

Congestion Games for V2G-Enabled EV Charging

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Abstract

A model of the problem of charging and discharging electrical vehicles as a congestion game is presented. A generalization of congestion games – feedback congestion games (FCG) – is introduced. The charging of grid-integrated vehicles, which can also discharge energy back to the grid, is a natural FCG application. FCGs are proven to be exact potential games and therefore converge to a pure-strategy Nash equilibrium by an iterated better-response process. A compact representation and an algorithm that enable efficient best-response search are presented. A detailed empirical evaluation assesses the performance of the iterated best-response process. The evaluation considers the quality of the resulting solutions and the rate of convergence to a stable state. The effect of allowing to also discharge batteries using FCG is compared to scenarios that only include charging and is found to dramatically improve the predictability of the achieved solutions as well as the balancing of load.

1 Introduction

Electric Vehicles (EVs) are an important part of the transition plan to a low carbon economy. New designs, such as plug-in hybrid vehicles and range-extended electric vehicles, are part of the expected future automotive DNA (Mitchell, Borroni-Bird, and Burns 2010). EVs need to be charged daily. When parked during office hours, EVs are expected to charge in a well-balanced pattern in order to avoid overloading the smart grid (Gerding et al. 2011; Vandael et al. 2011). EVs are expected to be parked a large fraction of the working day and may be able to charge part of the time and be used as storage (Kamboj et al. 2010). Consequently, it has been proposed that EVs could sell part of the energy stored in their batteries back to the grid. This concept is termed Vehicle-to-Grid (V2G). In the following, the term Grid-Integrated Vehicles (GIVs) will be used to describe EVs that support V2G sessions. Such vehicles may be used to balance the load on the grid by charging when demand is low and selling power back to the grid (discharging) when demand is high (Kempton and Tomić 2005b).

The present paper models the problem of charging (and discharging) EVs as a congestion game (CG) (Rosenthal

1973). A Congestion game consists of players and resources. The cost of each resource depends on the number of players that choose to use it. The overall cost of each player is simply the sum of the costs of all the resources that the player selects to use. The connection to the charging of EVs is clear – each player is an agent representing a single EV and the resources are the time-slots in which the agents are interested in charging their batteries.

Congestion games are closely related to another important class of games – potential games (Monderer and Shapley 1996). Particularly, Monderer and Shapley proved that every congestion game is an *exact* potential game. In a potential game there exists a global function (the potential function) that coincides with the incentives of all the players. More precisely, the set of pure-strategy Nash equilibria (PNE) in a potential game is equivalent to the local minima of the potential function. Potential games, inherently including congestion games, become interesting when the potential function has some desirable global meaning. In such games the actions of strategic, non-cooperative, players leads to a desirable global outcome.

The increasing popularity of GIVs introduces new opportunities to the EV charging/discharging domain. A GIV parked for long periods of time could sell power back at peak hours. Moreover, a fleet of cars, with heterogeneous parking times, could balance its charging loads and avoid charging at expensive peak hours. Consequently, GIVs create the need for a new class of games that enable both charging and discharging of the EVs batteries. The original version of congestion games falls short of describing the desired class of games, since Rosenthal only considered situations in which players consume resources (Rosenthal 1973). Here, players may also free up resources by discharging their batteries during some time-slots. To deal with this situation, a generalization of the congestion game model, that is termed here *feedback congestion games*, is introduced. The proposed generalization is shown formally to still satisfy the same connection to potential games as the original congestion games. More precisely, it is proven in Section 3 that every feedback congestion game is an exact potential game.

This is not the first time that a real-world problem motivates a generalization of congestion games. Liu, Ahmad, and Wu (2009) define congestion games with resource reuse (CG-RR), which include an interference set for each player.

Consequently, the cost of each user is a function of the number of interfering players. By using the CG-RR generalization, the authors were able to model the problem of resource competition in wireless communication.

A closely related research (Ibars, Navarro, and Giupponi 2010) models a distributed demand-side management system using traditional congestion games that do not allow selling power back to the grid. This research is similar to our proposed method in its applicability to the smart grid. However, distributed demand side management is mainly considered for residual areas; in such areas, by using our proposed generalization one can take advantages of micro-storage devices (Vytelingum et al. 2010; Voice et al. 2011).

Several game-theoretic approaches that rely on some central authority were recently proposed for the EV charging domain. In the vehicle-to-aggregator interaction game (Wu, Mohsenian-Rad, and Huang 2012) the aggregator controls the prices for the nearest time-slot in a V2G setting in a manner that enables achieving an optimal outcome for the grid in a distributed fashion. A different approach uses iterative Boolean games to solve a simpler version of the charging problem (Levit, Grinshpoun, and Meisels 2013). There, a principal manipulates the players into reaching a PNE in a dichotomous manner, i.e., without involving prices and discharging. Although these approaches are decentralized, they heavily rely on the involvement of a central entity, which is not the case in the proposed method of the present paper.

An empirical evaluation of the performance of congestion games and their respective feedback congestion games is presented. That is, the effect of allowing to also discharge batteries is compared to situations which include charging only scenarios. Both alternatives are also compared to a naïve approach in which each GIV starts charging at the moment it is connected to the grid. The evaluation considers the quality of the resulting solutions and the number of rounds until convergence. The experimentation of large problems was possible by using a compact representation and a novel algorithm that enable efficient best-response search.

The plan of the paper is as follows. The GIV charging problem is introduced in Section 2. Potential and congestion games, as well as the feedback congestion games generalization, are formally described in Section 3. The representation of the GIV charging problem as a feedback congestion game is presented in Section 4. A compact representation of the problem and an algorithm for finding the best response are introduced in Section 5. An extensive empirical evaluation of the proposed games is in Section 6. Section 7 outlines our conclusions and future work directions.

2 The GIV Charging Problem

Electric vehicles received a lot of attention in the recent years. Generally, EVs are associated with their positive effects over the environment and especially low carbon emissions and noise reduction (Kemp et al. 2010). However, their widespread use is also expected to place considerable strains on existing electricity distribution networks. Moreover, many EVs are expected to be charged during the same time phase (between the times that the majority of the population is driving to work and the time they are driving

back home, for example). This pattern may lead to large peaks, such that will have to be tackled by extending the grid infrastructure which in turn will reduce or even dismiss the positive effects on the environment (Stein et al. 2012; Sovacool and Hirsh 2009).

One solution for the EVs charging problem is to try to schedule the charging of EVs in a way that will reduce the peaks and balance the load. This scheduling however will have to take into consideration the fact that different consumers (EVs) may have different time constraints and willingness to pay. Grid-Integrated Vehicles are a special kind of EVs that support Vehicle-to-Grid sessions. In a V2G session a vehicle may sell power, stored in its battery, back into the grid (Kempton and Letendre 1997; Kempton and Tomić 2005a). Since most vehicles are parked over 90% of the time (Kamboj et al. 2010), some GIVs that have rather loose time constraints can sell energy stored in their battery back to the grid and in this way help to serve the charging needs of other, more tightly time-constrained, GIVs. Doing so in a smart way can be beneficial both to the GIVs owners and to the electrical grid operators. The GIV owners are being paid for helping distribute the load; this payment can then reduce the cost of the GIV charge.

Formally, the *GIV charging problem* takes the form of the tuple $\langle V, T, \{l_t\}_{t \in T}, \{S_v\}_{v \in V} \rangle$, where $V = \{1, 2, \dots, n\}$ is a set of vehicles (GIVs) and T is a set of time-slots. For each time-slot $t \in T$ one defines l_t to be the initial load on the power grid that exists as background to the problem (e.g., by residential homes or industry). For each $v \in V$, $S_v \subseteq \{\text{charge, do-nothing, discharge}\}^{|T|}$ is a set of assignments of actions (a strategy) for the different time-slots. Each assignment $s_v \in S_v$ encodes a valid combination of time-slots during which the GIV is available for charge/discharge and that coincides with its owners preferences. Given this input, the goal is to find a schedule (or a strategy profile) $S = \{s_1, s_2, \dots, s_n\}$, such that it balances the loads inflicted by the charging operations combined with the initial background load $\{l_t\}_{t \in T}$.

3 Potential and Congestion Games

The class of potential games is characterized as games that admit a potential function on the joint strategy space, such that the gradient of the potential function is the gradient of the constituents' private utility function (Monderer and Shapley 1996). A potential function has a natural interpretation as representing opportunities for improvement to a player that deviates from any given strategy profile (Chapman, Rogers, and Jennings 2008). A potential game with $I = \{1, 2, \dots, n\}$ players and a set of the available strategies for these players $\{S_i\}_{i \in I}$ has several unique properties.

1. The game has at least one PNE.
2. The local optima of the potential function are PNEs of the game.
3. Given a strategy profile $S = \{s_1, s_2, \dots, s_n\}$ which is a selection of strategies for each player in the game, an improvement step of player i is a change of its strategy from s_i to s'_i , such that the utility $u_i : S_i \rightarrow \mathbb{R}$ of

player i increases. In potential games, sequences of improvement steps do not run into cycles. Such sequences of improvement steps reach a PNE after a finite number of steps (Monderer and Shapley 1996). This is sometimes termed an *iterated better-response process* or the *finite improvement property*.

Definition 1 (exact potential game). A game is an exact potential game if there exists a function $\Phi : S \rightarrow \mathbb{R}$ such that for each player i and for any two strategies $s_i, s'_i \in S_i$ the following holds

$$u_i(s_i, s_{-i}) - u_i(s'_i, s_{-i}) = \Phi(s_i, s_{-i}) - \Phi(s'_i, s_{-i}) \quad (1)$$

where $s_{-i} = S \setminus \{s_i\}$ denotes the set of the selected strategies of every player except i .

The class of congestion games models scenarios in which players use congestible resources (Rosenthal 1973). The congestion level of resources is a function of the number of players that use them. Our definition of the classical congestion game as given below, is slightly different yet equivalent to the definition of Rosenthal (1973).

Definition 2 (congestion game). A congestion game is a tuple $\langle I, T, \{S_i\}_{i \in I}, \{c_t\}_{t \in T} \rangle$, where $I = \{1, 2, \dots, n\}$ is a set of players, T is a set of congestible resources, $S_i \subseteq 2^T$ is the strategy space of player i , and $c_t : \mathbb{N} \rightarrow \mathbb{R}$ is a cost function associated with resource $t \in T$ (Rosenthal 1973). The utility of a player for selecting a strategy s_i is assumed to be proportional to

$$u_i(s_i, s_{-i}) = -1 \cdot \sum_{t \in T} s_i[t] \cdot c_t(d_t + 1) \quad (2)$$

where $s_i[t]$ is 1 if the player consumes resource t when applying strategy s_i and 0 otherwise. d_t denotes the congestion over resource t as can be deduced from s_{-i} , and formally $d_t = \sum_{s \in s_{-i}} s[t]$.

It was proven that every congestion game in an exact potential game (Rosenthal 1973), since the following potential function always holds:

$$\Phi(S) = -1 \cdot \sum_{t \in T} \sum_{x=1}^{d_t} c_t(x) \quad (3)$$

Feedback Congestion Games (FCGs)

Let us consider an extended definition of the classical congestion game (as described by Rosenthal). The extension is termed *feedback congestion game* (FCG) and is defined to be a game similar to the classical congestion game with the exception that the players play the role of both producer and consumer. This means that each player is able to produce some resources and consume other resources.

A clear motivation for feedback congestion games is that they naturally model the GIV charging problem; an EV can choose to charge at one time-slot and to discharge at another. It can do so in order to reduce the total cost of its charging session and as a side effect it can also help balance the overall load.

Definition 3 (Feedback Congestion Game). A feedback congestion game is a tuple $\langle I, T, \{S_i\}_{i \in I}, \{c_t\}_{t \in T} \rangle$, where $I = \{1, 2, \dots, n\}$ is a set of producer/consumer players (henceforth termed agents); T is a set of congestible resources; each agent $i \in I$ has a set of strategies $S_i \subseteq \{-1, 0, 1\}^{|T|}$, each strategy $s_i \in S_i$ is an assignment of resources usage – 0 means no use, 1 means consume, and -1 means produce; and $c_t : \mathbb{N} \rightarrow \mathbb{R}$ is a cost function associated with resource $t \in T$. The utility agent i has for selecting strategy s_i is assumed to be proportional to

$$u_i(s_i, s_{-i}) = -1 \cdot \sum_{t \in T} s_i[t] \cdot c_t(d_t + \frac{s_i[t] + 1}{2}) \quad (4)$$

Theorem 1. A feedback congestion game is an exact potential game.

Proof. In order to show that a feedback congestion game is an exact potential game one needs to provide a potential function $\Phi : S \rightarrow \mathbb{R}$ that satisfies the condition of Equation 1. We will show that the potential function

$$\Phi(s_i, s_{-i}) = -1 \cdot \sum_{t \in T} \sum_{x=1}^{d_t + s_i[t]} c_t(x) \quad (5)$$

achieves this objective.

Consider an agent $i \in I$ and two arbitrary strategies $s_i, s'_i \in S_i$. In order to prove that Equation 5 is an exact potential function one must show that

$$\begin{aligned} \forall t \in T, s_i[t] \cdot c_t(d_t + \frac{s_i[t] + 1}{2}) - s'_i[t] \cdot c_t(d_t + \frac{s'_i[t] + 1}{2}) \\ = \sum_{x=1}^{d_t + s_i[t]} c_t(x) - \sum_{x=1}^{d_t + s'_i[t]} c_t(x) \end{aligned} \quad (6)$$

In order to prove the correctness of Equation 6 one must consider all possible cases. The same outcome results when switching between the values of $s_i[t]$ and $s'_i[t]$ (only the sign may flip). In what follows we use the term *without loss of generality (w.l.o.g.)* to refer to such cases.

Case 1. $s_i[t] = s'_i[t]$. This is the trivial case, since both sides of the equation are identically 0.

Case 2. w.l.o.g., $s_i[t] = 1$, $s'_i[t] = -1$. Inserting these values into Equation 6 results in the expression:

$$c_t(d_t + 1) + c_t(d_t) = \sum_{x=1}^{d_t + 1} c_t(x) - \sum_{x=1}^{d_t - 1} c_t(x)$$

This equality holds because it is an identity. It simply uses the elimination of similar elements from the right-hand side of the equation, resulting in its left-hand side.

Case 3. w.l.o.g., $s_i[t] = 0$, $s'_i[t] = -1$. Inserting these values into both sides of Equation 6 simplifies it to:

$$c_t(d_t) = \sum_{x=1}^{d_t} c_t(x) - \sum_{x=1}^{d_t - 1} c_t(x)$$

The equality holds with the same justification as in Case 2.

Case 4. w.l.o.g., $s_i[t] = 0$, $s'_i[t] = 1$. Inserting these values into both sides of Equation 6 simplifies it to:

$$-c_t(d_t + 1) = \sum_{x=1}^{d_t} c_t(x) - \sum_{x=1}^{d_t+1} c_t(x)$$

Again, the resulting equality is trivially true for the same reason as in the previous cases.

The fact that Equation 6 holds proves that Equation 5 is indeed an exact potential function. This proves that every feedback congestion game is an exact potential game. \square

4 Modeling GIV Charging as FCG

Modeling the GIV charging problem as an FCG is straightforward. Let $\langle V, T, \{l_t\}, \{S_v\} \rangle$ be an instance of the GIV charging problem. Every vehicle $v \in V$ can be represented as an agent $i \in I$. The set of resources T in the FCG is the set of time-slots. Finally, the set of available strategies $\{S_i\}$ represents the set of available GIV actions $\{S_v\}$. In order to support the initial background load one can add several “pseudo-agents”, each with a single strategy, so that together they will impose the congestion defined in $\{l_t\}_{t \in T}$. This modeling has several advantages:

1. Distributed iterated better-response playing is guaranteed to converge to a PNE.
2. In order to compute its utility, an agent only needs to know the congestion over the time-slots. The agent does not need to know any additional information about any other agent. This results in a compact representation of the game and preservation of the privacy of agents.
3. Since each turn in the better-response process improves the value of the potential function, one can execute this process as a distributed *anytime* hill-climbing algorithm.

Given the above transformation, one can design appropriate pricing schemes. Pricing schemes are designed to achieve global objectives which are inherent to issues of demand side management and the smart grid. Important examples are load balancing and peak reduction. The relevant pricing scheme for achieving load balancing is based on Shannon’s entropy (Shannon 1948). For achieving peak reduction, one can use a lexicographic-order pricing scheme. More details on these pricing schemes are excluded due to page limitation.

5 Representation and Runtime

In formal formats for specifying a game (Neumann and Morgenstern 1944; Kuhn, Arrow, and Tucker 1953) utility functions are represented explicitly by listing the values for each agent and for each combination of actions. The number of utility values that must be specified (i.e., the number of possible combinations of actions) is exponential in the number of players. The actions available to the agents can be represented by a set of variables and their respective domains in an Asymmetric Distributed Constraints Optimization Problem (Grinshpoun et al. 2013). This makes the utility functions exponential both in the number of agents and in the

number of variables controlled by the agents. For a large number of agents, as is the case in the GIV charging problem, the explicit representation is impractical. First, it needs exponential space. Second, computing a best-response strategy requires accessing all the utility values at least once, and hence would take exponential time.

While the above explicit representation yields exponential complexity, a property of real-life charging scenarios comes to our aid. Vehicle owners usually could not care less regarding some specific time-slots; rather, they want their EV to be charged within some time interval in which the vehicle is parked. This comprehension leads to a natural and remarkably compact representation.

Scalability of Representation

Each agent (GIV) $i \in I$ in the GIV charging problem has a set of strategies that encode a valid combination of time-slots during which agent i is able to charge/discharge; these strategies coincide with the vehicles owner’s preferences. We assume that agent i is able to charge/discharge within a time interval (a_i, d_i) , where a_i represents the arrival time and d_i the departure time. The vehicle’s owner expects that during this time interval the GIV will charge q_i energy units. This expectation enables to present the set of strategies S_i of agent i as a tuple $\langle a_i, d_i, q_i \rangle$. One may also notice that in the iterated best-response process an agent does not need to know the strategies chosen by other agents (i.e., s_{-i}) in order to calculate $u_i(s_i, s_{-i})$, but only the congestion of time-slots in the interval (a_i, d_i) . These properties make the size of the FCG representation size-scalable in the number of agents, which is an important property in this domain. Moreover, the proposed method for finding a PNE inherently preserves the privacy of agents’ preferences.

Finding Best Response

Running an iterated best (or better) response process requires numerous calculations of the best response for each agent. The naïve search process for the best response iterates over all the strategies available for the agent, and selects the one that yields the maximal utility. Following the problem definition in Section 2, each agent i which is “active” in time interval (a_i, d_i) has at most 3^{t_i} strategies, where $t_i = d_i - a_i$. Iterating over all these strategies yields exponential run-time. For this setting, Algorithm 1 finds a best-response strategy in time $O(t_i \cdot \log(t_i))$, reducing the run-time of the best-response process.

The algorithm receives as input the agents’ preferences and the current congestions d . The algorithm first sorts d with respect to the costs and then finds the first q_i minimal-cost time-slots to charge in. Next, the algorithm tries to find pairs of time-slots, such that charging in one and discharging in the other yields a profit. The agent has to verify that the battery is not already fully charged when charging, and that it has enough power in the battery when discharging.

Proposition 2. *The run-time of Algorithm 1 is $O(t_i \cdot \log(t_i))$.*

Proposition 3. *Algorithm 1 finds a best-response strategy.*

The proofs are omitted due to page limitation.

Algorithm 1 FindBestResponse (a_i, d_i, q_i, d)

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1: Let  $t_{charge} \subseteq T$  be the set of time-slots in  $(a_i, d_i)$  or-
   ordered with respect to the cost of (congestion + 1)
2: Let  $t_{discharge} \subseteq T$  be the set of time-slots in  $(a_i, d_i)$ 
   ordered with respect to the congestion cost
3:  $s \leftarrow 0^T$ 
4: for  $\min(q_i, t_i)$  times do
5:   find time-slot  $t \in t_{charge}$  with lowest cost s.t.  $s[t] = 0$ 
6:    $s[t] \leftarrow 1$ 
7: while  $\exists t \in t_{charge}, t' \in t_{discharge}$  s.t.
    $c(d[t']) > c(d[t] + 1)$  and  $s[t] = 0$  and  $s[t'] = 0$  do
8:    $s[t] \leftarrow 1$ 
9:    $s[t'] \leftarrow -1$ 
10: return  $s$ 

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6 Experimental Evaluation

In the following evaluation we generated a random set of GIV charging problems. These problems were then translated to both congestion games (by ignoring strategies that include discharging) and feedback congestion games. We tested the effectiveness of the iterated best-response process for both CG and FCG, as well as for a fixed pricing scheme. For CG and FCG we used an entropy-based pricing scheme.

Problem Generation

The problems used in this evaluation were randomly generated according to the following process. First, the number of agents V and time-slots T were given to each experiment as parameters. Next, a background power load was randomly selected for each time-slot from the range $[0, |V|/2]$. Then, the EVs preferences were generated by randomly selecting the arrival and departure times (in the range $[0, |T|]$), as well as the amount of energy units that each EV needs to charge. This amount was defined by a natural number randomly selected from the range $[0, 100]$. All selections were made with uniform distribution. Note that since EVs preferences are intervals, in the extreme time-slots (at the beginning and at the end), the resulting demand is not uniform. Finally, the congestion game and corresponding feedback congestion game that represent the generated GIV charging problem were constructed according to the transformation described in Section 4.

Solution Quality

The first experiment is designed to test the quality of the solutions achieved by using iterated best-response for both the congestion and feedback congestion games that correspond to the generated GIV charging problems. We also included in the experiment the results of a fixed pricing scheme in which the price for each time-slot is the same and it is not affected by the congestion over the time-slot; this pricing scheme corresponds to the naïve approach in which each GIV starts charging at the moment it is connected to the grid.

The motivation for this experiment stems from the fact that CG/FCG may include many different PNEs; while an iterated best-response process is guaranteed to find one of them, the quality of the found PNE may be far from optimal.

We present the results of 200 randomly generated problems, each with 500 agents and 200 time-slots. Figures 1, 2, and 3 show the average congestion over the time-slots that resulted from solving the generated problems using the fixed pricing scheme, CG, and FCG, respectively. Presenting only the mean values is not particularly informative in this context, since random values tend to average nicely. Thus, the standard deviation is also shown.

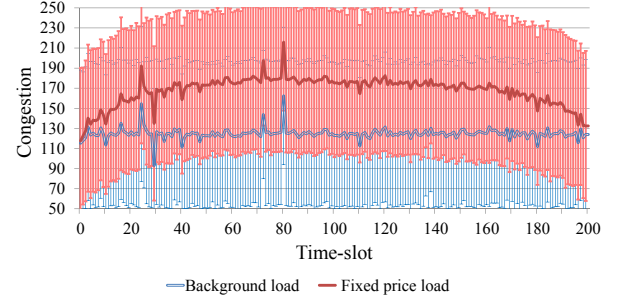


Figure 1: Congestion over time-slots – Background vs. Fixed pricing scheme

Figure 1 clearly shows that the demand when using the fixed pricing scheme is highly unpredictable, in the sense that the variance between problem instances corresponds to the variance of the background load. This is not a desirable property for both the electricity company and the consumers. The electricity company needs to plan the power generation in advance, whereas the consumers benefit from predictable electricity costs. Note that the average over all experiments maintains the locations of the background load peaks.

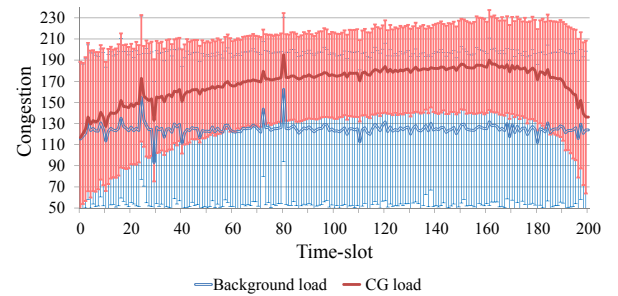


Figure 2: Congestion over time-slots – Background vs. CG

The CG results in Figure 2 show some improvement in their predictability (e.g., lower variance). Nevertheless, even in the average case CG was not able to flatten the demand, as the background load peaks still appear to some extent.

Considerable improvement is achieved when using FCG, as can be clearly seen in Figure 3. In the time-slots that have high GIV availability (roughly between time-slots 70 and 170), the average demand is virtually flattened. Moreover, the demand in this region is highly predictable, demonstrating a small variance.

Load Balancing

The second experiment is designed to measure the effect that the amount of consumers has on the resulting demand. The

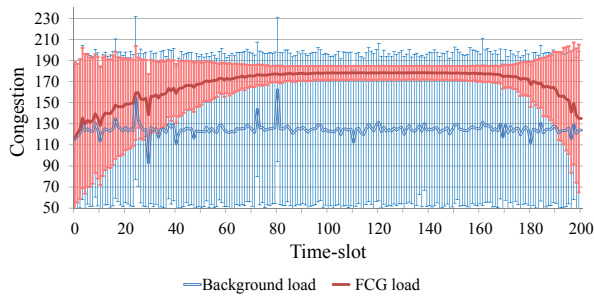


Figure 3: Congestion over time-slots – Background vs. FCG

objective is to achieve a balanced load among the time-slots, thus we measure the standard deviation of the resulting congestion over all the time-slots.

For this experiment we considered problems of different sizes, in which the number of consumers is taken from the set $\{100, 200, \dots, 1000\}$. The number of time-slots remains fixed (200) for all problems. For each problem size we generated 200 random instances. The values presented in Figure 4 are the averages for each problem size of the resulting standard deviations.

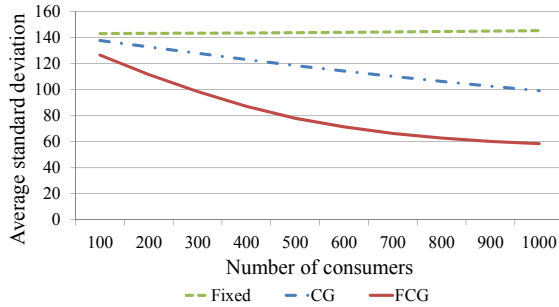


Figure 4: Load balancing between time-slots

It is clear that the results of the fixed pricing scheme are not affected by the number of consumers. This is expected, since the fixed pricing scheme basically amplifies the background load. In Contrast, when the number of consumers increases, both CG and FCG are able to produce much more balanced solutions. The ability of FCG to utilize V2G enables it to achieve considerably more balanced solutions than those achieved by CG.

Scalability and Player Ordering

To verify the scalability of the proposed solution we examine the number of turns until the players converge to a PNE. In each turn exactly one player is allowed to change its strategy or remain with its former strategy. For a problem with n consumers, the process is considered *converged* after n consecutive turns with no strategy changes.

Different player orderings may potentially affect the number of turns until convergence. The basic ordering, which was also used in the preceding experiments, is “Round-robin”, in which the same (random) ordering is used in each round. Another player ordering that we consider is “Expensive first”, in which the order changes each round according to the agents’ costs in the previous round. “Expensive”

agents use congested time-slots, therefore lowering their costs may lead to faster convergence.

Figure 5 presents the number of rounds until convergence for CG and FCG when using each of the two player orderings (same settings).

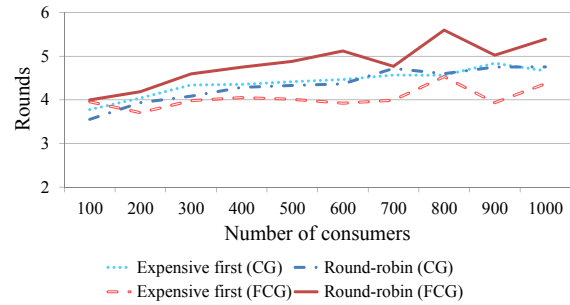


Figure 5: Rounds until convergence

For the “Round-robin” ordering CG is shown to converge faster than FCG. This result is not surprising since in FCG each player has more strategies, which leads to a larger solution space. As expected, the “Expensive first” ordering resulted in considerably faster convergence for FCG.

From the graph it is clear that the proposed solution scales well, although one must keep in mind that as the problem size grows, so does the number of turns in each round.

In most cases the different orderings converged to the same PNE. On the rare occasions that they converged to different PNEs, the changes in solution quality were marginal.

7 Conclusions

The problem of V2G-enabled EV charging and discharging is modeled as a congestion game. In order to incorporate the discharge operation, a generalized model of congestion games is proposed. The resulting feedback congestion games (FCGs) were proven to be exact potential games, as is the case with standard congestion games. Being a potential game, FCGs converge to a PNE by an iterated better-response process. This property along with an extremely compact representation that is presented, enable efficient better-response search for a PNE.

An extensive experimental evaluation demonstrates that the proposed model and its compact representation yield a highly effective and scalable process. The experiments also revealed that enabling the discharging operation (by using FCGs) results in considerably better outcomes in terms of their predictability as well as in the balance of loads that are imposed on the different time-slots.

In the present work, the best-response process is completely sequential. In future work it would be interesting to devise an algorithm in which all the agents act concurrently. Another interesting direction is to adjust the proposed scheme in order to enable an online mechanism, in which agents can come and go at any time (Gerding et al. 2011; Robu et al. 2011; Stein et al. 2012). Finally, the V2G-charging/discharging domain is an interesting playground for semi-cooperative agents, which may lead to more effective schemes.

Acknowledgments

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