Two-Dimensional Description Logics for Context-Based Semantic Interoperability

Szymon Klarman
Department of Computer Science
Vrije Universiteit Amsterdam
sklarman@few.vu.nl

Víctor Gutiérrez-Basulto
Department of Computer Science
Universität Bremen
victor@informatik.uni-bremen.de

Abstract

Description Logics (DLs) provide a clear and broadly accepted paradigm for modeling and reasoning about terminological knowledge. However, it has been often noted, that although DLs are well-suited for representing a single, global viewpoint on an application domain, they offer no formal grounding for dealing with knowledge pertaining to multiple heterogeneous viewpoints — a scenario ever more often approached in practical applications, e.g. concerned with reasoning over distributed knowledge sources on the Semantic Web. In this paper, we study a natural extension of DLs, in the style of two-dimensional modal logics, which supports declarative modeling of viewpoints as *contexts*, in the sense of McCarthy, and their *semantic interoperability*. The formalism is based on two-dimensional semantics, where one dimension represents a usual object domain and the other a (possibly infinite) domain of viewpoints, addressed by additional modal operators and a metalanguage, on the syntactic level. We systematically introduce a number of expressive fragments of the proposed logic, study their computational complexity and connections to related formalisms.

Introduction

Description Logics (DLs) are popular knowledge representation formalisms, whose most prominent application is the design of ontologies — formal models of terminologies and instance data, representative of particular domains of interest (Baader et al. 2003) — used extensively on the Semantic Web and in biomedical applications. Under the standard Kripkean semantics, a DL ontology forces a unique, global view on the represented world, in which the ontology axioms are interpreted as universally true. This philosophy is well-suited as long as everyone can share the same conceptual perspective on the domain or there is no need for considering alternative viewpoints. Alas, this is hardly ever the case and very often, same domains are modeled differently depending on the intended use of an ontology. In practice, effective representation and reasoning about knowledge pertaining to such multiple heterogenous viewpoints becomes the primary objective for many applications, e.g. those concerned with reasoning over distributed knowledge sources on the Semantic Web (Guha, McCool, and Fikes 2004; Bao et al. 2010).

The challenges above resemble clearly the problems which once inspired J. McCarthy to introduce his theory of formalizing contexts in knowledge representation systems, as a way of granting them more generality (McCarthy 1987; Guha 1991). The gist of his proposal, motivating several existing logics of context (Buvač and Mason 1993; Buvač 1996; Nossum 2003), is to replace logical formulas φ, as the basic knowledge carriers, with assertions $ist(\boldsymbol{c}, \varphi)$. Assertions of this form state that φ is true in $\boldsymbol{c}$, where $\boldsymbol{c}$ denotes an abstract first-order entity called a *context*, which on its own can be described in a first-order language. For instance:

$$ist(\boldsymbol{c}, Heart(a)) \wedge \boldsymbol{HumanAnatomy}(\boldsymbol{c})$$

states that the object a is a heart in some context described as ***HumanAnatomy***. Based on this foundation, the theory allows for defining models of *semantic interoperability* within a possibly unbounded space of contexts, i.e. generic rules guiding the information flow between contexts, such as e.g.:

$$\forall xy\ \boldsymbol{HumanAnatomy}(x) \wedge \boldsymbol{Anatomy}(y) \rightarrow \\ \forall z(ist(x, Heart(z)) \rightarrow ist(y, HumanHeart(z)))$$

which ensures that in every ***Anatomy*** context, the interpretation of $HumanHeart$ includes also all those objects which are instances of $Heart$ in any ***HumanAnatomy*** context.

The formalism proposed in this paper incorporates these fundamental ideas of McCarthy's theory into the DL framework by considering contexts as abstract, first-class citizens, and offering an expressive formal apparatus for modeling their semantic interoperability. As a result, we harmonize and give a uniform formal treatment to two seemingly diverse aspects of the problem of reasoning with contexts in DL: 1) how to extend DLs to support the representation of inherently contextualized knowledge; 2) how to use knowledge from coexisting classical DL ontologies while respecting its context-specific scope. Our logic is essentially a two-dimensional DL, in the style of product-like combinations of DLs with modal logics (Wolter and Zakharyaschev 1999; Kurucz et al. 2003), similar to e.g. temporal DLs (Lutz, Wolter, and Zakharyaschev 2008; Artale, Lutz, and Toman 2007). In particular, we extend the standard DL semantics with a second dimension, representing a possibly infinite domain of contexts, and include additional modal operators along with a separate metalanguage in the syntax, for quantifying and expressing properties over the context entities.

In the following sections, we systematically introduce and motivate a number of expressive fragments of the logic, study their computational complexity and highlight the connections to some related formalisms.

Description Logics: preliminaries

A DL language $\mathcal{L}$ is specified by a vocabulary $\Sigma = (N_C, N_R, N_I)$, where N_C is a set of *concept names*, N_R a set of *role names*, N_I a set of *individual names*, and a number of operators for constructing complex concept descriptions (Baader et al. 2003). The semantics of $\mathcal{L}$ is given through *interpretations* of the form $\mathcal{I} = (\Delta, \cdot^{\mathcal{I}})$, where Δ is a non-empty *domain* of individuals, and $\cdot^{\mathcal{I}}$ is an *interpretation function*. The meaning of the vocabulary is fixed via mappings: $a^{\mathcal{I}} \in \Delta$ for every $a \in N_I$, $A^{\mathcal{I}} \subseteq \Delta$ for every $A \in N_C$ and $r^{\mathcal{I}} \subseteq \Delta \times \Delta$ for every $r \in N_R$, and $\top^{\mathcal{I}} = \Delta$. Then the function is inductively extended over complex expressions according to the fixed semantics of the constructors. Table 1 contains the list of concept constructors and their semantics which are considered in the rest of this paper: (1) top concept, (2) concept intersection, (3) existential role restriction, (4) complement, (5) nominal, where C, D are concepts, $r \in N_R$ and $a \in N_I$. We abbreviate $\neg\top$ with $\bot$, $\neg(\neg C \sqcap \neg D)$ with $C \sqcup D$ and $\neg\exists r.\neg C$ with $\forall r.C$.

	Syntax	Semantics
(1)	$\top$	Δ
(2)	$C \sqcap D$	$\{x \mid x \in C^{\mathcal{I}} \cap D^{\mathcal{I}}\}$
(3)	$\exists r.C$	$\{x \mid \exists y : (x, y) \in r^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
(4)	$\neg C$	$\{x \mid x \notin C^{\mathcal{I}}\}$
(5)	$\{a\}$	$\{a^{\mathcal{I}}\}$

Table 1: Concept constructors and their semantics.

A *knowledge base* (or an *ontology*) $\mathcal{K}$ is a finite set of axioms of three possible forms:

$$C \sqsubseteq D \mid C(a) \mid r(a, b) \qquad (\dagger)$$

where C, D are concepts, $a, b \in N_I$ and $r \in N_R$. We write $C \equiv D$, whenever $C \sqsubseteq D$ and $D \sqsubseteq C$ are both in $\mathcal{K}$. Typically, the formulas of the first type are denoted as *TBox axioms*, whereas the remaining two as *ABox axioms*. An interpretation $\mathcal{I}$ *satisfies* an axiom in either of the cases:

- $\mathcal{I} \models C \sqsubseteq D$ *iff* $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$,
- $\mathcal{I} \models C(a)$ *iff* $a^{\mathcal{I}} \in C^{\mathcal{I}}$,
- $\mathcal{I} \models r(a, b)$ *iff* $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$.

Finally, $\mathcal{I}$ is a *model* of $\mathcal{K}$ whenever it satisfies all its axioms. The computational complexity of reasoning in DLs varies depending on the expressiveness of the language. In the logic $\mathcal{EL}$, comprising only constructors (1-3), the central reasoning problem, *deciding concept subsumption* (i.e. verifying whether $\mathcal{K} \models C \sqsubseteq D$), is in PTIME (Baader, Brandt, and Lutz 2005). For $\mathcal{ALC}$ (1-4) and $\mathcal{ALCO}$ (1-5), the main reference problem of *deciding knowledge base satisfiability* (i.e. verifying whether $\mathcal{K}$ has a model) is EXPTIME-complete (Baader et al. 2003).

Interoperability systems

The DL semantics is extensional in its nature, in the sense that the meaning of an expression is its denotation in the object domain. Consequently, we define *semantic interoperability* also in a strictly extensional way. We say that it is the ability of a knowledge system to interpret expressions in different contexts via shared extensions, according to the declared constraints. For instance, the constraint $\alpha^{\mathcal{I}(\boldsymbol{c})} = \beta^{\mathcal{I}(\boldsymbol{d})}$ entails that the expression α has the same meaning in the context $\boldsymbol{c}$ as β in $\boldsymbol{d}$. A formal representation of the context-specific domain knowledge together with the interoperability constraints is denoted here as an *interoperability system*.

We introduce our framework in several steps. First, we demonstrate the basic interoperation mechanism in the simplest scenario involving a fixed number of explicitly named contexts. Next, we generalize the approach to account for a possibly infinite domain of contexts and include a lightweight metalanguage for describing them. Finally, we consider a few expressive extensions to the framework.

Simple interoperability systems

A *simple interoperation language* $SL_{\mathcal{L}}$ consists of a finite set M_I of individual context names, and an object language, which extends a DL $\mathcal{L}$ with special *context operators* applied to all constructs of $\mathcal{L}$.

Definition 1 ($SL_{\mathcal{L}}$-object language) *Let $\mathcal{L}$ be a DL with vocabulary $\Sigma = (N_C, N_R, N_I)$. Then the* object language *of $SL_{\mathcal{L}}$ is the smallest language containing $\mathcal{L}$ and closed under the constructors of $\mathcal{L}$ and the operators $\langle \boldsymbol{c} \rangle$, for $\boldsymbol{c} \in M_I$:*

$$\langle \boldsymbol{c} \rangle C \mid \langle \boldsymbol{c} \rangle r \mid \langle \boldsymbol{c} \rangle a$$

where C is a concept of the object language, $r \in N_R$ and $a \in N_I$. The resulting expressions are a concept, a role and an individual name of the object language, respectively.

Intuitively, the operator $\langle \boldsymbol{c} \rangle$ 'imports' the meaning of the bounded expression from the context denoted by name $\boldsymbol{c}$, to the context of occurrence. Formally, the semantics of $SL_{\mathcal{L}}$ is defined via extended interpretations.

Definition 2 ($SL_{\mathcal{L}}$-interpretations) *An $SL_{\mathcal{L}}$-*interpretation *is a tuple $\mathfrak{M} = (\mathfrak{C}, \cdot^{\mathcal{J}}, \Delta, \{\cdot^{\mathcal{I}(i)}\}_{i \in \mathfrak{C}})$, where:*

- *$\mathfrak{C}$ is a non-empty domain of contexts,*
- *$\cdot^{\mathcal{J}} : M_I \mapsto \mathfrak{C}$,*
- *$(\Delta, \cdot^{\mathcal{I}(i)})$, for every $i \in \mathfrak{C}$, is an interpretation of the object language, such that for every $\boldsymbol{c} \in M_I$ and expression α, $(\langle \boldsymbol{c} \rangle \alpha)^{\mathcal{I}(i)} = \alpha^{\mathcal{I}(\boldsymbol{c}^{\mathcal{J}})}$.*

Finally, we define the notion of Simple Interoperability System (SIS) and its $SL_{\mathcal{L}}$-model.

Definition 3 (Simple Interoperability System) *A* Simple Interoperability System *in $SL_{\mathcal{L}}$ is a finite set of formulas:*

$$\boldsymbol{c} : \varphi$$

where $\boldsymbol{c} \in M_I$ and φ is an axiom of the object language, in any of the forms ($\dagger$).

Definition 4 ($SL_{\mathcal{L}}$-models) *An $SL_{\mathcal{L}}$-interpretation $\mathfrak{M} = (\mathfrak{C}, \cdot^{\mathcal{J}}, \Delta, \{\cdot^{\mathcal{I}(i)}\}_{i \in \mathfrak{C}})$ is a* model *of a SIS $\mathcal{O}$* iff *for every $\boldsymbol{c} : \varphi \in \mathcal{O}$, $(\Delta, \cdot^{\mathcal{I}(\boldsymbol{c}^{\mathcal{J}})})$ satisfies φ.*

Formulas $\boldsymbol{c} : \varphi$, corresponding to McCarthy's $ist(\boldsymbol{c}, \varphi)$, have a straightforward reading: φ holds (or is an axiom) in the context denoted by name $\boldsymbol{c}$. A SIS can be also viewed as a collection of ontologies $\{\mathcal{O}_{\boldsymbol{c}}\}_{\boldsymbol{c} \in M_I}$ in $SL_{\mathcal{L}}$, where each $\mathcal{O}_{\boldsymbol{c}} = \{\varphi \mid \boldsymbol{c} : \varphi \in \mathcal{O}\}$ represents the knowledge relevant to one context, related to others by means of operators $\langle \cdot \rangle$, e.g.:

$\mathcal{O}_{\boldsymbol{c}}$:	$Patient \sqsubseteq \exists hasPart.\langle \boldsymbol{d} \rangle HumanHeart$
$\mathcal{O}_{\boldsymbol{d}}$:	$HumanHeart \sqsubseteq Heart$
	$Heart \sqsubseteq Organ$

It is easy to observe, that $SL_{\mathcal{L}}$ can serve as a language for integrating a set of standard DL ontologies $\{\mathcal{O}_{\boldsymbol{c}}\}_{\boldsymbol{c} \in M_I}$ in $\mathcal{L}$, which supports simple, logic-based mappings aligning the semantics of concepts, roles and individual names used in the ontologies:

$$\langle \boldsymbol{c} \rangle C \sqsubseteq \langle \boldsymbol{d} \rangle D \mid \langle \boldsymbol{c} \rangle r \sqsubseteq \langle \boldsymbol{d} \rangle s \mid \langle \boldsymbol{c} \rangle \{a\} \sqsubseteq \langle \boldsymbol{d} \rangle \{b\} \qquad (\ddagger)$$

where $\boldsymbol{c}, \boldsymbol{d} \in M_I$, C, D are concepts of $\mathcal{L}$, $r, s \in N_R$ and $a, b \in N_I$. For instance, two ontologies with partly overlapping information, e.g:

$\mathcal{O}_{\boldsymbol{c}}$:	$Staff \sqsubseteq \exists isEmployed.Company$
	$Staff(J.Smith)$
$\mathcal{O}_{\boldsymbol{d}}$:	$Employee \sqsubseteq \exists employedIn.\top$
	$Employee(JohnSmith)$

might be integrated by means of constraints:

$$\langle \boldsymbol{c} \rangle Staff \equiv \langle \boldsymbol{d} \rangle Employee$$
$$\langle \boldsymbol{c} \rangle isEmployed \sqsubseteq \langle \boldsymbol{d} \rangle employedIn$$
$$\langle \boldsymbol{c} \rangle \{J.Smith\} \equiv \langle \boldsymbol{d} \rangle \{JohnSmith\}$$

Not surprisingly, the language $SL_{\mathcal{L}}$ bares some obvious similarities with other known formalisms for connecting/integrating ontologies, such as $\mathcal{E}$-connections (Kutz et al. 2003), Distributed DLs (DDLs) (Borgida and Serafini 2003) and Package-based DLs (P-DLs) (Bao et al. 2009). In particular, mappings (‡) have exactly the same function as *bridge rules* in DDLs, i.e. lifting information from one context to another. The major difference from the first two approaches is that integration in $SL_{\mathcal{L}}$ is achieved by interpreting the aligned elements of the language directly over the same domain objects, without involving intermediary link relations such as $\mathcal{E}$-connections or directional semantic mappings (DDLs). This renders our integration mechanism in principle stronger. In the case of P-DLs, it is not difficult to show that $SL_{\mathcal{L}}$, although based on a more natural semantics, can be mapped on the corresponding P-DL $\mathcal{LP}$. Analogically to P-DLs, reasoning in $SL_{\mathcal{L}}$ is polynomially reducible to reasoning in $\mathcal{L}$, which guarantees the same worst case complexity.

Theorem 1 *The complexity of reasoning in $SL_{\mathcal{L}}$ is the same as in $\mathcal{L}$.*

The full proof, along others from this paper, can be found in the technical report (Klarman and Gutiérrez-Basulto 2011).

Abstract interoperability systems

The expressive power of $SL_{\mathcal{L}}$ is strongly limited by restricting the representation to a fixed number of contexts. In this section, we dispose of this constraint and permit an unbounded space of context entities, thus shifting towards a full-fetched two-dimensional semantics. This natural generalization stems from the introduction of a quantification mechanism over the context domain, often advocated by the continuators of McCarthy (Guha 1991; Buvač 1996) as a mean of constructing more abstract and generic interoperability constraints. On the philosophical side, this step might be interpreted as a manifestation of the Open World Assumption on the level of contexts (or knowledge sources), which in some open-ended environments such as the Web can be often justified.

An *abstract interoperation language* $AL_{\mathcal{L}}$ consists of a metalanguage, supporting atomic concept assertions and taxonomies of concept names, and an object language, equipped with generalized context operators over concepts. To distinguish between the atoms of the two languages, we use a ***bold font*** for writing the former and a *regular font* for the latter.

Definition 5 ($AL_{\mathcal{L}}$-metalanguage) *The* metalanguage *of $AL_{\mathcal{L}}$ consists of a set M_C of concept names, the top concept $\top$, and a set M_I of individual names. The axioms of the metalanguage are formulas:*

$$\boldsymbol{A} \sqsubseteq \boldsymbol{B} \mid \boldsymbol{A}(\boldsymbol{c})$$

where $\boldsymbol{A}, \boldsymbol{B}$ are concepts and $\boldsymbol{c} \in M_I$.

Definition 6 ($AL_{\mathcal{L}}$-object language) *Let $\mathcal{L}$ be a DL language. Then the* object language *of $AL_{\mathcal{L}}$ is the smallest language containing $\mathcal{L}$, and closed under the constructors of $\mathcal{L}$ and two concept-forming operators:*

$$\langle \boldsymbol{A} \rangle C \mid [\boldsymbol{A}] C$$

where $\boldsymbol{A}$ is a concept of the metalanguage and C a concept of the object language.

Informally, the concept $\langle \boldsymbol{A} \rangle C$ denotes all objects which are C in *some* context of type $\boldsymbol{A}$, whereas $[\boldsymbol{A}]C$ objects which are C in *all* such contexts. For instance, $\langle$***HumanAnatomy***$\rangle Heart$ refers to the concept $Heart$ in some ***HumanAnatomy*** context, which corresponds to McCarthy's: $\exists x(ist(x, Heart(y)) \wedge$ ***HumanAnatomy***$(x))$. The two context operators behave almost as the usual **S5** modalities, in particular preserving the duality $[\boldsymbol{A}] = \neg \langle \boldsymbol{A} \rangle \neg$, with the sole difference that an additional (metalanguage) condition is imposed on the accessed possible worlds.

Further, we define the notion of Abstract Interoperability System (AIS) in $AL_{\mathcal{L}}$.

Definition 7 (Abstract Interoperability System) *An* Abstract Interoperability System *in $AL_{\mathcal{L}}$ is a pair $\mathcal{K} = (\mathcal{C}, \mathcal{O})$, where $\mathcal{C}$ is a set of axioms of the metalanguage and $\mathcal{O}$ is a set of formulas:*

$$\boldsymbol{c} : \varphi \mid \boldsymbol{A} : \varphi$$

where φ is an axiom of the object language in any of the forms (†), $\boldsymbol{c} \in M_I$ and $\boldsymbol{A}$ is a concept of the metalanguage.

A formula $\boldsymbol{A} : \varphi$ states that the axiom φ must hold in all contexts of type $\boldsymbol{A}$. The semantics is given through the corresponding $AL_{\mathcal{L}}$-interpretations and $AL_{\mathcal{L}}$-models.

Definition 8 ($AL_{\mathcal{L}}$-interpretations) *An $AL_{\mathcal{L}}$-interpretation is a tuple $\mathfrak{M} = (\mathfrak{C}, \cdot^{\mathcal{J}}, \Delta, \{\cdot^{\mathcal{I}(i)}\}_{i \in \mathfrak{C}})$ where:*

- $\mathfrak{C}$ *is a non-empty domain of contexts,*
- $\cdot^{\mathcal{J}}$ *is an interpretation function of the metalanguage, which maps* $\boldsymbol{A}^{\mathcal{J}} \subseteq \mathfrak{C}$*, for every* $\boldsymbol{A} \in M_C$*,* $\top^{\mathcal{J}} = \mathfrak{C}$*, and* $\boldsymbol{c}^{\mathcal{J}} \in \mathfrak{C}$*, for every* $\boldsymbol{c} \in M_I$*,*
- $(\Delta, \cdot^{\mathcal{I}(i)})$*, for every* $i \in \mathfrak{C}$*, is an interpretation of the object language, such that for every* $\langle \boldsymbol{A} \rangle C$ *and* $[\boldsymbol{A}]C$*:*
  - $(\langle \boldsymbol{A} \rangle C)^{\mathcal{I}(i)} = \{x \mid \exists j \in \mathfrak{C} : \ j \in \boldsymbol{A}^{\mathcal{J}} \wedge x \in C^{\mathcal{I}(j)}\}$,
  - $([\boldsymbol{A}]C)^{\mathcal{I}(i)} = \{x \mid \forall j \in \mathfrak{C} : \ j \in \boldsymbol{A}^{\mathcal{J}} \rightarrow x \in C^{\mathcal{I}(j)}\}$.

Definition 9 ($AL_{\mathcal{L}}$-models) *An* $AL_{\mathcal{L}}$*-interpretation* $\mathfrak{M} = (\mathfrak{C}, \cdot^{\mathcal{J}}, \Delta, \{\cdot^{\mathcal{I}(i)}\}_{i \in \mathfrak{C}})$ *is a* model *of an AIS* $\mathcal{K} = (\mathcal{C}, \mathcal{O})$ iff*:*

- *for every* $\boldsymbol{A}(\boldsymbol{c}) \in \mathcal{C}$*,* $\boldsymbol{c}^{\mathcal{J}} \in \boldsymbol{A}^{\mathcal{J}}$*,*
- *for every* $\boldsymbol{A} \sqsubseteq \boldsymbol{B} \in \mathcal{C}$*,* $\boldsymbol{A}^{\mathcal{J}} \subseteq \boldsymbol{B}^{\mathcal{J}}$*,*
- *for every* $\boldsymbol{c} : \varphi \in \mathcal{O}$*,* $(\Delta, \cdot^{\mathcal{I}(\boldsymbol{c}^{\mathcal{J}})})$ *satisfies* φ*,*
- *for every* $\boldsymbol{A} : \varphi \in \mathcal{O}$ *and* $i \in \mathfrak{C}$*, if* $i \in \boldsymbol{A}^{\mathcal{J}}$ *then* $(\Delta, \cdot^{\mathcal{I}(i)})$ *satisfies* φ*.*

Application scenarios

Similarly to $SL_{\mathcal{L}}$, $AL_{\mathcal{L}}$ can be used both as a native language for constructing contextualized knowledge bases or as an external layer for imposing generic interoperability constraints over standard DL representations. However, unlike in $SL_{\mathcal{L}}$, the context operators in $AL_{\mathcal{L}}$ govern the semantic interoperation not only among a fixed number of explicitly introduced contexts, but rather within an entire space of possible contexts — some of which might be only logically entailed. Hence, the operators $\langle \cdot \rangle$ and $[\cdot]$ serve an analogical purpose to $\exists$ and $\forall$ in the object dimension: they restrict the set of possible (two-dimensional) models only to those in which certain entities — here contexts with specific object knowledge — are present. In the following paragraphs, we present a few sample applications of $AL_{\mathcal{L}}$.

Contextualized knowledge base. We model a piece of information presented on the disambiguation website of Wikipedia on querying for the term $Ring$[1].

$\boldsymbol{disambiguation} : Ring \sqsubseteq \langle \boldsymbol{Math} \rangle Ring \sqcup \langle \boldsymbol{People} \rangle Ring$
$\boldsymbol{Math} : Ring \sqsubseteq AlgebStruct \sqcup \langle \boldsymbol{Geometry} \rangle Annulus$
$\boldsymbol{People} : Ring \sqsubseteq \{nickRing\}$

Observe, that the named context ***disambiguation*** provides basic distinction on $Ring$ in some ***Math*** context and in some ***People*** context. This is further enhanced, by the distinction defined on the level of all ***Math*** contexts. There, $Ring$ denotes either $AlgebStruct$ or, in some further ***Geometry*** context, $Annulus$. In case of ***People*** context, $Ring$ actually denotes an individual $nickRing$.

Interoperation constraints for ontology alignment and reuse. Consider an infrastructure such as the NCBO BioPortal project[2], which gathers numerous published biohealth ontologies, and categorizes them via simple thematic tags ***Cell***, ***Health***, ***Anatomy***, etc., organized in a simple concept hierarchy. The intention of the project is to facilitate the reuse of the collected resources in new applications. We assume that each ontology name is interpreted as a distinct context name in $AL_{\mathcal{L}}$. Note, that the division between the metalanguage and the object language is already present in the architecture of the BioPortal, which can be immediately utilized, for example to state:

$\top : \langle \boldsymbol{HumanAnat.} \rangle Heart \sqsubseteq [\boldsymbol{Anat.}] HumanHeart$	(1)
$\boldsymbol{Anatomy} : Heart \sqsubseteq Organ$	(2)
$\boldsymbol{HumanAnatomy} \sqsubseteq \boldsymbol{Anatomy}$	(3)

where (1) fixes the translation from $Heart$ to $HumanHeart$ (cf. Introduction); (2) imposes an axiom $Heart \sqsubseteq Organ$ of an upper anatomy ontology over all ontologies tagged with ***Anatomy***, which due to the metalanguage taxonomy (3) carries over to all ontologies tagged with ***HumanAnatomy***.

More generally, $AL_{\mathcal{L}}$ provides logic-based explications of some interesting notions, relevant to the problem of ontology alignment and reuse, such as:

concept alignment: $\top : \langle \boldsymbol{A} \rangle C \sqsubseteq [\boldsymbol{B}] D$
every instance of C in any ontology of type $\boldsymbol{A}$ is D in every ontology of type $\boldsymbol{B}$

semantic importing: $\boldsymbol{c} : \langle \boldsymbol{A} \rangle C \sqsubseteq D$
every instance of C in any ontology of type $\boldsymbol{A}$ is D in ontology $\boldsymbol{c}$

upper ontology axiom: $\boldsymbol{A} : C \sqsubseteq D$
axiom $C \sqsubseteq D$ holds in every ontology of type $\boldsymbol{A}$

Ontology versioning management and change analysis. The context operators can be also interpreted as change operators, in the style of DL of Change (Artale, Lutz, and Toman 2007), for instance, for representing and studying dynamic aspects of ontology versioning, especially when evolutionary constraints apply to a whole collection of semantically interoperable ontologies. Some central issues arising in this setup are integrity (constraining the scope of changes allowed due to versioning), evolvability (ability of coordinating the evolution of ontologies) and formal analysis of differences between the versions (Huang and Stuckenschmidt 2005). In the examples below, we assume that contexts represent possible versions, while each metalanguage concept refers to all versions of the same ontology.

version-invariant concepts: $\top : \langle \boldsymbol{A} \rangle C \equiv [\boldsymbol{A}] C$
C is a version-invariant concept within the scope of versions of type $\boldsymbol{A}$

dynamic analysis: $\top : \langle \boldsymbol{A} \rangle C \sqcap \langle \boldsymbol{A} \rangle \neg C \sqsubseteq C^{\star}$
$C^{\star}$ retrieves all instances which are C in some versions of type $\boldsymbol{A}$ and $\neg C$ in some others

evolvability constraints: $\top : \langle \boldsymbol{A} \rangle C \sqsubseteq \langle \boldsymbol{B} \rangle D$
every instance of C in a version of type $\boldsymbol{A}$ has to evolve into D in some version of type $\boldsymbol{B}$

Complexity and expressiveness

Interestingly, reasoning in $AL_{\mathcal{L}}$ is not significantly harder than reasoning in the underlying DLs.

Theorem 2 *The complexity of reasoning in* $AL_{\mathcal{L}}$ *ranges as in Table 2.*

[1] See http://en.wikipedia.org/wiki/Ring.
[2] See http://bioportal.bioontology.org/.

$\mathcal{L}$	
$\mathcal{EL}$	PTIME
$\mathcal{ALC}$	EXPTIME-complete
$\mathcal{ALCO}$	NEXPTIME-complete

Table 2: Complexity of reasoning in $AL_{\mathcal{L}}$.

Only in the case of $\mathcal{L} = \mathcal{ALCO}$ we encounter a jump from EXPTIME to NEXPTIME-completeness. The interaction of nominals with the context operators enables encoding of the usual 2^n-tiling problem, known to be NEXPTIME-complete (Kurucz et al. 2003). The result holds already when the metalanguage is trivialized by setting $M_C = M_I = \emptyset$.

As the next results show, $AL_{\mathcal{L}}$ is closely related to the product-like combination of DL with modal logic $\mathbf{S5}$ — a formalism well-studied in the literature (Kurucz et al. 2003), also used as the foundation for the DL of Change (Artale, Lutz, and Toman 2007) and connected to the Probabilistic DL (Lutz and Schröder 2010).

Theorem 3 *If $M_C = M_I = \emptyset$ and only TBox axioms are allowed, then $AL_{\mathcal{L}}$ is a notational variant of $\mathbf{S5}_{\mathcal{L}}$ with global TBoxes.*

Proof. Observe, that only axioms $\top : C \sqsubseteq D$ are allowed, for arbitrary concepts C, D in $AL_{\mathcal{L}}$. Replace every $\langle \top \rangle$ with $\Diamond$, every $[\top]$ with $\Box$ and every $\top : C \sqsubseteq D \in \mathcal{O}$ with $C \sqsubseteq D$. It is easy to see that the semantics of $AL_{\mathcal{L}}$ coincides with that of $\mathbf{S5}_{\mathcal{L}}$. Note, that a TBox is considered global *iff* its every axiom is satisfied in all possible $\mathbf{S5}$-worlds. ❑

Theorem 4 *If $M_I = \emptyset$ and only TBox axioms are allowed, then reasoning in $AL_{\mathcal{L}}$ is polynomially reducible to reasoning in $\mathbf{S5}_{\mathcal{L}}$ with global TBoxes and concepts from M_C interpreted globally.*

Proof. First note, that a concept C is interpreted globally *iff* for every possible $\mathbf{S5}$-world w, $C^{\mathcal{I}(w)} = \Delta$ or $C^{\mathcal{I}(w)} = \emptyset$. Observe also, that only axioms $\boldsymbol{A} : C \sqsubseteq D$ are allowed, for $\boldsymbol{A} \in M_C$ and arbitrary concepts C, D in $AL_{\mathcal{L}}$. Translate every occurrence $\langle \boldsymbol{A} \rangle C$ to $\Diamond(A \sqcap C)$, every $[\boldsymbol{A}]D$ to $\Box(\neg A \sqcup C)$ and every $\boldsymbol{A} : C \sqsubseteq D \in \mathcal{O}$ to $A \sqcap C \sqsubseteq D$. Clearly, the resulting set of formulas is satisfiable in $\mathbf{S5}_{\mathcal{L}}$ *iff* the original one was in $AL_{\mathcal{L}}$. ❑

The corresponding $\mathbf{S5}_{\mathcal{L}}$ logics are obviously not full $\mathbf{S5} \times \mathcal{L}$ products, as we deliberately do not allow the roles of $\mathcal{L}$ to be interpreted rigidly across the context dimension, i.e. such that $r^{\mathcal{I}(c)} = r^{\mathcal{I}(d)}$ for every pair $c, d \in \mathfrak{C}$. Hence, in the landscape of combinations of modal logics (Kurucz et al. 2003), $AL_{\mathcal{L}}$ classifies as an 'approximation' of modal products, i.e. a combination considerably more expressive than fusion of logics, but weaker from those based on full product semantics. We also do not consider here context operators over roles, which allow for emulating such behavior. As it turns out, adding constructs $\langle \boldsymbol{A} \rangle r, [\boldsymbol{A}]r$ to $AL_{\mathcal{L}}$, with the expected semantics, immediately rises the lower complexity bounds to PSPACE-hard for $\mathcal{L} = \mathcal{EL}$ and 2EXPTIME-hard for $\mathcal{L} = \{\mathcal{ALC}, \mathcal{ALCO}\}$, which follows by immediate reductions from the corresponding variants of $\mathbf{S5}_{\mathcal{L}}$ with modalized roles in (Lutz and Schröder 2010; Artale, Lutz, and Toman 2007).

A formalism similar in the spirit to $AL_{\mathcal{L}}$, both in the formal design and in the underlying motivation, has been studied in (Klarman and Gutiérrez-Basulto 2010) as a Context DL $\mathcal{ALC}_{\mathcal{ALC}}$. There, however, the combination of DLs with the context operators is based on $(\mathbf{K_n})_{\mathcal{L}}$-frames, rather than $\mathbf{S5}_{\mathcal{L}}$. Consequently, $\mathcal{ALC}_{\mathcal{ALC}}$ seems less suitable for applications dealing with semantic interoperability between loosely coexisting DL representations, which are more natural to represent as possible worlds in a universal frame. Moreover, $(\mathbf{K_n})_{\mathcal{L}}$ exhibits much worse computational behavior, with 2EXPTIME-complete satisfiability problem already for $\mathcal{L} = \mathcal{ALC}$ with no rigid roles, and undecidable when rigid (or modalized) roles are included (Klarman and Gutiérrez-Basulto 2010).

Expressive metalanguages

For many applications, particularly relevant for the Semantic Web, a practical metalanguage for describing knowledge sources requires not only concept tags but also properties, e.g. for describing the provenance (authorship, date, place, relationships to other sources, etc.) (Bao et al. 2010). A natural way to support such requirements in the presented setting is to employ a standard DL in the role of the metalanguage.

Definition 10 (**$AL_{\mathcal{L}}^{\mathcal{M}}$-metalanguage**) *The* metalanguage *of $AL_{\mathcal{L}}^{\mathcal{M}}$ is a DL language $\mathcal{M}$ based on vocabulary $\Gamma = (M_C, M_R, M_I^\star)$, where M_C is a set of concept names, M_R a set of role names and $M_I^\star$ a set of individual names, with a designated subset $M_I \subseteq M_I^\star$. Axioms of the metalanguage are formulas of the form (†).*

Observe, that the context names M_I are here only a subset of all individual names $M_I^\star$ which might be used in context descriptions. Further, we also allow possibly complex concepts $\boldsymbol{C}$ of $\mathcal{M}$ inside the operators $\langle \boldsymbol{C} \rangle D$, $[\boldsymbol{C}]D$ and axioms $\boldsymbol{C} : \varphi$. Presence of roles in the metalanguage allows for effective reasoning with such information as:

$$\frac{\textit{hasAuthor}(\textit{anatomy\_ont}, \textit{johnSmith})}{\exists \textit{maintainedBy}.\textit{University}(\textit{anatomy\_ont})}$$

where ***anatomy_ont*** $\in M_I$, ***johnSmith*** $\in M_I^\star$, ***hasAuthor***, ***maintainedBy*** $\in M_R$, ***University*** $\in M_C$.

To accommodate the interpretation of $\mathcal{M}$ in the semantics, without damaging its original architecture, we pose a new domain of the metalanguage Θ, with the set of context domain being a subset of it, and extend the interpretation function accordingly.

Definition 11 (**$AL_{\mathcal{L}}^{\mathcal{M}}$-interpretations**) *An $AL_{\mathcal{L}}^{\mathcal{M}}$-interpretation is a tuple $\mathfrak{M} = (\Theta, \mathfrak{C}, \cdot^{\mathcal{J}}, \Delta, \{\cdot^{\mathcal{I}(c)}\}_{c \in \mathfrak{C}})$, where:*

- *Θ is a non-empty metalanguage domain,*
- *$\mathfrak{C} \subseteq \Theta$ is a non-empty context domain,*
- *$\cdot^{\mathcal{J}}$ is an interpretation function which maps $\boldsymbol{A}^{\mathcal{J}} \subseteq \Theta$, for every $\boldsymbol{A} \in M_C$, $\boldsymbol{r}^{\mathcal{J}} \subseteq \Theta \times \Theta$, for every $\boldsymbol{r} \in M_R$, $\boldsymbol{c}^{\mathcal{J}} \in \Theta$, for every $\boldsymbol{c} \in M_I^\star$, with $\boldsymbol{c}^{\mathcal{J}} \in \mathfrak{C}$, whenever $\boldsymbol{c} \in M_I$,*
- *$(\Delta, \cdot^{\mathcal{I}(i)})$ as in Definition 8.*

$\mathcal{L}$ \ $\mathcal{M}$	$\mathcal{EL}$	$\mathcal{ALC}, \mathcal{ALCO}$
$\mathcal{EL}$	as in $AL_{\mathcal{L}}$	ExpTime-hard
$\mathcal{ALC}$	as in $AL_{\mathcal{L}}$	NExpTime-complete
$\mathcal{ALCO}$	as in $AL_{\mathcal{L}}$	NExpTime-complete

Table 3: Complexity of reasoning in $\mathrm{AL}_{\mathcal{L}}^{\mathcal{M}}$.

The notions of AIS and $AL_{\mathcal{L}}^{\mathcal{M}}$-model remain exactly the same as in the case of $AL_{\mathcal{L}}^{\mathcal{M}}$ (Definition 7 and 9).

It turns out, that a shift from simple taxonomies to much more convenient $\mathcal{EL}$, as the metalanguage of AISs, does not entail a further increase in the complexity, which remains the same as in the corresponding $AL_{\mathcal{L}}$. Pushing the metalanguage envelope, however, has its limits. The use of $\mathcal{ALC}$ and $\mathcal{ALCO}$ in the same role, noticeably affects the complexity. The ExpTime-hardness for $\mathcal{L} = \mathcal{EL}$, transfers directly from the lower bound of the involved metalanguages. The non-determinism involved in the other two cases can be interpreted by the need of guessing the interpretation of the metalanguage first, before finding the model of the object component of the combination.

Theorem 5 *The complexity of reasoning in $AL_{\mathcal{L}}^{\mathcal{M}}$ ranges as in Table 3.*

The lower bound of $AL_{\mathcal{ALC}}^{\mathcal{ALC}}$ is again obtained by an encoding of the $2^n \times 2^n$ tiling problem. For the upper bounds for $\mathcal{L} \in \{\mathcal{ALC}, \mathcal{ALCO}\}$ we devise a variant of a type elimination algorithm, whereas for $\mathcal{L} = \mathcal{EL}$ a completion algorithm in the style of (Baader, Brandt, and Lutz 2005). In most cases the results are robust enough to allow generalizations to more expressive DLs (Klarman and Gutiérrez-Basulto 2011).

Conclusions

The problems of 1) representing inherently contextualized knowledge within the paradigm of DLs and 2) reasoning with multiple heterogenous, but semantically interoperating, DL representations, are both interesting and important issues, motivated by numerous practical application scenarios. It is our belief that these two challenges are in fact two sides of the same coin and, consequently, they should be approached within the same, unifying formal framework. In this paper, we have argued that two-dimensional DLs incorporating the principles of McCarthy's theory of contexts achieve this objective to a great extent, by providing sufficient syntactic and semantic means to support both functionalities. As our results show, such an extension of the standard DLs does not necessarily entail an increase in the computational complexity of reasoning, nor does it affect the generally adopted knowledge representation methodology of DLs. We therefore consider the approach a worthwhile subject to further research. In particular, we intend to investigate how certain basic notions, which are essential for practical use and maintenance of multi-context knowledge systems (e.g. inconsistency handling), can be meaningfully restated within the presented framework.

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References

Artale, A.; Lutz, C.; and Toman, D. 2007. A description logic of change. In *Proc. of IJCAI-07*, 218–223.

Baader, F.; Calvanese, D.; Mcguinness, D. L.; Nardi, D.; and Patel-Schneider, P. F. 2003. *The description logic handbook: theory, implementation, and applications*. Cambridge University Press.

Baader, F.; Brandt, S.; and Lutz, C. 2005. Pushing the $\mathcal{EL}$ envelope. In *Proc. of IJCAI-05*.

Bao, J.; Voutsadakis, G.; Slutzki, G.; and Honavar, V. 2009. Modular ontologies. chapter Package-Based Description Logics, 349–371.

Bao, J.; Tao, J.; McGuinness, D. L.; and Smart, P. 2010. Context representation for the semantic web. *Proc. of Web Science Conference*.

Borgida, A., and Serafini, L. 2003. Distributed description logics: Assimilating information from peer sources. *Journal of Data Semantics* 1:2003.

Buvač, S., and Mason, I. A. 1993. Propositional logic of context. In *Proc. of AAAI-93*.

Buvač, S. 1996. Quantificational logic of context. In *Proc. of AAAI-96*.

Guha, R.; McCool, R.; and Fikes, R. 2004. Contexts for the semantic web. In *Proc. of ISWC-04*, 32–46.

Guha, R. 1991. *Contexts: a formalization and some applications*. Ph.D. Dissertation, Stanford University.

Huang, Z., and Stuckenschmidt, H. 2005. Reasoning with multi-version ontologies: A temporal logic approach. In *Proc. of ISWC-05*. 398–412.

Klarman, S., and Gutiérrez-Basulto, V. 2010. $\mathcal{ALC}_{\mathcal{ALC}}$: A context description logic. In *Proc. of JELIA-10*.

Klarman, S., and Gutiérrez-Basulto, V. 2011. Two-dimensional description logics for context-based semantic interoperability. Technical report, http://klarman.synthasite.com/resources/KlaGutAAAI11.pdf.

Kurucz, A.; Wolter, F.; Zakharyaschev, M.; and Gabbay, D. M. 2003. *Many-Dimensional Modal Logics: Theory and Applications*. Elsevier.

Kutz, O.; Lutz, C.; Wolter, F.; and Zakharyaschev, M. 2003. $\mathcal{E}$-connections of description logics. In *Proc. of DL-03*.

Lutz, C., and Schröder, L. 2010. Probabilistic description logics for subjective uncertainty. In *Proc. of KR-10*.

Lutz, C.; Wolter, F.; and Zakharyaschev, M. 2008. Temporal description logics: A survey. In *Proc. of TIME-08*.

McCarthy, J. 1987. Generality in artificial intelligence. *Communications of the ACM* 30:1030–1035.

Nossum, R. 2003. A decidable multi-modal logic of context. *Journal of Applied Logic* 1(1-2):119 – 133.

Wolter, F., and Zakharyaschev, M. 1999. Multi-dimensional description logics. In *Proc. of IJCAI-99*, 104–109.