

## Combining CP-Nets with the Power of Ontologies

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### Abstract

The Web is currently shifting from data on linked Web pages towards less interlinked data in social networks on the Web. Therefore, rather than being based on the link structure between Web pages, the ranking of search results needs to be based on something new. We believe that it can be based on user preferences and ontological background knowledge, as a means to personalized access to information. There are many approaches to preference representation and reasoning in the literature. The most prominent qualitative ones are perhaps CP-nets. Their clear graphical structure unifies an easy representation of preferences with nice properties when computing the best outcome. In this paper, we introduce *ontological CP-nets*, where the knowledge domain has an ontological structure, i.e., the values of the variables are constrained relative to an underlying ontology. We show how the computation of Pareto optimal outcomes for such ontological CP-nets can be reduced to the solution of constraint satisfaction problems. We also provide several complexity and tractability results.

### Introduction

During the recent years, several revolutionary changes are taking place on the classical Web. First, the so-called Web of Data is more and more being realized as a special case of the Semantic Web. Second, as part of the social Web, users are acting more and more as first-class citizens in the creation and delivery of contents on the Web. The combination of these two technological waves is called the *Social Semantic Web* (or also *Web 3.0*), where the classical Web of interlinked documents is more and more turning into (i) semantic data and tags constrained by ontologies, and (ii) social data, such as connections, interactions, reviews, and tags.

The Web is thus shifting away from data on linked Web pages towards less interlinked data in social networks on the Web relative to underlying ontologies. This requires new technologies for search and query answering, where the ranking of search results is not based on the link structure between Web pages anymore, but on the information available in the Social Semantic Web, in particular, the underlying ontological knowledge and the preferences of the users.

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As for preferences, there are approaches to (a) quantitative preferences, which are associated with a number representing their worth (e.g., “my preferences for WiFi and cable connections are 0.8 and 0.4, respectively”), and (b) qualitative preferences, which are related to each other via pairwise comparisons (e.g., “I prefer WiFi over cable connection”). The qualitative approach is commonly regarded as being the more natural way of representing preferences, since humans are not very comfortable in expressing their “wishes” in terms of a numerical value. To have a quantitative representation of her preferences, the user needs to explicitly determine a value for a large number of alternatives usually described by more than one attribute. It is generally much easier to provide information about preferences as pairwise qualitative comparisons (Domshlak et al. 2011).

One of the most powerful qualitative frameworks for preference representation and reasoning are perhaps CP-nets (Boutilier et al. 2004). They are a graphical language that unifies an easy representation of user preferences with nice properties when computing the best outcome. In this paper, towards defining a ranking of search results for the Social Semantic Web, which is based on user preferences and ontological knowledge, we combine CP-nets with ontologies.

### Ontological CP-Nets

We now introduce ontological CP-nets, which combine CP-nets (Boutilier et al. 2004) with ontologies represented in description logics (DLs) (Baader et al. 2003). Intuitively, the values of the variables in such CP-nets are satisfiable DL concepts relative to an underlying DL ontology.

More formally, an *ontological CP-net*  $(N, \mathcal{T})$  consists of a CP-net  $N$  and a DL ontology  $\mathcal{T}$  such that: (i) for every variable  $A$  in  $N$ , the domain of  $A$ , denoted  $\text{dom}(A)$ , is a set  $\{\alpha, \neg\alpha\}$ , where both  $\alpha$  and  $\neg\alpha$  are satisfiable DL concepts relative to  $\mathcal{T}$ , and (ii) all the conditional preferences in  $N$  are pairwise not equivalent relative to  $\mathcal{T}$ . Given an ontological CP-net  $(N, \mathcal{T})$ , an outcome  $\mathcal{I}$  is *feasible* iff  $\mathcal{I} \models \mathcal{T}$ . A feasible outcome  $\mathcal{I}$  is *undominated* iff there exists no feasible outcome  $\mathcal{I}'$  such that  $\mathcal{I}' \succ \mathcal{I}$ .

Observe that even if there are no explicit hard constraints among the variables of the CP-net, due to the background ontology, there is a set of implicit constraints among the values  $\alpha_i$  and  $\neg\alpha_i$  of the variables  $\mathcal{V}$  in the CP-net.

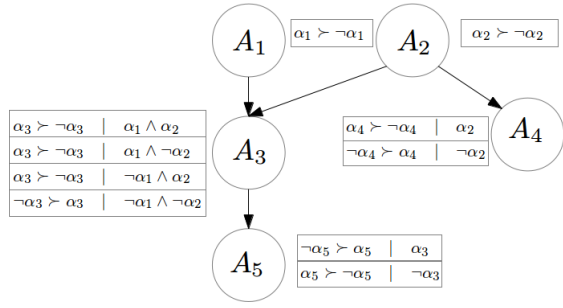


Figure 1: CP-net over five binary variables  $A_1, \dots, A_5$ .

**Example 1 (Hotel)** Consider the following simple ontology, describing the services offered by a hotel:

$$\begin{aligned}
\text{Scooter} &\sqsubseteq \text{Motorcycle}; \\
\text{Motorcycle} &\sqsubseteq \neg \text{Bike}; \\
\exists \text{rent.Scooter} &\sqsubseteq \exists \text{facilities.}(\text{Parking} \sqcap \\
&\quad \exists \text{payment} \sqcap \forall \text{payment.Free}).
\end{aligned}$$

Let the CP-net be given by Fig. 1, where the domains of  $A_3$ ,  $A_4$ , and  $A_5$  are given by  $\{\alpha_3, \neg\alpha_3\}$ ,  $\{\alpha_4, \neg\alpha_4\}$ , and  $\{\alpha_5, \neg\alpha_5\}$ , respectively:

$$\begin{aligned}
\alpha_3 &= \exists \text{rent} \sqcap \forall \text{rent.Scooter}; \\
\alpha_4 &= \exists \text{facilities.Parking}; \\
\alpha_5 &= \exists \text{rent.Bike}.
\end{aligned}$$

Then,  $\mathcal{T} \models \alpha_3 \sqcap \alpha_5 \sqsubseteq \perp$  and  $\mathcal{T} \models \alpha_3 \sqsubseteq \alpha_4$ . Thus,  $A_3$  and  $A_5$  constrain each other, and so do  $A_3$  and  $A_4$  as well. ■

## Algorithms and Complexity

**Algorithms.** The set of all undominated feasible outcomes for an ontological CP-net  $(N, \mathcal{T})$  can be computed on top of the HARD-PARETO algorithm in (Prestwich et al. 2005).

This computation is based on the following Boolean encoding of both the ontology  $\mathcal{T}$  and of the clauses corresponding to the conditional preferences encoded in  $N$  (along with the transformation back into DL representation of the results computed by the HARD-PARETO algorithm):

1. for each  $A_j$  in  $N$  with  $\text{Dom}(A_j) = \{\alpha_j, \neg\alpha_j\}$ , select a fresh concept name  $V_j$ ;
2. define the ontology  $\mathcal{T}' = \mathcal{T} \cup \{V_j \sqsubseteq \alpha_j \mid j = 1, \dots, |\mathcal{V}|\}$ ;
3. define the ontological CP-net  $(N', \mathcal{T}')$ , where  $N'$  is the same CP-net as  $N$ , but for the domain of its variables. In particular, in  $N'$ , we have  $\text{Dom}(A_j) = \{V_j, \neg V_j\}$ ;
4. define  $\mathcal{F} = \{V_j \mid j = 1, \dots, |\mathcal{V}|\}$ , where the  $V_j$ 's are the concept names introduced in step 1;
5. compute the ontological closure of  $\mathcal{F}$  relative to  $\mathcal{T}'$ ;
6. introduce a Boolean variable  $v_j$  for each  $V_j \in \mathcal{F}$ ;
7. transform the ontological closure of  $\mathcal{F}$  relative to  $\mathcal{T}'$  into the corresponding set of Boolean clauses  $\mathcal{C}$  by replacing  $V_j$  with the corresponding Boolean variable  $v_j$ ;
8. generate the Boolean encoding of  $N'$  as clauses by replacing  $V_j \in \text{Dom}(A_j)$  by the variable  $v_j$ .

**Complexity.** For tractable ontology languages (i.e., those for which deciding knowledge base satisfiability is tractable), the complexity of ontological CP-nets is dominated by the complexity of CP-nets. That is, deciding (a) consistency, (b) whether a given outcome is undominated, and (c) dominance of two given outcomes are all PSPACE-complete. In particular, this is the case if the ontological CP-net is defined over a DL of the *DL-Lite* family (Calvanese et al. 2007) (which all allow for deciding knowledge base satisfiability in polynomial time, such as *DL-Lite<sub>R</sub>*, which stands behind the important OWL 2 QL profile (W3C 2012b)). The same complexity results hold for ontology languages with PSPACE-complete knowledge base satisfiability checks, and even computing the set of all undominated outcomes (generalizing (b)) is PSPACE-complete under the condition that there are only polynomially many of them.

**Theorem 1** *Given an ontological CP-net  $(N, \mathcal{T})$  over a tractable ontology language, (a) deciding whether  $(N, \mathcal{T})$  is consistent, (b) deciding whether a given outcome  $o$  is undominated, and (c) deciding whether  $o \prec o'$  for two given outcomes  $o$  and  $o'$  are all PSPACE-complete.*

For EXP (resp., NEXP) complete ontology languages (i.e., those for which knowledge base satisfiability is complete for EXP (resp., NEXP)), the complexity of ontological CP-nets is dominated by the complexity of ontology languages. That is, deciding (a) inconsistency, (b) whether a given outcome is dominated, and (c) dominance of two given outcomes are all complete for EXP (resp., NEXP). In particular, this is the case if the ontological CP-net is defined over the expressive DL *SHIF(D)* (resp., *SHOIN(D)*) (Horrocks and Patel-Schneider 2003) (which stands behind OWL Lite (resp., OWL DL) (W3C 2004; 2012a)).

**Theorem 2** *Given an ontological CP-net  $(N, \mathcal{T})$  over an EXP (resp., NEXP) complete ontology language, (a) deciding whether  $(N, \mathcal{T})$  is inconsistent, (b) deciding whether a given outcome  $o$  is dominated, and (c) deciding whether  $o \prec o'$  for two given outcomes  $o$  and  $o'$  are all complete for EXP (resp., NEXP).*

**Tractability.** If the ontological CP-net is a polytree and defined over a tractable ontology language, deciding dominance of two outcomes is possible in polynomial time, which follows from dominance being decidable in polynomial time for standard polytree CP-nets (Boutilier et al. 2004). Note that polytree ontological CP-nets are always consistent. In particular, if the ontological CP-net is a polytree and defined over a DL of the *DL-Lite* family, deciding dominance of two outcomes can be done in polynomial time.

**Theorem 3** *Given an ontological CP-net  $(N, \mathcal{T})$  over a tractable ontology language, where  $N$  is a polytree, deciding whether  $o \prec o'$  for two given outcomes  $o$  and  $o'$  can be done in polynomial time.*

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