm (DSA), called VSR-DSA. The algorithm consists of two main phases: reduction and assignment. The Reduction phase is a 3-state cyclic process, discussed below, for discovering and assigning surrogates that terminates when no further reductions are found. The Assignment phase assigns a value to variables and leverages the surrogate assignments provided by the reduction phase.

The reduction phase is performed locally by agents that are currently active, with all agents begin initialized with active = true. When an agent is not active, it is because that agent has relinquished control of its variable assignment.
Our first evaluation is with respect to the reduction in density of the constraint graphs using only the reduction capabilities of VSR. We present the resulting density measures after running VSR on constraint graphs of different sizes and starting densities in Figure 1. Our results show that using VSR on problems with original densities that are relatively low (i.e. 2.0) we see very little gain. This is due to the lack of core structures that create frozen pairs in the problem. Without a sufficient number of frozen pairs, significant reduction cannot be found. In Figure 1 we see that as density rises, more reduction occurs, ultimately converging to 1, a k-clique.

To test performance, we evaluate the VSR-DSA algorithm on the MaxSAT 3-colorability DCOP using randomized graph instances. Given a set of cost functions $f = \{f_1, \ldots, f_m\}$ where each $f_i(d_i, d_j)$ returns 0 iff $d_i \neq d_j$, 1 otherwise, the goal is to find a set of assignments $A = \{d_1, \ldots, d_n\}$ such that global cost $G(A) = \sum_{i=1}^{m} f_i(A)$ is minimized.

We show the experimental results regarding solution quality (Figure 2) and time (Figure 3) required to solve 2.7 density DCOPs. The two algorithms shown are DSA and VSR-DSA. In Figure 2, the VSR-DSA algorithm convergence slightly slower in the beginning (in cycles), but ultimately converges to a better minimization of the cost functions in $f$. The convergence speeds up once structural collapsing is nearing complete. In Figure 3, we see that VSR-DSA drastically outperforms the basic DSA algorithm in terms of real computation time. This performance gain is due to the reduced number of agents that the system has to wait on for negotiation to occur. Due to agents that have relinquished control of their variable assignments, there is less communication and a hastened turnaround time until the set of agents in the system get to re-negotiate.

**Conclusions**

We have introduced a fully distributed approach for reducing problem density in constraint graphs called Virtual Structure Reduction (VSR). We have shown empirical results that the resulting reduction of density can lead to increased performance using the VSR-DSA algorithm. Our preliminary results suggest that other existing solvers might benefit from the VSR technique as either an integration or pre-processing step.
References